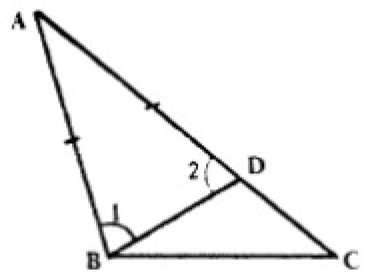
THEOREM 13.1.1

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.



Given

In ΔABC, $m\overline{AC} > m\overline{AB}$

To Prove

$m\angle ABC > m\angle ACB$

Construction

On AC take a point D such that $\overline{AD} \cong \overline{AB}$. Join B to D so that ΔADB is an isosceles triangle.

Proof

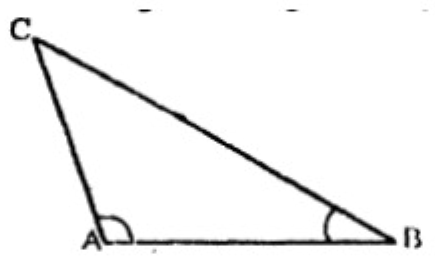
Statements	Reasons
In ΔABD $m\angle 1 = m\angle 2$ ⋯⋯ (i)	Angles opposite to congruent sides,
In ΔBCD $m\angle 2 > m\angle ACB$ ⋯⋯ (ii)	An exterior angle of a triangle is greater than a non-adjacent interior angle

$\therefore m\angle 1 > m\angle ACB$ ⋯⋯ (iii)	By (i) and (ii)
$m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of angles.
$\therefore m\angle ABC > m\angle 1$ ⋯⋯ (iv)	By (iii) and (iv)
Hence $m\angle ABC > m\angle ACB$	Transitive property of inequality of real numbers

THEORM 13.1.2

Converse of THEORM 13.1.1

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.



Given

In ΔABC, $m\angle A > m\angle B$

To Prove

$m\overline{BC} > m\overline{AC}$

Proof

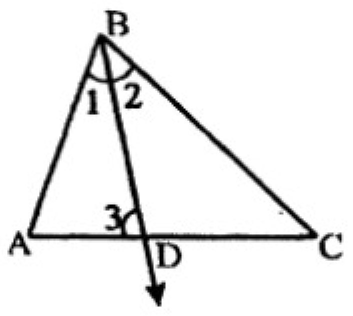
Statements	Reasons
If, $m\overline{BC} \ngtr m\overline{AC}$, then	
either (i) $m\overline{BC} = m\overline{AC}$	Trichotomy property of real numbers
or (ii) $m\overline{BC} < m\overline{AC}$	
From (i) if $m\overline{BC}$ = mAC, then	
$m\angle A = m\angle B$	Angles opposite to congruent sides are congruent
which is impossible.	Contrary to the given.

From (ii) if $m\overline{BC} < m\overline{AC}$, then	
$m\angle A < m\angle B$	The angle opposite to longer side is greater than angle opposite to smaller side
This is also impossible to contrary to what is given	
$\therefore m\overline{BC} \neq m\overline{AC}$	
and $m\overline{BC} \nless m\overline{AC}$	Trichotomy property of real numbers.
Hence $m\overline{BC} > m\overline{AC}$	

THEORM 13.1.3

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Solution



Given

ABC is triangle

To Prove

(i) m$\overline{AB}$ + m$\overline{BC}$ > m$\overline{AC}$

(ii) mAC + m$\overline{AB}$ > m$\overline{BC}$

(iii)mAC + m$\overline{BC}$ > m$\overline{AB}$

Construction

Take a point D on CA such that $\overline{AD} \cong \overline{AB}$. Join B to D and name the angles. ∠ 1, ∠2 as shown in the given figure.

