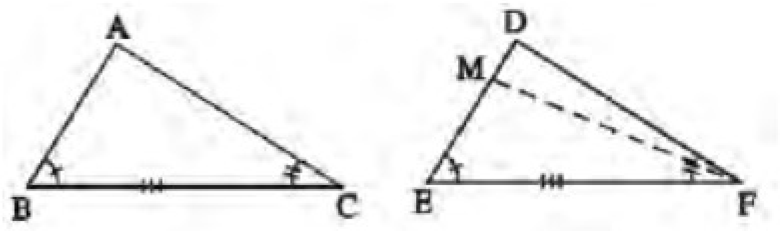
THEOREM 10.1.1

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, then the triangles are congruent.

(A.S.A. ≅ A.S.A.)



Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E,\ \overline{BC} \cong \overline{EF}, \angle C \cong \angle F.$

To prove:

$\Delta ABC \cong \Delta DEF$

Construction

Suppose $\overline{AB} \neq \overline{DE}$, take a point M on DE such that $\overline{AB} \cong \overline{ME}$. Join M to F

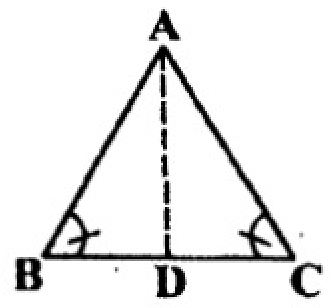
Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta D'EF$	
$\overline{AB} \cong \overline{D'E}$ ...... (i)	Construction
$\overline{BC} \cong \overline{EF}$ ...... (ii)	Given
	Given

$\angle B \cong \angle E$ ...... (iii)	S.A.S. postulate
$\therefore \Delta ABC \cong \Delta D'EF$	(Corresponding angles of congruent triangles)
So, $\angle C \cong \angle D'FE$	
	Given
	Both congruent to ∠C
But, $\angle C \cong \angle DFE$	
$\therefore \angle DFE \cong \angle D'FE$	
This is possible only if D and	
D are the same points, and	Proved that D and D' are the same points
$D'E \cong DE$	
So, $\overline{AB} \cong \overline{DE}$ .... (iv)	
Thus from (ii), (iii) and (iv), we	S.A.S. postulate
Have	
$\Delta ABC \cong \Delta DEF$	

THEOREM 10.1.2

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.



Given:

In ΔABC,

$\angle B \cong \angle C$

To prove:

$\overline{AB} \cong \overline{AC}$

Construction:

Draw the bisector of $\angle A$, to meet BC at point D.

Proof:

Statements	**Reasons**
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore \Delta ABD \cong \Delta ACD$	S.A.A. ≅ S.A.A.
Hence $\overline{AB} \cong \overline{AC}$	(Corresponding angles of congruent triangles)

THEOREM 10.1.3

If in a given correspondence of two triangles, the three sides of one triangle are congruent to the corresponding three sides of the other triangle then the triangles are congruent.

Solution:

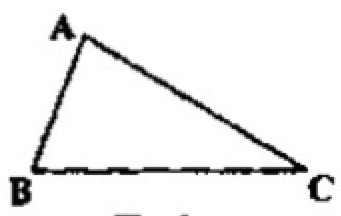


Fig. 1

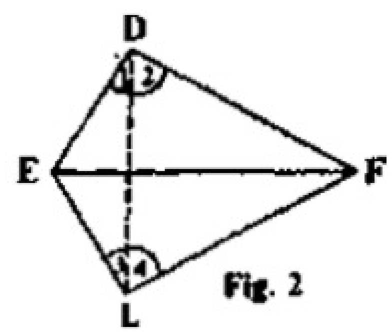


Fig. 2

Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔLEF in which, $\angle FEL \cong \angle B$ and $\overline{LE} \cong \overline{AB}$. Join D and M. As shown in the above figures we label some of the angles as 1, 2, 3 and 4.

