Exercise 3.1

Q1. Express each of the following numbers in scientific notation.

(1) 5700 **(2) 49,800,000**

(3) 96,000,000 **(4) 416.9**

(5) 83,000 **(6) 0.00643**

(7) 0.0074 **(8) 60,000,000**

(9) 0.00000000395 **(10)** $\frac{275,000}{0,0025}$

Note:

A number written in the form $a\times10^{n}$, where $1\le a\ge 10$ and n is an integer, is called scientific notation.

Solution:

1) 5700

$$\frac{5700}{1000}\times1000=5.70\times10^{3}$$

2) 49,800,000

$$\frac{49,800,000}{10000000}\times10000000=4.98\times10^{7}$$

3) 96,000,000

$$\frac{96,000,000}{10000000}\times10000000=9.6\times10^{7}$$

4) 416.9

$$\frac{416.9}{10}=4169\times10^{-1}$$

$$\frac{416.9}{1000}=1000\times10^{-1}=4.169\times10^{3-1}=4.169\times10^{2}$$

5) 83,000

$$\frac{83,000}{10000}\times10000=8.3\times10^{4}$$

6) 0.00643

$$0.00643=\frac{643}{100000}=643\times10^{-5}$$

$$\frac{643}{100}\times100\times10^{-5}=6.43\times10^{2-5}=6.43\times10^{-3}$$

7) 0.0074

$$\frac{.0074}{10000}=74\times10^{-4}$$

$$\frac{74}{10}\times10\times10^{-4}=7.4\times10^{1-4}=7.4\times10^{-3}$$

8) 60,000,000

$$\frac{60000000}{10000000}\times10000000=6.0\times10^{7}$$

9) 0.00000000395

$$\frac{00000000395}{100000000000}=395\times10^{-11}$$

$$\frac{395}{100}\times100\times10^{-11}=3.95\times10^{2-11}=3.95\times10^{-9}$$

10) $\frac{275,000}{0.0025}$

$$=\frac{275,000}{0.0025}$$

$$=\frac{275\times10^{3}}{25\times10^{-4}}$$

$$=11\times10^{3+4}$$

$$=\frac{11}{10}\times10\times10^{7}=1.1\times10^{7+1}$$

$$=1.1\times10^{8}$$

Q2 Express the following numbers in ordinary notation

(1) 6×10^{-4}

(2) 5.06×10^{10}

(3) 9.018×10^{-6}

(4) 7.865×10^{8}

Solution:

1) 6×10^{-4}

$$=\frac{6}{10^{4}}$$

$$=\frac{6}{10000}$$

$$=0.0006$$

2) 5.06×10^{10}

$$=\frac{506}{100}\times10^{10}$$

$$=506\times10^{10-2}$$

$$=506\times10^{8}$$

$$=50,600,000,000$$

3) 9.018×10^{-6}

$$=\frac{9018}{1000}\times10^{-6}$$

$$=9018\times10^{-6-3}$$

$$=9018\times10^{-9}$$

$$=\frac{9018}{100000000}$$

$$=0.000009018$$

4) 7.865×10^{8}

$$=\frac{7865}{1000}\times10^{8}$$

$$=7865\times10^{-3}\times10^{8}$$

$$=7865\times10^{8-3}$$

$$=7865\times10^{5}$$

$$=785,500,000$$

Exercise 3.2

Q1. Find the common logarithm of each of the following numbers:

(1) 232.92 **(2) 29.326**

(3) 0.00032 **(4) 0.3206**

Note: For Finding the common logarithm of any given number, use the following steps:

(i) **Round off the numbers to four significant digits**
(ii) **Find the characteristics of the logarithm of the number by inspection**
(iii) **Find the mantissa of the logarithm of the number from the log tables**
(iv) **Combine the two**

Solution:

(1) 232.92

Rounding off 232.92 to four significant digits, we get: 232.9

The characteristic of 232.9 is 2 as there are 3 digits

To find mantissa, we use the log table, and follow the row of 23. For row 23, the value at column of 2 is 3655. In the same row in the difference column of 9, the value is 17. Adding 3655 and 17, we get the mantissa .3672

Combining the two values of characteristic and mantissa, we get:

So, $\log\ 232.92 = 2.3672$

(2) 29.326

Rounding off 29.326 to four significant digits, we get: 29.33

The characteristic of 29.33 is 1 as there are 2 digits.

