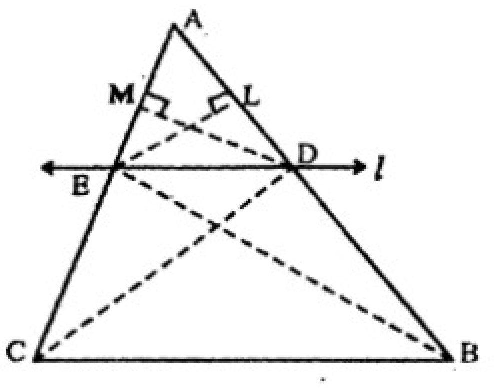
THEOREM 14.1.1

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.



Given

In ΔABC, the line l is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$.

To prove

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$.

Construction

Join B to E and C to D and draw $\overline{DM} \perp \overline{EL}$ perpendiculars from D and E on $\overline{AC} \perp \overline{AB}$ to meet at the points M and L respectively.

Proof

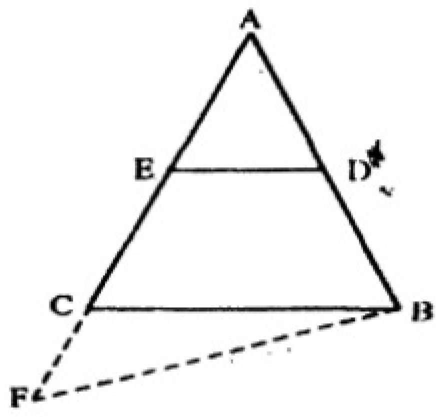
Statements	Reasons
In triangles BED and AED $m\overline{EL}$ is the common perpendicular.	
$\therefore \Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$ .... (i)	Area of a triangle = $\frac{1}{2}$ (base)(height)

And $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$ .... (ii)	
$\frac{\Delta BED}{\Delta AED} = \frac{m\overline{BD}}{m\overline{AD}}$ .... (a)	Dividing (i) by (ii)
Similarly	
$\frac{\Delta CDE}{\Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$ .... (b)	
But $\Delta BED = \Delta CDE$	Areas of triangles with common base and same altitudes are equal. $\overline{ED} \parallel \overline{CB}$ Given, that so altitudes are equal.
$\therefore$ From (a) and (b), we have $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	
$\therefore$ Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	Taking reciprocal of both sides

Theorem 14.1.2

(Converse of Theorem 14.1.1)

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.



Solution

Given

In ΔABC, $\overline{ED}$ intersects $\overline{AB}$ and $\overline{AC}$ such that $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

To prove

$\overline{ED} \parallel \overline{CB}$

Construction

If $\overline{ED} \nparallel \overline{CB}$ then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ produced at F.

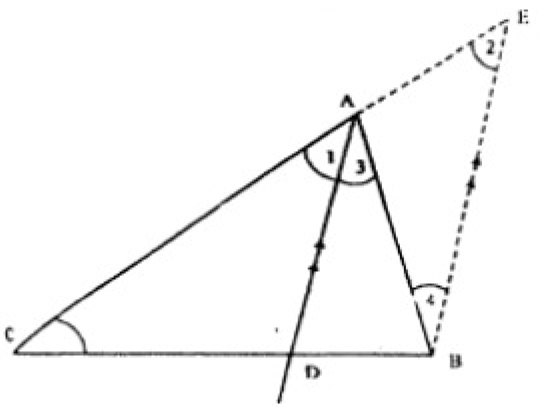
Proof

Statements	Reasons
In ΔABF $\overline{DE} \parallel \overline{BF}$	Construction

$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$ (i)	A line parallel to one side of a triangle divides the other two sides proportionally (Theorem 4.1)
But $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ (ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = m\overline{EC}$ Which is possible only if point F is coincident with C.	Property of real numbers
∴ Our supposition is wrong Hence $\overline{ED} \parallel \overline{CB}$	

THEOREM 14.1.3

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the angle.



Given

In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the point D.

To prove

$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction

Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ produced at E.