Proof

Statements	**Reasons**
In Δ CBD	
mz3 > mz2 (i)	Exterior angle is greater than non-adjacent interior angle
mz2 = mz1 (ii)	Construction

∴ mz3 > mz1	By (i) and(ii)
and m$\overline{AB}$ > mAD (iii)	In Δ ABD the side opposite to the larger angle is greater than that of the side opposite to the smaller angle.
similarly	
mBC > mDC	Adding (iit) and (iv)
m$\overline{AB}$ + m$\overline{BC}$ >m$\overline{AD}$ + m$\overline{DC}$	∴ m$\overline{AD}$ + m$\overline{DC}$= m$\overline{AC}$
or m$\overline{AB}$ + m$\overline{BC}$ > m$\overline{AC}$	
Similarly, by drawing angle bisector of ZA and zC it can be proved that	
m$\overline{AC}$ + m$\overline{AB}$ > m$\overline{BC}$	
and m$\overline{AC}$ + m$\overline{BC}$> m$\overline{AB}$	

Theorem 13.1.4

From a point, outside a line, the perpendicular is the shortest distance from the point to the line.

Given

A line $\overleftrightarrow{AB}$ and a point C (not lying on $\overleftrightarrow{AB}$) and a point D on $\overleftrightarrow{AB}$ such that $\overline{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

To Prove

m$\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$

Construction

Take a point E on $\overleftrightarrow{AB}$. Join C and E to form a ΔCDE.

Proof

Statements	Reasons
In ΔCDE	
m∠CDB > m∠CED	An exterior angle of a triangle is greater than non-adjacent interior angle.
But m∠CDB = m∠CDE	Supplement of right angle.
∴ m∠CDE > m∠CED	
or m∠CED < m∠CDE	Reflexive property if inequality
or m$\overline{CD}$ < m$\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on AB	
Hence m$\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$.	

EXERCISE 13.1

Q1. Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?

(a)5 cm (b) 20 cm (c) 25 cm (d) 30 cm

Solution:

(a) Measure of sides are 10 cm, 15 cm and 5 cm

As $10 + 5 = 15$

Since the 'sum of two sides is equal to the third side therefore:

So, **5 cm is not possible.**

(b) Sides are 10 cm, 15 cm and 20 cm

$10 + 15 > 20$

$10 + 20 > 15$

$15 + 20 > 10$

Since the sum of two sides is greater than third side therefore:

20 cm is possible for third side.

(c) Sides are 10 cm, 15 cm and 25 cm

Since the sum of two sides is equal to the third side therefore:

So, **25 cm is not possible.**

(d) Sides are 10 cm, 15 cm and 30 cm

$10 + 15 < 30$

Since the sum of two sides is equal to the third side therefore:

So, **30 cm is not possible.**

Q2. O is interior point of the ΔABC. Show that

$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$

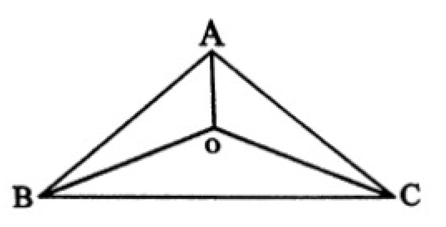
Solution:

Given:

O is a point in side a triangle ABC. O is joined with A, B and C.

To prove:

$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$



Proof

Statements	Reasons
In ΔOAB $mOA + mOB > mAB$ In ΔOBC $mOB + mOC > mAC$ In ΔCOA	In a triangle the sum of measure of two dies is greater than measure of third side

$mOC + mOA > mAC$ Adding (i), (ii), (iii) $2(mOA + mOB + mOC)$ $> mAB + mBC + mAC$ $mOA + mOB + mOC$ $> (mAB + mBC + mCA)$	Dividing both sides by 2

3. In the ΔABC, $m\angle B = 70°$ and $m\angle C = 45°$. Which of the sides of the triangle is longest and which is the shortest?

Solution:

$m\angle B = 70°$
$m\angle C = 45°$
$m\angle A + m\angle B + m\angle C = 180°$
$m\angle A + 70° + 45° = 180°$
$m\angle A + 115° = 180°$
$m\angle A = 180° - 115° = 65°$

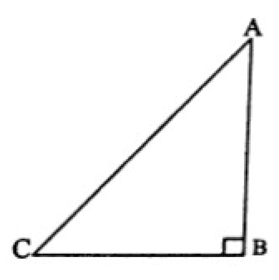
Since the largest angle is B. So, the longest side is opposite to B is $\overline{AC}$

(Longest)