Proof

Statements	**Reasons**
In $\Delta ABC \leftrightarrow \Delta LEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B = \angle FEL$	Construction
$\overline{AB} \cong \overline{LE}$	Construction
$\therefore \Delta ABC \cong \Delta LEF$	S.A.S postulate
And $\overline{CA} \cong \overline{FL}$ .... (i)	corresponding sides of congruent triangles)
Also $\overline{CA} \cong \overline{FD}$ .... (ii)	Given
$\therefore$ $\overline{FL} \cong \overline{FD}$	From (i) and (ii)
In ΔFDL	
$\angle 2 \cong \angle 4$ .... (iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly, $\angle 1 \cong \angle 3$ .... (iv)	{from (iii) and (iv)
$\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	
Now, in $\Delta DEF \leftrightarrow \Delta LEF$	
$\overline{FD} \cong \overline{FL}$	Proved
And $m\angle EDF \cong m\angle ELF$	Proved
$\overline{DE} \cong \overline{LE}$	Each one $\cong \overline{AB}$
$\therefore \Delta DEF \cong \Delta LEF$	S.A.S. postulate
Also $\Delta ABC \cong \Delta LEF$	Proved
Hence $\Delta ABC \cong \Delta LEF$	Each $\Delta \cong \Delta LEF$ (Proved)

THEOREM 10.1.4

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent.

Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (Right angles)

$\overline{CA} \cong \overline{FD}$, $\overline{AB} \cong \overline{DE}$

To Prove:

$\Delta ABC \cong \Delta DEF$

Construction:

Produce $\overline{EF}$ to point L such that $EL \cong BC$ and join points D and L.

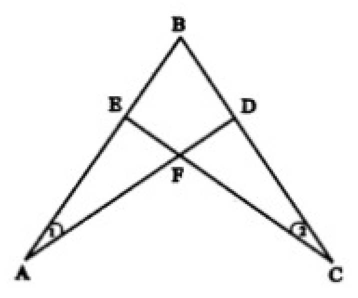
Proof:

Statements	Reasons
$m\angle DEF + m\angle DEL = 180°$ (i)	Supplementary angles
Now $m\angle DEF = 90°$ (ii)	Given
$\therefore m\angle DEL = 90°$	From (i) and (ii)
In $\Delta ABC \leftrightarrow \Delta DEL$	
$BC \cong EL$	Construction
$\angle ABC \cong DEL$	Each equal to 90°
$\overline{AB} \cong \overline{DE}$	Given
$\therefore \Delta ABC \cong \Delta DEL$	S.A.S. postulate
And $\angle C \cong \angle L$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{LD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\therefore \overline{LD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In ΔDLF	
$\angle F \cong \angle L$	$\overline{FD} \cong \overline{LD}$ (proved)
But $\angle C \cong \angle L$	Proved
$\angle C \cong \angle F$	Each is congruent to ∠L.
In $\Delta ABC \leftrightarrow \Delta ADEF$	
$\overline{AB} \cong \overline{DE}$	Given
$\angle ABC \cong \angle DEF$	Given
$\angle C \cong \angle F$	Proved
$\therefore \Delta ABC \cong \Delta DEF$	S.A.A. S.A.A

Exercise 10.1

Q1. In the given figure, AB ≅ CB, ∠1 ≅ ∠2. Prove that Δ ABD ≅ ΔCBE.

Solution:



Given:

In the given figure $\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

To prove:

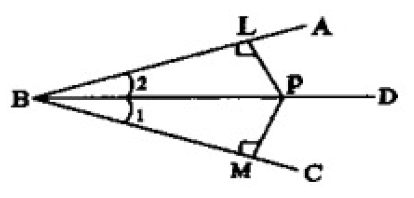
$\Delta ABD \cong \Delta CBE$

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$AB \cong CB$	Given
$\angle BAD \cong \angle BCE$	Given ∠1 ≅ ∠2
$\angle ABD \cong \angle CBE$	Common
$\therefore \Delta ABD \cong \Delta CBE$	S.A.A ≅ S.A.A

Q2. From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.