Proof

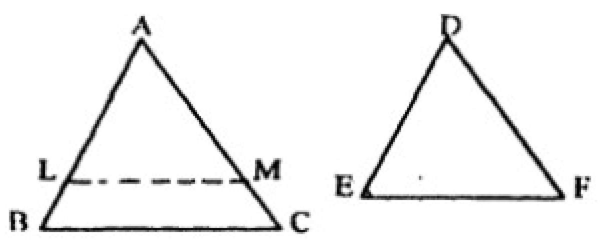
Statements	Reasons
$\therefore \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersects there at A and E,	Construction
So $m\angle 1 = m\angle 2$ ......(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$	
and $\overline{AB}$ intersects them.	
So, $m\angle 3 = m\angle 4$ ....(ii)	Alternate angles

But $m\angle 1 = m\angle 3$	Construction (Given)
$\therefore m\angle 2 = m\angle 4$	
and $\overline{AE} \cong \overline{AB}$	
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally,
Or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	$\therefore m\overline{EA} = m\overline{AB}$ (proved)
$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$	

THEOREM 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional.

Solution



Given

$\Delta ABC \leftrightarrow \Delta DEF$

i.e., $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$.

To prove

$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$

Construction

(a) Suppose that $m\,\overline{AB} > m\,\overline{DE}$

(b) $m\,\overline{AB} < m\,\overline{DE}$

On $\overline{AB}$ take a point L such that $m\,\overline{AL} = m\,\overline{DE}$
On $\overline{AC}$ take a point M such that $m\,\overline{AM} = m\,\overline{DF}$. Join L and M by the line segment $\overline{LM}$.

Proof

Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$ $\angle A \cong \angle D$	Given

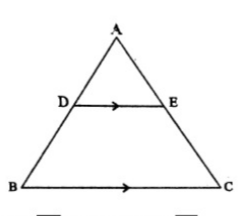
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
and $\angle L \cong \angle E$, $\angle M \cong \angle F$	Corresponding angles of congruent triangles
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore \angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal.
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AB}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$	$m\,\overline{AL} = m\,\overline{DE}$ and $m\,\overline{AM} = m\,\overline{DF}$ (construction)
Similarly, by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that	
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ (ii)	
Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals.

(b) If $m\,\overline{AB} < m\,\overline{DE}$, it can Similarly, be proved by taking intercepts on the sides of ΔDEF.	
If $m\,\overline{AB} = m\,\overline{DE}$	
Then in the correspondence of $\Delta ABC \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
and $\overline{AB} \cong \overline{DE}$	Construction
so $\Delta ABC \cong \Delta DEF$	A.S.A ≅ A.S.A
Thus = $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{EC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
Hence the result is true for all cases.	

EXERCISE 14.1

Q1. In ΔABC, $\overline{DE} \parallel \overline{BC}$.

Solution



(i) $m\overline{AD} = 1.5$cm, $m\overline{BD} = 3$cm, $m\overline{AE} = 1.3$cm, find $m\overline{CE}$.

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$

$\frac{1.5}{3} = \frac{1.3}{m\overline{EC}}$

$1.5\, m\overline{EC} = (1.3)(3)$

$m\overline{EC} = \frac{1.3 \times 3}{1.5} = \frac{13 \times 3 \times 10}{10 \times 15}$

$= \frac{13 \times 3}{15} = \frac{13}{5} = 2.6$ **cm**

(ii) $m\overline{AD} = 2.4$cm, $m\overline{AE} = 3.2$cm, $m\overline{EC} = 4.8$ cm, find $m\overline{AB}$.

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$

$\frac{2.4}{m\overline{DB}} = \frac{3.2}{4.8}$

$3.2\, m\overline{DB} = (2.4)(4.8)$

$m\overline{DB} = \frac{2.4 \times 4.8}{3.2}$

$= \frac{18}{5} = 3.6$ cm

$m\overline{AB} = m\overline{AD} + m\overline{DB} = 2.4 + 3.6$

$= 6.0$ cm

(iii) $\frac{m\overline{AD}}{m\overline{DB}} = \frac{3}{5}$, $m\overline{AC} = 4.8$cm, find $m\overline{AE}$.