Since the smallest angle is C. So, the shortest side is opposite to C is $\overline{AB}$
(Shortest)

Q4. Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

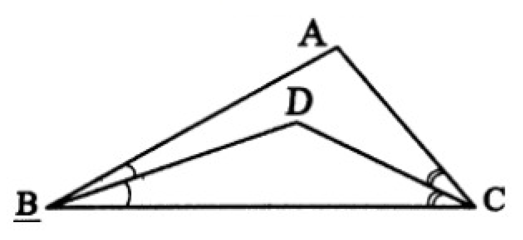
Solution:



In the right-angled triangle ABC, $m\angle B = 90°$ and $\overline{AC}$ is hypotenuse.
$\angle A$ and $\angle C$ both are acute angles
$\therefore m\angle B > m\angle A$
And $m\angle B\ m\angle C$
$\therefore \angle B$ is the largest angle
$\therefore$ Side opposite to $\angle B$
Which is hypotenuse is longer than each of the other two sides.

Q5. In the triangle figure, $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of ∠B and ∠C respectively. Prove that $\overline{BD} > \overline{DC}$.

Solution:



$\overline{BD}$ is bisector of $\angle B$

$\therefore m\angle ABD = m\angle CBD$

So, $m\angle CBD = m\angle ABC$

Similarly, as $\overline{CD}$ is bisector of $\angle C$.

So, $m\angle BCD = m\angle ACB$

It is given that $\overline{AB} > \overline{AC}$

$\therefore ACB < ABC$ Opposite angles

=> $\frac{1}{2}mACB > \frac{1}{2}mABC$

$ACB < ABC$ Opposite angles

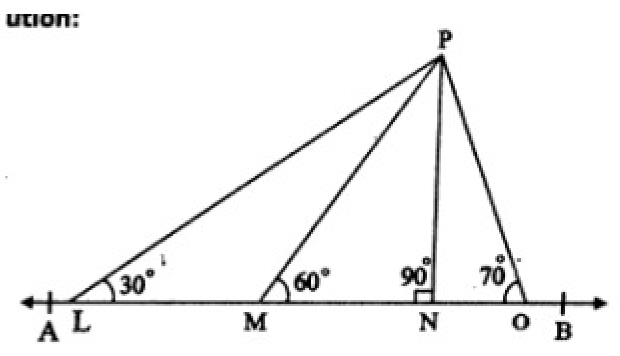
$m\angle BCD > \frac{1}{2}m\angle CBD$

EXERCISE 13.2

Q1. In the figure, P is any point and $\overline{AB}$ is a line. Which of the following is the shortest distance between the point P and the line $\overline{AB}$?

(a) $m\overline{PL}$ (b) $m\overline{PM}$ (c) $m\overline{NP}$ (d) $m\overline{PO}$

ution:



We all know that form a point outside the line, the perpendicular is the shortest distance from the point to the.

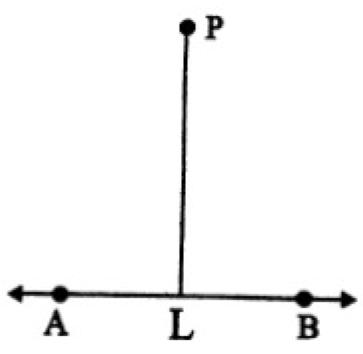
As $\overline{PN}$ is perpendicular to $\overline{AB}$.

So $\overline{PN}$ is the shortest distance.

Q2. In the figure, P is any point lying away from the line AB. Then m $\overline{PL}$ will be the shortest distance if

(a) $m\angle PLA$ = 80° (b) $m\angle PLB$ = 100° (c) $m\angle PLA$ = 90°

Solution



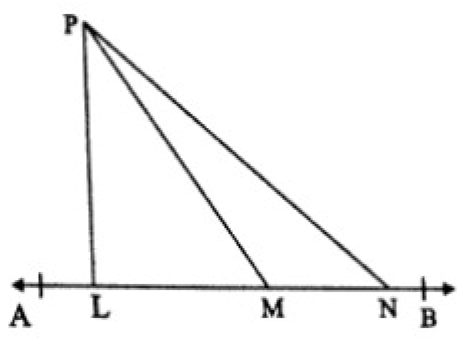
We know that for a point outside a line, the shortest distance from the point to the line is perpendicular to the line.