To find mantissa using the log table, we follow the row of 29 and reach the column of 3 to get 4669. In the same row in the difference column of 3 we see 4. Adding 4669 and 4, we get the value of mantissa .4673.

So, $\log 29.326 = 1.4673$

(3) 0.00032

The number 0.00032 can be written as 3.2×10^{-4}, therefore the characteristic is -4, which is written as $\bar{4}$.

To find mantissa using the log table, we follow the row of 32 and reach the column of 0 to get 5051. So, the value of mantissa is .5051

So, $\log 0.00032 = \bar{4}.5051$

(4) 0.3206

The number 0.3206 can be written as 3.206×10^{-1}, therefore the characteristic is -1, as which is written as $\bar{1}$.

To find the mantissa using the log table, we follow the row of 32 and reach the column if 0 to get 5051. In the same row in the difference column of 6 we see 8. Adding 5051 and 8, we get the value of mantissa as .5059

So, $\log 0.3206 = \bar{1}.5059$

Q2. If log 31.09 = 1.4926. Find values of the following.

i. **log 3.109**
ii. **log 310.9**
iii. **log 0.003109**
iv. **log 0.3109**

Hint: Since the digits in all the above numbers are same, therefore the mantissa will remain the same as .4926

Solution:

Given: log 31.09 =1.4926

1) log 3.109

log 3.109 can also be written as 3.109×10^{0}, therefore characteristic is 0 and mantissa is .4926

So, $\log 3.109 = 0.4926$

2) 310.9

log 310.9 can also be written as 3.109×10^{2}, therefore characteristic is 2 and mantissa is .4926

So, $\log 310.9 = 2.4926$

3) 0.003109

log 0.003109 can also be written as 3.109×10^{-3}, therefore the characteristic is $\bar{3}$ and the mantissa is .4926

So, $\log 0.003109 = \bar{3}.4926$

4) 0.3109

log 0.3109 can also be written as 3.109×10^{-1}, therefore the characteristic is $\bar{1}$ and mantissa is 0.4926

So, $\log 0.3109 = \bar{1}.4926$

Q3. Find the numbers whose common logarithm are

(i) 3.5621

(ii) $\bar{1}.7427$

Solution:

(i) 3.5621

Reading along the row corresponding to .56 (as mantissa is .5621), we get 3648 at the intersection of this row and the column of 2. The number at the intersection of this row and the mean difference column corresponding to 1 is 1. Adding 3648 and 1, we get 3649.

Since the characteristics is 3, the number will have four digits before decimal.

Hence, antilog of 2.5621 is 3649

(ii) $\bar{1}.7427$

Reading along the row corresponding to .74 (as mantissa is .7427), we get 5521 at the intersection of this row and the column of 2. The number at the intersection of this row and the mean difference column corresponding to 7 is 9. Adding 5521 and 9, we get 5530.

Since the characteristics is $\bar{1}$ or -1, the number will be written as

Hence, antilog of $\bar{1}.7427$ is 0.5530

Q4. What replacement for the unknown in each of following will make the statemen true?

(i) $\log_3 81 = L$

(ii) $\log_a 6 = 0.5$

(iii) $\log_5 n = 2$

(iv) $10^P = 40$

Solution:

(i) $\log_3 81 = L$

$3^L = 81$

$3^L = 3^4$

L=4

(ii) $\log_a 6 = 0.5$

$a^{0.5} = 6$

$a^{\frac{1}{2}} = 6$

$\sqrt{a} = 6$

Squaring both sides, we get

$a = 36$

(iii) $\log_5 n = 2$

$5^2 = n$

$n = 25$

(iv) $10^P = 40$

Changing into logarithm form, we get

$P = \log_{10} 40$

$P = \log 40$

$P = 1.6021$

Q5. Evaluate.