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$

$\frac{3}{5} = \frac{m\overline{AE}}{m\overline{EC}}$

$\frac{3}{5} + 1 = \frac{m\overline{AE}}{m\overline{EC}} + 1$

$\frac{8}{5} = \frac{m\overline{AE} + m\overline{EC}}{m\overline{EC}} = \frac{m\overline{AC}}{m\overline{EC}}$

$\frac{8}{5} = \frac{4.8}{m\overline{EC}}$

$8\, m\overline{EC} = 4.8 \times 5 = 24$

$m\overline{EC} = \frac{24}{8} = 3$

$m\overline{AE} = m\overline{AC} - m\overline{EC} = 4.8 - 3 = 1.8$ **cm**

(iv) $m\overline{AD} = 2.4$cm, $m\overline{AE} = 3.2$cm, $m\overline{DE} = 2$cm, $m\overline{BC} = 5$cm, find $m\overline{AB}$, $m\overline{DB}$, $m\overline{AC}$, $m\overline{CE}$.

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{DE}}{m\overline{BC}}$

$\frac{2.4}{m\overline{AB}} = \frac{2}{5}$

$2(m\overline{AB}) = 5(2.4) = 12$

$m\overline{AB} = \frac{12}{2} = 6$ **cm**

$m\overline{DB} = m\overline{AB} - m\overline{AD}$

$= 6 - 2.4 = 3.6$ **cm**

$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{DE}}{m\overline{BC}}$

$2(m\overline{AC}) = 5(3.2) = 16$

$m\overline{AC} = \frac{16}{2} = 8$ **cm**

$m\overline{CE} = m\overline{AC} - m\overline{AE}$

$= 8 - 3.2 = 4.8$ **cm**

(v) $m\overline{AD} = 4x-3$, $m\overline{AE} = 8x-7$, $m\overline{BD} = 3x-1$, $m\overline{CE} = 5x-3$, find x.

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{CE}}$

$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$(4x-3)(5x-3) = (8x-7)(3x-1)$

$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$\Rightarrow 20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$-4x^2 + 2x + 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + 1(x-1) = 0$

$(x-1)(2x+1) = 0$

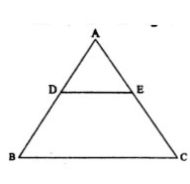
$x = 1, -\frac{1}{2}$

For $x = -\frac{1}{2}$ sides become negative.

So, $x = 1$.

Q2. If ΔABC is an isosceles triangle, ∠A is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$. Prove that ΔADE is also an isosceles triangle.

Solution



Given

In ΔABC, $\overline{AB} \cong \overline{AC}$

and $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$

To prove

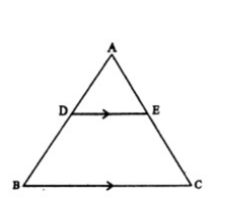
ΔADE is isosceles.

Proof

Statements	Reasons
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Given
$\frac{m\overline{AD}}{m\overline{DB}} + 1 = \frac{m\overline{AE}}{m\overline{EC}} + 1$	
Or $\frac{m\overline{AD} + m\overline{DB}}{m\overline{DB}} = \frac{m\overline{AE} + m\overline{EC}}{m\overline{EC}}$	
i.e. $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$	Given $\overline{AB} \cong \overline{AC}$
=> $m\overline{DB} = m\overline{EC}$	From figure
$m\overline{AB} - m\overline{DB} = m\overline{AC} - m\overline{EC}$	
$m\overline{AD} = m\overline{AE}$	
or $\overline{AD} = \overline{AE}$	
$\therefore \Delta ADE$ is isosceles.	

Q3. In an equilateral triangle ABC shown in the figure, $m\overline{AE} : m\overline{AC} = m\overline{AD} : \overline{AB}\, m$. Find all the three angles of ΔADE and name it also.

Solution:



Given

ABC is equilateral triangle

$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$

To prove

All angles of XΔADE.

