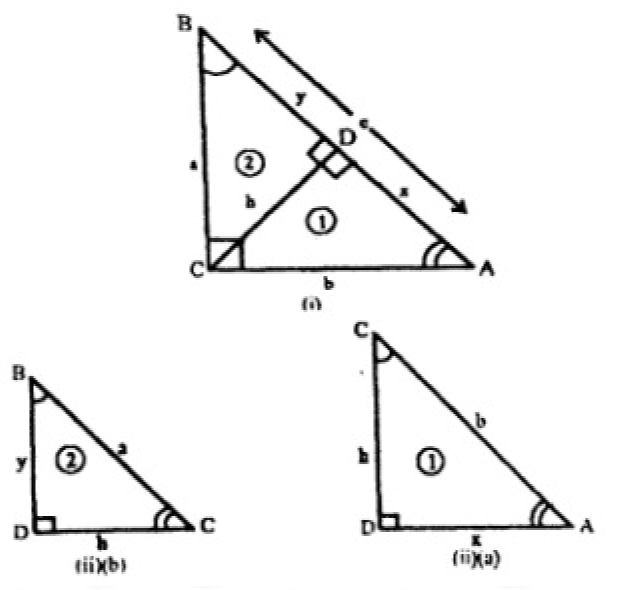
THEORM 15.1.1

PYTHAGORAS' THEOREM

In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



Given

ΔACB is a right-angled triangle in which $m\angle C = 90°$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AC} = c$.

To prove

$c^2 = a^2 + b^2$

Construction

Draw CD perpendicular from C on AB.

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two Δs ADC and BDC which are separately shown in the figures (ii) -a and (ii) b respectively.

Proof

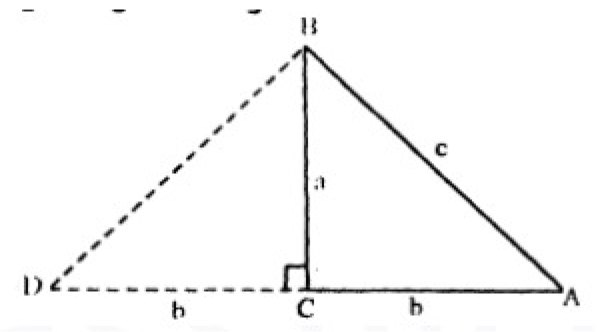
Statements	Reasons
In $\Delta ADC \leftrightarrow \Delta ACB$	Refer to figure ii (a) and (i) common-self congruent
$\angle A \cong \angle A$	
$\angle ADC \cong \angle ACB$	Construction given both measure 90°
$\angle C \cong \angle B$	$\angle$C and $\angle$B, complements of $\angle$A
$\therefore \Delta ADC \cong \Delta ACB$	Congruency of three angles
$\therefore = \frac{x}{b} = \frac{b}{c}$	(Measure of corresponding sides of similar triangles is similar)
or $x = \frac{b^2}{c}$ .....(i)	
	Refer to figure ii(b) and (i)
Again, in correspondence	
$\Delta BDC \leftrightarrow \Delta BCA$	Common self-congruent
$\angle B \cong \angle B$	Construction given, both measure 90°
$\angle BDC \cong \angle BCA$	$\angle$C and $\angle$A at complements of $\angle$M
$\angle C \cong \angle A$	Congruency of three angles.
$\therefore \Delta BDC \sim \Delta BCA$	
$\therefore = \frac{y}{a} = \frac{a}{c}$	Sides of similar triangles are proportional. (Theorem 6)
$\therefore y = \frac{a^2}{c}$ .....(ii)	
	Supposition

But $y + x = c$	
$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	Multiplying both sides with c.
i.e. $c^2 = a^2 + b^2$	

THEOREM 15.1.2

(CONVERSE OF PYTHAGORAS' THEOREM 15.1.1)

In a triangle if the sum of the squares of the measure of two sides is equal to the square of the measure of the third side, the triangle is a right-angled triangle.



Given

In a Δ ABC, $m\overline{AB} = c$, $m\overline{BC} = a$ and $m\overline{AC} = b$ such that $a^2 + b^2 = c^2$

To prove

$m\angle ACB = 90°$, ΔACB is a right-angled triangle.