As $m\overline{PL}$ is shortest,

So, $\overline{PL}$ is perpendicular to $\overline{AB}$.

So, $m\angle PLA = 90°$

Q3. In the figure, $\overline{PL}$ is perpendicular to the line AB and $m\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.



Given

$\overline{PL}$ is perpendicular to $\overline{AB}$ and $m\overline{LN} > m\overline{LM}$

To prove

$m\overline{PN} > m\overline{PM}$

Proof

Statements	Reasons
In ΔLPN	Given
$m\angle PLN = 90°$	Angle of a triangle
$\therefore m\angle PLN < 90°$ (i)	
In ΔPLM	Exterior angle
$m\angle PMN > m\angle PLM$	$\angle PLM = 90°$
$\therefore m\angle PMN < 90°$ (ii)	from (i) and (ii)
In ΔPMN	opposite sides
$m\angle PMN > m\angle PNL$	
$m\overline{PN} > m\overline{PM}$	

REVIEW EXERCISE 13

Q1. Which of the following are true and which are false?

(i) The angle opposite to the longer side is greater.

(ii) In a right-angled triangle greater angle is of 60°.

(iii) In an isosceles right-angled triangle, angles other than right angles are each of 45°.

(iv) A triangle having two congruent sides is called equilateral triangle.

(v) A perpendicular from a point to line is shortest distance.

(vi) Perpendicular to line form an angle of 90°.

(vii) A point outside the line is collinear

(viii) Sum of two sides of triangle is greater than the third.

(ix) The distance between a line and a point on it is zero.

(x) Triangle can be formed of lengths 2 cm, 3 cm and 5 cm.

Answers:

(i) T	**(ii)** F	**(iii)** T	**(iv)** F	**(v)** T	**(vi)** T
(vii) F	**(viii)** T	**(ix)** T	**(x)** F		

Q2. What will be angle for shortest distance from an outside point to the line?

Solution:

The shortest distance between a point and a line is perpendicular from the point to the line. So, the angle for shortest distance from an outside point is

90°.

Q3. If 13 cm, 12 cm, and 5 cm are the lengths of a triangle, then verify that difference of measures of any two sides of a triangle is less than the measure of the third side.

Solution

Let the three sides be

$a = 13\ cm, b = 12\ cm, c = 5\ cm$
$a - b - 13 - 12 = 1 < 5 = c$

i.e. $a - b < c$ (i)

$12 - 5 = 7 < 13$

$b - c < a$ (ii)

$13 - 5 = 8 < 12$

i.e. $a - c < b$ (iii)

From (i), (ii) and (iii) we find that the difference of measures of any two sides of a triangle is less than the measure of the third side.

Q4. If 10 cm, 6 cm and 8 cm are the lengths of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side.

Solution

Let the measure of the sides of the triangle be

$a = 10\ cm, b - 6\ cm, c = 8\ cm$

$10 + 6 = 16 > 8$

$a + b < c$ (i)

$6 + 8 = 14 > 10$

$b + c < a$ (ii)

$8 + 10 = 18 > 6$

$c + a > b$

From (i), (ii) and (iii) we conclude that the sum of measures of two sides of a triangle is greater than the third side.

Q5. 3 cm, 4 cm and 7 cm are not the lengths of the triangle. Give the reason.

Solution

Let $a = 3\ cm, b = 4\ cm, c = 7\ cm$

$a + b = 3+4 = 7 = c$

i.e. $a + b = c$

Since we know that for a triangle sum of measures of two sides should be greater than measure of the third side.

3 + 4è 7

So, $3\ cm$, $4\ cm$ and $7\ cm$ are not the, lengths of the triangle

Q6. If 3 cm and 4 cm are the lengths two sides of a right-angle triangle then what should be the third length of the triangle.

Solution

Let the three sides be

$3^2 + 4^2 = a^2$ (Pythagoras theorem)

$9 + 16 = a^2$

$a^2 = 25$

$=> a = 5$

The third Length is 5 cm.