Proof

Statements	Reasons
$\frac{m\overline{AC}}{m\overline{AE}}. = \frac{m\overline{AB}}{m\overline{AD}}$	Given
$\frac{m\overline{AC}}{m\overline{AE}} - 1 = \frac{m\overline{AB}}{m\overline{AD}} - 1$	
$\frac{m\overline{AC} - mAE}{m\overline{AE}} = \frac{m\overline{AB} - m\overline{AD}}{m\overline{AD}}$	
$\frac{m\overline{EC}}{m\overline{AE}} = \frac{m\overline{DB}}{m\overline{AD}}$	Given

$or \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ $\therefore \overline{DE} \parallel \overline{BC}$ (i) $m\angle A = m\angle B = m\angle C = 60°$ $\angle AED \cong \angle ACD \cong 60°$ ∴ Each angle of $\Delta\, ADE$ has measure of 60° So $\Delta\, ADE$ is equiangular or equilateral	From (i)

Q4. Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.

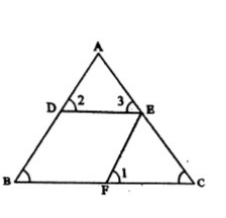
Solution:

Given

In ΔABC, D is mid-point of $\overline{AB}$. $\overline{DE} \parallel \overline{BC}$

To prove

$\overline{EA} \cong \overline{EC}$



Construction

Take $\overline{EF} \parallel \overline{AB}$

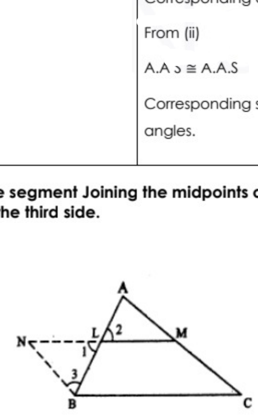
Proof

Statements	Reasons

$\overline{DE} \parallel \overline{BF}$	Given
$\overline{EF} \parallel \overline{BD}$	Construction
∴ DBEF is a parallelogram	
$\overline{EF} \cong \overline{DB}$ (i)	Opposite sides
$\overline{AD} \cong \overline{DB}$ (ii)	Given
$\therefore \overline{EF} \cong \overline{AD}$ (iii)	From (i), (ii)
∠1 ≅ ∠LB	Corresponding angles
And ∠2 ≅ ∠B	
∴ ∠1 ≅ ∠2 (iv)	
In correspondence	
Δ ADE ↔ ΔEFC	
∠2 ≅∠1	From (iv)
∠3 ≅∠C	Corresponding angles
$\overline{AD} \cong \overline{IF}$	From (ii)
Hence ΔADE ≅ ΔEFC	A.A ɔ ≅ A.A.S
$\therefore \overline{EA} \cong \overline{EC}$	Corresponding sides of congruent angles.

Q5. Prove that the line segment Joining the midpoints of any two sides of a triangle is parallel to the third side.

Solution:



Given

In Δ ABC, mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

To prove

$\overline{LM} \parallel \overline{BC}$

Construction

Join M to L and produce ML to N such that $\overline{ML} \cong \overline{LN}$. Join N to B and in the figure, name the angles as ∠1, ∠2 and ∠3.

Proof

Statements	Reasons
In ΔBLN ↔ ΔALM	
$\overline{BL} \cong \overline{AL}$	Given
∠1 ≅ ∠2	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
∴ ΔBLN ≅ ΔALM	S.A.S. postulate
And ∠A ≅ ∠3 .... (i)	
$\overline{NB} \cong \overline{AM}$ .... (ii)	Corresponding angles of congruent triangles
$\overline{NB} \parallel \overline{AM}$	Corresponding angles of congruent triangles
Thus $\overline{NB} \parallel \overline{MC}$ .... (iii)	(M is mid-point of $\overline{AC}$)
	Given
$\overline{MC} \cong \overline{AM}$ ..... (iv)	from (ii) and (iv)
$\overline{NB} \cong \overline{AM}$ .... (V)	From (i) and {v)
∴ BCMN is a parallelogram	Opposite sides of a
$\overline{BC} \parallel \overline{LM}$	parallelogram (BCMN)