Construction

Draw $\overline{CD}$ Perpendicular to $\overline{BC}$ such that $\overline{CD} = \overline{CA}$. Join B and D.

Proof

Statements	**Reasons**
ΔDCB is a right-angled triangle	Construction
$\therefore (m\overline{BD})^2 - a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given

$\therefore (m\overline{BD})^2 = c^2$	
or $m\overline{BD} = c$	Taking square root of both sides.
Now in	
$\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each is equal to c
$\therefore \Delta DCB \cong \Delta ACB$	S.S.S ≅ S.S.S
$\therefore \angle DCB \cong \therefore \angle ACB$	Corresponding angles of congruent triangles.
But $m\angle DCB = 90°$	Construction
$\therefore m\angle ACB = 90°$	
and the Δ ACB is a right-angled triangle.	

EXERCISE 15.1

Q1. Verify that the Δs having the following measures of sides are right-angled.

(i) a = 5cm, b = 12cm, c = 13cm

Solution:

By Pythagoras theorem

$a^2 + h^2 = (5)^2 + (12)^2$
$= 25 + 144 = 169$
$c^2 = (13)^2 = 169$
$\therefore a^2 + b^2 = c^2$

Thus, the triangle is right angled triangle.

(ii) a = 1.5 cm, b = 2cm, c = 2.5cm

Solution:

By Pythagoras theorem

$a^2 + b^2 = (1.5) + (2)$
$= 2.25 + 4 = 6.25$
$c^2 = (2.5)^2 = 6.25$
$\therefore a^2 + b^2 = c^2$

Thus, the triangle is right angled triangle.

(iii) a = 9cm, b = 12cm, c = 15 cm

Solution:

By Pythagoras theorem

$a^2 + b^2 = (9)^2 + (12)^2$
$= 81 + 144 = 225$
$c^2 = (15) = 225$
$\therefore a^2 + b^2 = c^2$

Hence the triangle is right angled triangle.

(iv) a = 16cm, b = 30 = cm, c = 34cm

Solution:

By Pythagoras theorem

$\alpha^2 + b^2 = (16)^2 + (2)^2$
$= 256 + 900 = 1156$
$c^2 = (34)^2 = 1156$
$\therefore a^2 + b^2 = c^2$

Hence the triangle is right angled triangle.

Q2. Verify that $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are the measures of the sides of a right-angled triangle where a and b are any two real numbers (a > 6)

Solution:

Let ABC be triangle such that

$\overline{AB} = a^2 + b^2,$

$\therefore \overline{BC} = a^2 - b^2,$

$\overline{AC} = 2ab$

By Pythagoras theorem

$|\overline{AB}|^2 = (a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2$

and

$|\overline{AC}|^2 + |\overline{BC}|^2 = (2ab)^2 + (a^2 - b^2)^2$

$= 4a^4 + \alpha^4 + b^4 - 2a^2b^2$
$= a^4 + b^4 + 2a^2b^2$

So $|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$

Hence ABC is a right-angled triangle.

Q3. The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right-angled triangle?

Solution:

If x is the base of right-angled triangle then -17 is the measure of hypotenuse.

By Pythagoras Theorem

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(17)^2 = (x)^2 + (8)^2$

$289 = x^2 + 64$

$x^2 = 289 - 64$

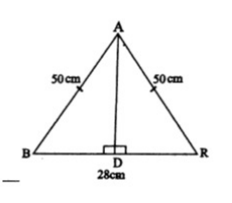
$x^2 = 225$

$x = 15$

Q4. In an isosceles Δ, the base $m\overline{BC}$ = 28 cm, and $m\overline{AB} = m\overline{AC}$ = 50 cm. If $m\overline{AD} \perp m\overline{BC}$, then find

(i) length of AD (ii) Area of ΔABC

Solution:



(i) $\overline{AD} \perp \overline{BC}$

$\therefore$ D is mid-point for BC

So $m\overline{BD} = \frac{1}{2}(28) = 14\ cm$

From right angled ΔABD

$(m\overline{AB})^2 = (m\overline{BD})^2 + (m\overline{AD})^2$

$(50)^2 = (14)^2 + (m\overline{AD})^2$

$(m\overline{AD})^2 = (50)^2 - (14)^2 = 2500 - 196 = 2304$

$m\overline{AD} = 48cm$

(ii) Area of ΔABC

$= (m\overline{BC})(m\overline{AD})$

$= (28)(48)$
$= 672cm^2$

Q5. In a quadrilateral ABCD, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. Prove that $m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$