Solution:



Given:

$\overline{BD}$ is bisector of ∠ABC. P is point on $\overline{BD}$ and $\overline{PL}$ and $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively

To prove:

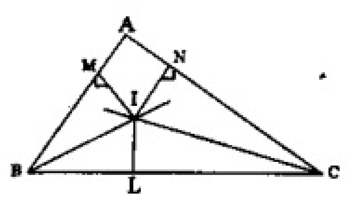
$\overline{PL} \cong \overline{PM}$

Proof:

Statements	Reasons
In $\Delta BLP \leftrightarrow \Delta BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \equiv \angle BMP$	Each right angle (given)
$\angle LBP \equiv \angle MBP$	Given BD is bisector of angle B
$\Delta BLP \cong \Delta BMP$	S.A.A ≅ S. A. A.
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of ≅ Δ's.

Q3. In a triangle ABC, the bisectors of ∠B and ∠C meet in a point I. Prove that I is equidistant from the three sides of △ABC.

Solution:



Given:

In ΔABD the bisector of ∠B and ∠C meet at I, IL, IM and IN are perpendiculars to the three sides of ΔABD.

To prove:

$\overline{IL} \cong \overline{IM} \cong \overline{IN}$

Proof:

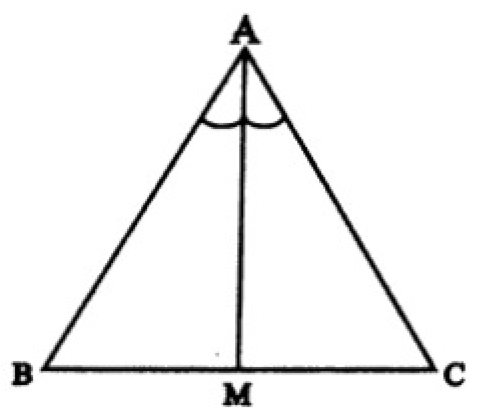
Statements	Reasons
In $\Delta ILB \leftrightarrow \Delta IMB$	
$\overline{BI}$ $\overline{BI}$	Common
$\angle lBL \cong \angle lBM$	Given BI is bisector of ∠B
$\angle lLB \cong \angle lMB$	Given each ∠ is right angles.
$\Delta ILB \cong \Delta IMB$	S.A.A ≅ S.A.A.
$\overline{IL}$ $\overline{IM}$ (i)	Corresponding sides of ≅ Δs.
Similarly	
$\Delta IAC \cong \Delta INC$	
So $IL \cong IN$ (ii)	Corresponding sides of ≅ Δs.

From (i)and (ii)	
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
I is equidistant from	
the three sides of ΔABC.	

Exercise 10.2

Q1. Prove that any two medians of an equilateral triangle are equal in measure.

Solution:



Given:

In ΔABC, $\overline{AB} \cong \overline{AC}$ and M is mid point of BC.

To prove:

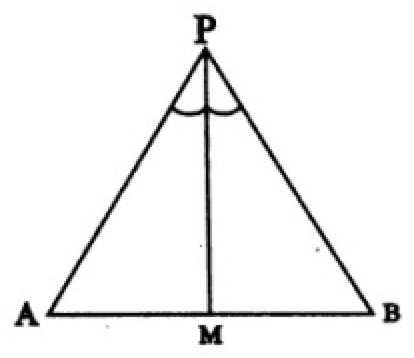
$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$.

Proof:

Statements	Reasons
In $\Delta ABM \leftrightarrow \Delta ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is mid point of $\overline{BC}$.
$\overline{BM} \cong \overline{AM}$	Common
$\therefore\ \Delta ABM \cong \Delta ACM$	S. S. S. ≅ S. S. S.
	Corresponding sides of ≅ Δ's.
So $\angle BAM \cong \angle CAM$	
$\therefore\ \overline{AM}$ bisects $\angle A$	Corresponding sides of ≅ Δ's.
Also $\angle AMB \cong \angle AMC$	
but $m\angle AMB \cong \angle AMC = 180°$	$\overline{BC}$ is a straight line.
$\therefore m\angle AMB = \angle AMC = 90°$	
I. e. $\overline{AM}$ is perpendicular to $\overline{BC}$.	

Q2. Prove that a point, which is equidistant from the end points of a line segment, is on the right bisector of the line segment.

Solution:



Given:

$\overline{AB}$ is a line segment and P is a point such that

$\overline{PA} \cong \overline{PB}$

To prove:

P is on right bisector of $\overline{AB}$

Construction:

Draw $\overline{PM}$ bisector of $\angle P$ meeting $\overline{AB}$ at M.