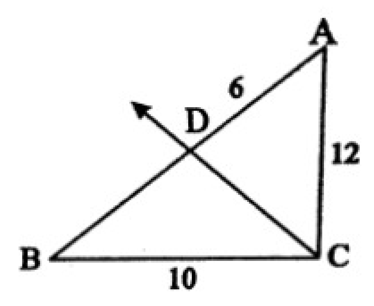
Or $\overline{BC} \parallel \overline{NL}$ ....(Vi)	

EXERCISE 14.2

Q1. In ΔABC as shown in the figure, CD bisects $\angle C$ and meets AB at D. $m\overline{BD}$ is equal to

(a) 5 **(b) 16** **(c) 10** **(d) 18**

Solution:



In ΔABC, CD bisect ∠C meets AB at D.

As CD is the internal bisector of ∠C

So $\frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$

$\frac{m\overline{BD}}{6} = \frac{10}{12}$

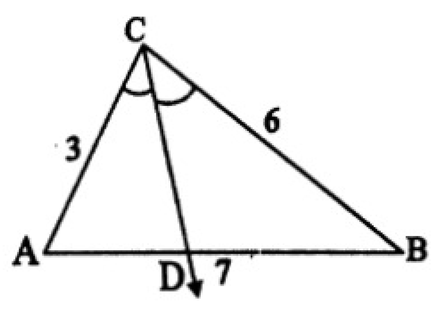
$m\overline{BD} = 6 \times \frac{10}{12}$

$= \boxed{5}$

The correct answer is **(a).**

Q2. In ΔABC shown in the figure, $\overline{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.

Solution:



In ΔABC, $m\overline{AC} = 3, m\overline{CB} = 6$ and $m\overline{AB} = 7 = 7$

Let $m\overline{AD} = x$

then $m\overline{DB} = 7 - x$

As $\overline{CD}$ is internal bisector of C

So $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{CB}}$

$\frac{x}{7-x} = \frac{3}{6}$

$6x = 21 - 3x$

$9x = 21$

$x = \frac{21}{9}$

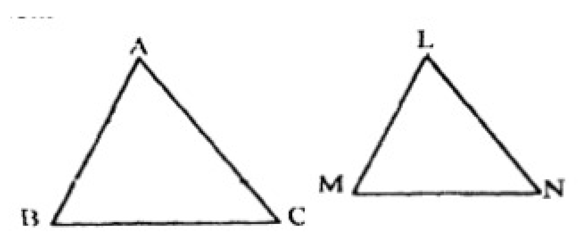
$m\overline{AD} = \frac{21}{9} = \frac{7}{3}$

$m\overline{DB} = m\overline{AB} = m\overline{AD}$

$m\overline{DB} = 7 - \frac{21}{9} = \frac{63-21}{9} = \frac{42}{9} = \frac{14}{3}$

Q3. Show that in any correspondence of two triangles, if two angles of one triangle is congruent to the corresponding angles of the other, then the triangles are similar.

Solution:



Let the two triangles be ΔABC and ΔLMN

It is given that,

$m\angle A = m\angle L$

$m\angle B = m\angle M$

As sum of the angles of a triangle is 180°

$m\angle A + m\angle B + m\angle C = 180°$

$m\angle L + m\angle M + m\angle C = m\angle L + m\angle M + m\angle N$

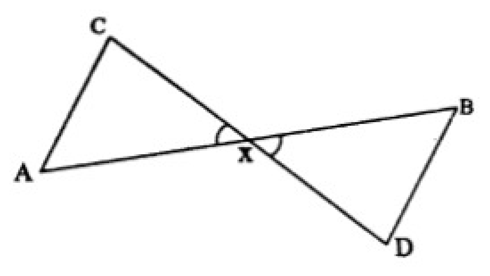
$m\angle L + m\angle M + m\angle C = m\angle L + m\angle M + m\angle N$

$m\angle C = m\angle N$

∴ The two triangles ABC and LMN are similar.