Solution:

The diagram AC and BD of the quadrilateral ABCD meet at O perpendicular in the right triangles ΔAOB

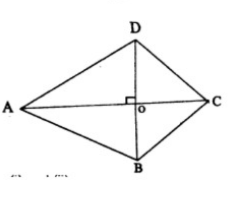
$m\overline{AB}^2 = m\overline{AO}^2 + m\overline{OB}^2$ (i)

In the right triangle ΔBOC

$m\overline{BC}^2 = m\overline{OB}^2 + m\overline{OC}^2$ (ii)

In the right triangle ΔDOC

$m\overline{DC}^2 = m\overline{OC}^2 + m\overline{OD}^2$ (iii)



Adding (i) and (ii)

$m\overline{AB}^2 + m\overline{BC}^2 = m\overline{AO}^2 + m\overline{OB}^2 + m\overline{OC}^2 + m\overline{OD}^2$

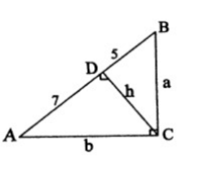
Adding (i) and (iv)

$m\overline{AD}^2 + m\overline{BC}^2 = m\overline{OA}^2 + m\overline{OB}^2 + m\overline{OC}^2 + m\overline{OD}^2$

Hence $m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$

Q6. (i) In the ΔABC as shown in the figure, $m\angle ACB$ = 90° and $\overline{CD} \perp \overline{AE}$. Find the lengths a, h and b if m $\overline{BD}$ = 5 units and $m\overline{AD}$ = 7 units.

Solution:



$m\overline{AB} = 5 + 7 = 12$

In right angled ΔBDC

$a^2 = 25 + h^2$ (1)

In right angled ΔADC

$b^2 = 49 + h^2$ (2)

In right angled ΔABC

$a^2 + b^2 = 144$ (3)

Adding (1) and (2)

$a^2 + b^2 = 74 + 2h^2$

from (3) and (4)

$74 + 2h^2 = 144$

$2h^2 = 144 - 74 = 70$

$h^2 = 35$

$h = \sqrt{35} units$

Put $h^2 = 35$ in (1)

$a^2 = 25 + 35 = 60$

$a = \sqrt{60}$

$a = 2\sqrt{15}\ units$

Put $h^2 = 35$ in (2)

$b^2 = 49 + 35$

$b^2 = 84$

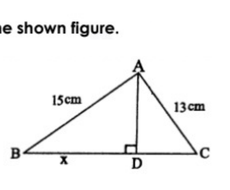
$b = \sqrt{84} = 2\sqrt{21} units$

So, $a = 2\sqrt{15} units$

$h = \sqrt{35} units$

$b = 2\sqrt{21} units$

(ii) Find the value of x in the shown figure.



From ΔADC

$(m\overline{AC})^2 = (m\overline{AD})^2 + (m\overline{DQ})^2$

$(13)^2 = (m\overline{AD})^2 + (5)^2$

$169 = (m\overline{AD})^2 + 25$

$(m\overline{AD})^2 = 169 - 25 = 144$

$\therefore \quad m\overline{AD} = 12cm$

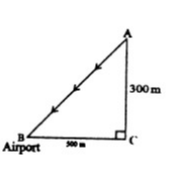
From ΔABD

$(m\overline{AB})^2 = (m\overline{AD})^2 + (m\overline{BD})^2$

$(15)^2 = (12)^2 + (x)^2$
$225 = 144 + x^2$
$x^2 = 225 - 144 = 81$
$\therefore \quad x = 9\ cm$

Q7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land the airport?