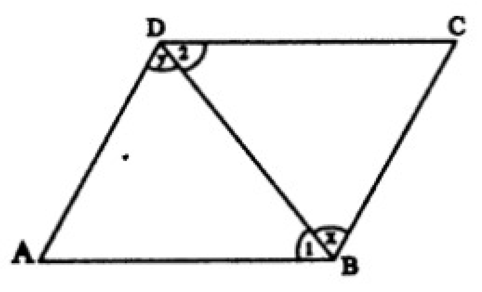
Proof:

Statements	Reasons
In $\Delta APM \leftrightarrow \Delta BPM$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle APM \cong \angle BPM$	Construction.
$\overline{PM} \cong \overline{PM}$	Common
$\therefore\ \Delta APM \cong \Delta BPM$	S. S. S. ≅ S. S. S.
So $\overline{AM} \cong \overline{BM}$	Corresponding sides of ≅ Δ's.
$\angle PMA \cong \angle PMB$	
but $m\angle PMA \cong \angle PMB = 180°$	$\overline{BC}$ is a straight line.
$\therefore m\angle PMA = \angle PMB = 90°$	
So $\overline{PM}$ is right bisector of $\overline{AB}$	
or P is on right bisector of $\overline{AB}$	

Exercise 10.3

Q1. In the given figure, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, Prove that ∠A ≅ ∠C, ∠ABC ≅ ∠ADC.

Solution:



Given:

In the figure, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

To prove:

$\angle A \cong \angle C$
$\angle ABC \cong \angle ADC$

Construction:

Join B to D

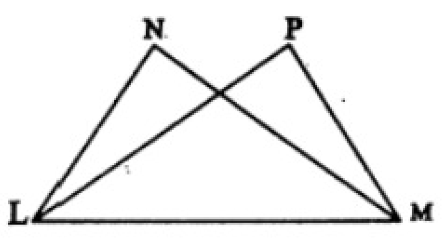
Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{CD}$	Given

$\overline{BD} \cong \overline{DB}$	Common
$\therefore\ \Delta ABC \cong \Delta CDB$	S. S. S. ≅ S. S. S.
$\therefore\ \angle A \cong \angle C$	Corresponding sides of ≅ Δ's.
$\hat{1} \cong \hat{2}$ and $\angle x \cong \angle y$	Corresponding sides of ≅ Δ's.
∴ by adding above equations $\hat{1} + \hat{x} = \hat{2} + \hat{y}$	Addition of angles
or $\angle ABC \cong \angle ADC$	

Q2. In tne figure, $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$. Prove that ∠N ≅ ∠P, ∠NML ≅ ∠PLM.

Solution:



Given:

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$

To prove:

$\angle N \cong \angle P$
$\angle NML = \angle PLM$

Proof

Statements	Reasons
In $\Delta LMN \leftrightarrow \Delta MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\therefore \Delta LMN \cong \Delta MLP$	S. S. S. ≅ S. S. S.
$\therefore \angle N \cong \angle P$	Corresponding sides of ≅ Δ's.
$\angle NML \cong \angle PLM$	Corresponding sides of ≅ Δ's.

Q3. Prove that the median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base.

Solution:

Given:

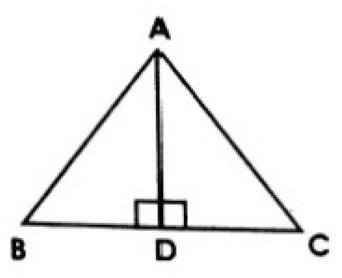
An isosceles triangle ABC with the base $\overline{BC}$ and $\overline{AD}$ bisects at point D.

i.e. $\overline{BD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{AC}$

To prove:

$\angle BAD \cong \angle CAD$

and $\overline{AD} \perp \overline{BC}$

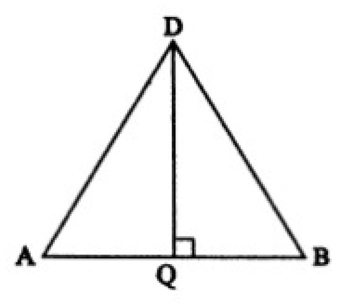


Proof

Statements	Reasons
In the correspondence of $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\overline{BD} \cong \overline{DC}$	Given
$\therefore \Delta ABD \cong \Delta ACD$	S. S. S postulate
Thus $\angle BAD \cong \angle CAD$	Corresponding angles of congruent triangle
$m\angle ADB = m\angle ADC = 180°$ $m\angle ADC = m\angle ADC = 180°$	Supplementary angles
$\Rightarrow 2\, m\angle ADC = 180°$ $\Rightarrow m\angle ADC = 90°$	As $m\angle ADC = m\angle ADB$
Hence $\overline{AD} \perp \overline{BC}$	

Exercise 10.4

Q1. In ΔPAB of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$ proved that $\overline{AQ} \cong \overline{BQ}$ and ∠APQ ≅ ∠BPQ.



Solution:

Given:

In ΔPAB,

$\overline{PQ} \perp \overline{AB}$, and $\overline{PA} \cong \overline{PB}$

To prove:

$\overline{AQ} \cong \overline{BQ}$

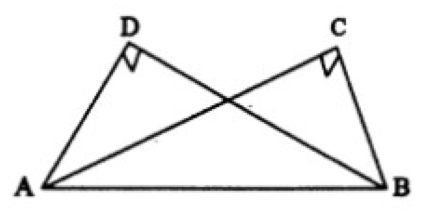
$\angle APQ \cong \angle BPQ$

Proof:

Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$ $\overline{PQ} \cong \overline{PQ}$	Given $PQ \perp AB$
	Common

Statements	Reasons
$\therefore \Delta APQ \cong \Delta BPQ$	H.S ≅ H.S
so $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of ≅ Δ's.
$\angle APQ \cong \angle BPQ$	Corresponding sides of ≅ Δ's.

Q2. In the figure, m∠C = m∠D = 90° and $\overline{BC} \cong \overline{AD}$. prove that $\overline{AC} \cong \overline{BD}$ and ∠BAC ≅ ∠ABD.



Solution:

Given:

In the figure

$m\angle C = m\angle D = 90°$ and $\overline{BC} \cong \overline{AD}$

To prove:

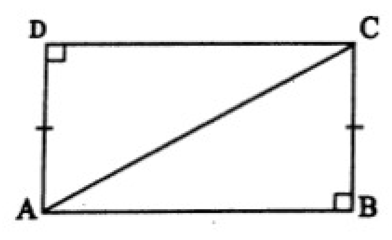
$\angle ABC \cong \angle ABD$

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta BAC$	
$\overline{D} \cong \overline{C}$	Given
$\overline{AD} \cong \overline{BC}$	Given
	Common

Statements	Reasons
$\overline{AB} \cong \overline{BA}$	H.S ≅ H.S
$\therefore \Delta ABD \cong \Delta BAC$	Corresponding sides of ≅ Δ's.
so $\overline{AC} \cong \overline{BD}$	Corresponding sides of ≅ Δ's.
$\angle BAC \cong \angle ABD$	

Q3. In the figure, m∠B = m∠D = 90° and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is rectangle.



Solution:

Given:

In the figure,

$m\angle B = m\angle D = 90°$ and $\overline{AD} \cong \overline{BC}$

To prove:

ABCD is rectangle.

Construction:

Join A to C.

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta CDA$	

Statements	Reasons
$\overline{B} \cong \overline{D}$	Given each angle = 90°.
$\overline{AC} \cong \overline{CA}$	Given
$\overline{BC} \cong \overline{DA}$	Common
$\therefore \Delta ABC \cong \Delta CDA$	H.S ≅ H.S
$\therefore$ So $\overline{AB} \cong \overline{CD}$	Corresponding sides of ≅ Δ's.
$\angle ACB \cong \angle CAD$ Hence ABCD is a rectangle.	Corresponding sides of ≅ Δ's.

Review Exercise 10

1. Which of the following are true and which are false?

(i) A ray has two end points

(ii) In a triangle, there can be only one right angle

(iii) Three points are said to be collinear, if they lie on same line

(iv) Two parallel lines intersect at a point.