Q4. If line segments $\overline{AB}$ and $\overline{CD}$ are intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ then show that ΔAXC and ΔBXD are similar.

Solution:



Given

Line segments $\overline{AB}$ and $\overline{CD}$ are intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

To prove:

ΔAXC and ΔBXD are similar.

Proof

Statements	Reasons
$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$	Given
SO, $\overline{AC} \parallel \overline{BD}$	
In ΔAXC and ΔBXD	Vertical angles
$m\angle AXC = m\angle BXD$	Alternate angles
$m\angle A = m\angle B$	Alternate angles
$m\angle C = m\angle D$	
So, the triangles are similar.	

REVIEW EXERCISE 14

Q1. Which of the following are true and which are false?

(i) Congruent triangles are of same size and shape.

(ii) Similar triangles are of same shape but different sizes.

(iii) Symbol used for congruent is '~'.

(iv) Symbol used for similarity is '≅'.

(v) Congruent triangles are similar.

(vi) Similar triangles are congruent.

(vii) A line segment has only one mid-point.

(viii) One and only one line can be drawn through two points.

(ix) Proportion is non-equality of two ratios.

(x) Ratio has no unit.

Answers:

(i) T	**(ii)** T	**(iii)** F	**(iv)** F	**(v)** T	**(vi)** F
(vii) T	**(viii)** T	**(ix)** F	**(x)** T		

Q2. Define the following:

(i) Ratio (ii) Proportion (iii) Congruent Triangles (iv) Similar Triangle

Solution:

(i) Ratio

The ratio of two quantities a and b of same kind is denoted as a: b and is defined as:

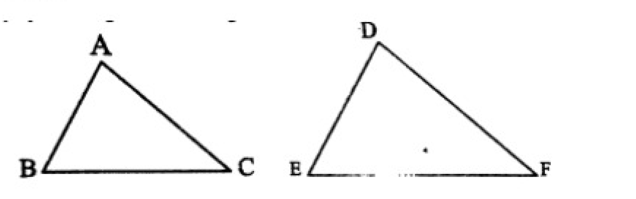
The ratio $a:b=\frac{a}{b}$ is the Comparison of two like a and b quantities 'a' and 'b' are called terms of 'a' ratio 'b'. Terms must be expressed in the same units.

(ii) Proportion

The statement of equality of two ratios is called proportion.

i.e. if $a:b = c:d$ then a, b, c and d are said to be in proportion.

(iii)Congruent Triangle



Two triangles said to be congruent written symbolically as ≅, if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

$$\text{if}\begin{cases}AB\cong DE\\ BC\cong EF\\ CA\cong FD\end{cases} \text{ and } \begin{matrix}\angle A\cong\angle D\\ \angle B\cong\angle E\\ \angle C\cong\angle F\end{matrix}$$

Then $\Delta ABC \cong \Delta DEF$

(iv) Similar Triangle

if in $\Delta ABC \leftrightarrow \Delta DEF$

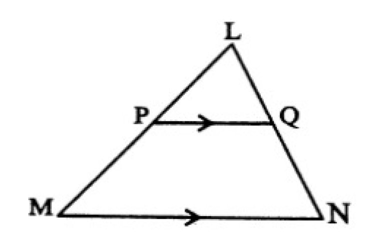
$\angle A\cong\angle D$, $\angle B\cong\angle E$, $\angle C\cong\angle F$

and $\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{BC}}{\overline{EF}} = \frac{\overline{CA}}{\overline{FD}}$

Then ΔABC and ΔDEF are called a similar triangle which is symbolically written as $\Delta ABC \sim \Delta DEF$.

Q3. In ΔLMN shown in the figure, $\overline{MN} \parallel \overline{PQ}$

Solution:



(i) If $m\overline{LM}$ = 5cm, $m\overline{LP}$ = 2.5cm, $m\overline{LQ}$ = 2.3cm, $m\overline{LN}$ =?