Solution:



$m\overline{BC}$ = 500 m, $m\overline{AC}$ = 300 m

By Pythagoras theorem

$m\overline{AB}^2 = m\overline{BC}^2 + m\overline{AC}^2$

$m\overline{AB}^2 = (500)^2 + (300)^2$

$m\overline{AB}^2 = 250000 + 90000$

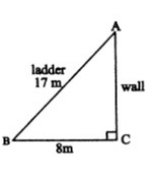
$m\overline{AB}^2 = 340000$

$m\overline{AB}^2 = \sqrt{34 \times 10000}$

$m\overline{AB} = 100\sqrt{34}$ m

Q8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach?

Solution:



By Pythagoras Theorem

$(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2$

$(17)^2 = (m\overline{AC})^2 + (8)^2$

$(m\overline{AC})^2 = (17)^2 - (8)^2$

$(m\overline{AC})^2 = 289 - 64 = 225$

$m\overline{AC} = \sqrt{225} = 15$ cm

Q9. A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

Solution:

A is house, B is bus stop and D is school. Produce $\overline{AB}$ and draw $\overline{DL} \parallel \overline{DC}$ to meet $\overline{AB}$ produced at L.

We have to find AD

$m\overline{LD} = m\overline{BC}$ = 6km

$m\overline{BL} = m\overline{CD}$ = 3km

$m\overline{AL} = m\overline{AB} + m\overline{BL} = 2 + 3 = 5$ km

ALD is a right angled Δ

By Pythagoras theorem

$m\overline{AD}^2 = m\overline{AL}^2 + m\overline{LD}^2$

$= (5)^2 + (6)^2 = 25 + 36 = 61$

$m\overline{AD} = \sqrt{61}$ km

REVIEW EXERCISE 15

Q1. Which of the following is true and which are false?

(i) In a right-angled triangle greater angle is of 90°.

(ii) In a right-angled triangle right angle is of 60°.

(iii) In a right triangle hypotenuse is a side opposite to right angle.

(iv) If a, b, c are sides of right-angled triangle with c as longer side then $c^2 = a^2 + b^2$

(v) If 3 cm and 4 cm are two sides of a right-angled triangle, then hypotenuse is 5 cm.

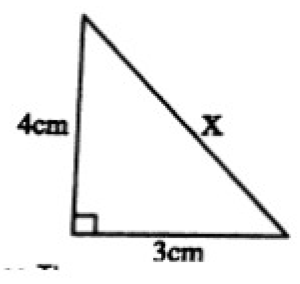
(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm.

Answers:

(i) T	**(ii)** F	**(iii)** T	**(iv)** T	**(v)** T	**(vi)** F

Q2. Find the unknown value in each of the following figures.

(i)



By Pythagoras Theorem

$x^2 = 4^2 + 3^2$

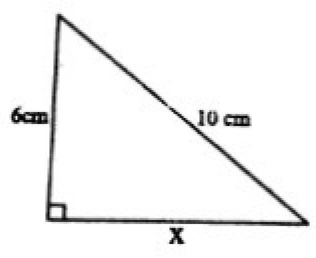
$x^2 = 16 + 9$

$x^2 = 25$

$x^2 = \sqrt{25}$

$x = 5cm$

(ii)



By Pythagoras Theorem

$(10)^2 = (6)^2 + (x)^2$

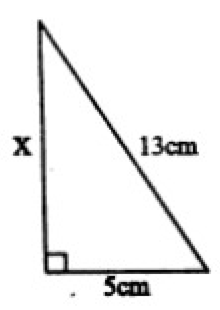
$100 = 36 + x^2$

$x^2 = 100 - 36 = 64$

$x^2 = \sqrt{64}$

$x^2 = 8cm$

(iii)



By Pythagoras Theorem

$(13)^2 = (x)^2 + (5)^2$

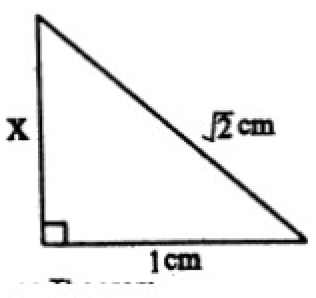
$169 = x^2 + 25$

$x^2 = 169 - 25$

$x^2 = \sqrt{144}$

$x = 12cm$

(iv)



By Pythagoras Theorem

$(\sqrt{2})^2 = (x)^2 + (1)^2$

$2 = x^2 + 1$

$x^2 = \sqrt{1} = 1cm$