(v) Two lines can intersect only at one point.

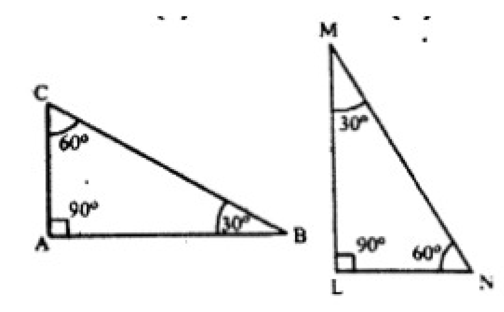
(vi) A triangle of congruent sides has non-congruent angles.

Answers:

(i) F	**(ii)** T	**(iii)** T	**(iv)** F	**(v)** T	**(vi)** F

2. If $\Delta ABC \cong \Delta LMN$, then

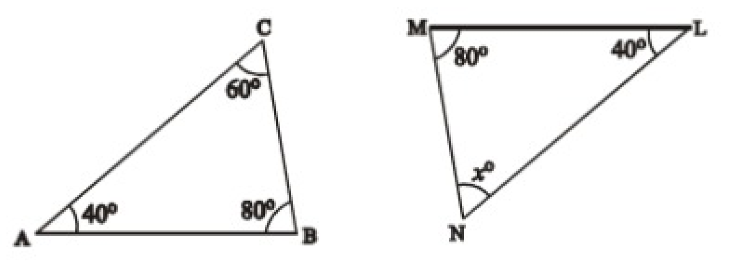
(i) m∠M ≅ ....
(ii) m∠N ≅ ....
(iii) m∠A ≅ ....



Solution:

(i) F	**(ii) T**	**(iii) T**

3. If $\Delta ABC \cong \Delta LMN$, then find the unknown x.



Solution:

Given that: $\Delta ABC \cong \Delta LMN$

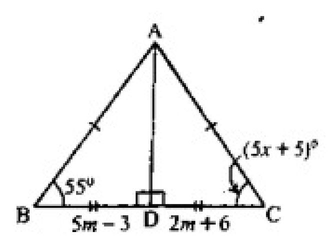
$\angle C \cong \angle M$

or $m\angle C \cong m\angle M$

=> $60° \cong x$
=> $x \cong 60°$

Q4. Find the value unknowns for the given congruent triangles.

Solution:



$\Delta ADB \cong \Delta ADC$
$BD \cong CD$

Corresponding sides of = Δ's.

=> $mBD \cong mCD$
=> $5m - 3 = 2m + 6$

or $5m - 2m = 6 + 3$

$3m = 9$
$\angle B \cong \angle C$

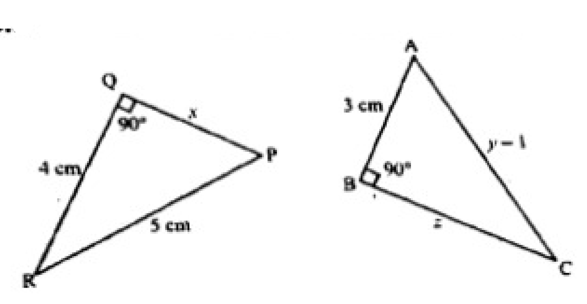
Corresponding sides of ≅ Δ's.

=> $m\angle B \cong m\angle C$

$55 = (5x + 5)$
$55 = 5x + 5$
$5x = 55 - 5 = 50$
$x = 10°$

Q5. If PQR ≅ ABC, then find the unknowns

Solution:



$\Delta PQR \cong ABC$

$\therefore \quad \overline{BD} \cong \overline{CD}$

Corresponding sides of ≅ Δ s.

=> $x = 3\ cm$

$\overline{PR} \cong \overline{AC}$

Corresponding sides of ≅ Δ's.

$5 = y - 1$
$y = 5 + 1$
=> $y = 6$

Also $\overline{QR} \cong \overline{BC}$

Corresponding sides of ≅ Δ's.

$m\overline{QR} \cong m\overline{BC}$

$4\ cm = z$

or $z = 4\ cm$