As $\overline{PQ} \parallel \overline{MN}$

$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$

$\frac{2.5}{5} = \frac{2.3}{m\overline{LN}}$

$2.5\left(m\overline{LN}\right) = 2.3\times5 = 11.5$

$m\overline{LN} = \frac{11.5}{2.5} = \frac{115}{25} = \frac{23}{5} = 4.6cm$

(ii) If $m\overline{LM}$ = 6cm, $m\overline{LQ}$ = 2.5cm, $m\overline{QN}$ = 5cm, $m\overline{LP}$ =?

As $\overline{PQ} \parallel \overline{MN}$

$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$

$\frac{m\overline{LP}}{6} = \frac{2.5}{7.5}$

$m\overline{LP} = \frac{2.5}{7.5}\times6 = \frac{25}{75}\times6 = 2cm$

Q4. In the shown figure, let $m\overline{PA}$ = 8x - 7, $m\overline{PB}$ = 4x - 3, $m\overline{AQ}$ = 5x - 3, $m\overline{BR}$ = 3x - 1. Find the value of x if $\overline{AB} \parallel \overline{QR}$.

Solution:

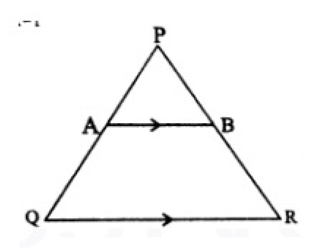
$m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$

$m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$

As $\overline{AB} \parallel \overline{QR}$

$\frac{m\overline{PA}}{m\overline{AQ}} = \frac{m\overline{PB}}{m\overline{BR}}$

$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$



$(8x-7)(3x-1) = (4x-3)(5x-3)$

$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$24x^2 - 20X^2 - 29x + 27x + 7 = 9$

$4x^2 - 2x + 7 = 9$

$4x^2 - 2x - 2 = 0$

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + (x-1) = 0$

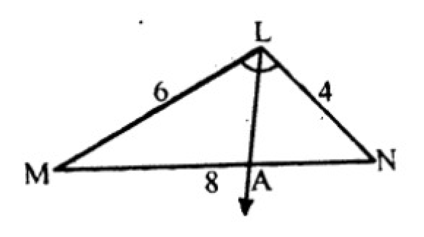
$(x-1)(2x+1) = 0$

$\left(x = 1, -\frac{1}{2}\right)$

$x=1$ is the required value.

Q5. In ΔLMN shown in the figure, $\overrightarrow{LA}$ bisects ∠L. If $m\overline{LN}$ = 4, $m\overline{LM}$ = 6, $m\overline{MN}$ = 8, then find $m\overline{MA}$ and $m\overline{AN}$.

Solution:



$m\overline{LN} = 4$, $m\overline{LM} = 6$, $m\overline{MN} = 8$

$\overrightarrow{LA}$ bisects $\angle L$

$\frac{m\overline{MA}}{m\overline{NA}} = \frac{m\overline{LM}}{m\overline{LN}} = \frac{6}{4}$

i.e. $m\overline{MA} : m\overline{NA} = 6:4$

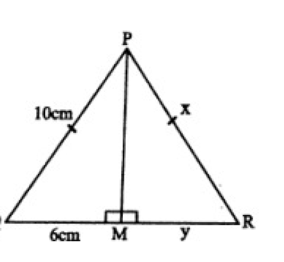
but $m\overline{MN} : m\overline{MA} + m\overline{AN} = 8$

$m\overline{MA} = \frac{6}{10}\times8 = \frac{48}{10} = 4.8$

and $m\overline{AN} = \frac{4}{10}\times8 = \frac{32}{10} = 3.2$

Q6. In isosceles ΔPQR shown in the figure, find the value of x and y.

Solution:



$\overline{PQ} \cong \overline{PR}$

$\Rightarrow x = 10cm$

$\overline{PM} \perp \overline{QR}$ where PQR is an isosceles triangle

$\therefore m\overline{MQ} = m\overline{MR}$

$\Rightarrow y = 6cm$