(i) $\log_2 \frac{1}{128}$

(ii) $\log 512$ to the base $2\sqrt{2}$

Solution:

(i) $\log_2 \frac{1}{128}$

Converting into exponential form, we get

$\log_2 \frac{1}{128} = x$

$2^x = \frac{1}{128}$

$2^x = \frac{1}{128}$

$2^x = \frac{1}{2^7}$

$2^x = 2^{-7}$

$x = -7$

(ii) $\log 512$ to the base $2\sqrt{2}$

$\log_{2\sqrt{2}} 512$

Converting into exponential form, we get

$\log_{2\sqrt{2}} 512 = x$

$\left(2\sqrt{2}\right)^x = 512$

$\left(2\times2^{\frac{1}{2}}\right)^x = 2^9$

$\left(2^{1+\frac{1}{2}}\right)^x = 2^9$

$\left(2^{\frac{3}{2}}\right)^x = 2^9$

$2^{\frac{3x}{2}} = 2^9$

Comparing the integral part, we get

$\frac{3x}{2} = 9$

$x = \frac{9\times2}{3}$

$= \frac{18}{3}$

$= 6$

Q6. Find the value of x from the following statements.

i. $\log_2 x = 5$

ii. $\log_{81} 9 = x$

iii. $\log_{64} 8 = \frac{x}{2}$

iv. $\log_x 64 = 2$

v. $\log_3 x = 4$

Solution:

(i) $\log_2 x = 5$

$(2)^5 = x$

$x = 32$

(ii) $\log_{81} 9 = x$

$(81)^x = 9$

$(9^2)^x = 9$

$9^{2x} = 9^1$

$2x = 1$

$x = \frac{1}{2}$

(iii) $\log_{64} 8 = \frac{x}{2}$

$(64)^{\frac{x}{2}} = 8$

$(8^2)^{\frac{x}{2}} = 8$

$8^x = 8^1$

$x = 1$

(iv) $\log_x 64 = 2$

$(x)^2 = 64$

$x^2 = 8^2$

Taking square root of both sides we get,

$\sqrt{x^2} = \sqrt{8^2}$

$x = 8$

(v) $\log_3 x = 4$

$(3)^4 = x$

$x = 3^4$

$x = 81$

Exercise 3.3

Q1. Write the following into sum or difference.

(i) $\log(A \times B)$

(ii) $\log \frac{15.2}{30.5}$

(iii) $\log \frac{21 \times 5}{8}$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

(vi) $\log \frac{25 \times 47}{29}$

Solution:

(i) $\log(A \times B) = \log A + \log B$

(ii) $\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5$

(iii) $\log \frac{21 \times 5}{8} = \log 21 \times 5 - \log 8 = \log 21 + \log 5 - \log 8$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

$= \log\left(\frac{7}{15}\right)^{\frac{1}{3}}$

$= \frac{1}{3}\log\left(\frac{7}{15}\right)$

$= \frac{1}{3}[\log 7 - \log 15]$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

$= \log(22)^{\frac{1}{3}} - \log 5^3$

$= \frac{1}{3}\log 22 - 3\log 5$

(vi) $\log \frac{25 \times 47}{29}$

$= \log(25 \times 47) - \log 29$

$= \log 25 + \log 47 - \log 29$

Q2. Express $\log x - 2\ \log x + 3\ \log(x+1) - \log(x^2 - 1)$ **as a single logarithm.**

Solution:

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

$= \log x(1-2) + 3\log(x+1) - \log(x+1)(x-1)$

$= \log x(-1) + 3\log(x+1) - [\log(x+1) + \log(x-1)]$

$= -\log x + 3\log(x+1) - \log(x+1) - \log(x-1)$

$= 2\log(x+1) - \log x - \log(x-1)$

$= 2\log(x+1) - [\log x + \log(x-1)]$

$= \log(x+1)^2 - \log x(x-1)$

$= \log \frac{(x+1)^2}{x(x-1)}$

Q3. Write the following in the form of single logarithm.

Solution:

i. $\log 21 + \log 5$

$= \log 21 \times 5$

ii. $\log 25 - 2\log 3$

$= \log 25 - \log 3^2$

$= \log \frac{25}{3^2}$

iii. $2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log \frac{x^2}{y^3}$

iv. $\log 5 + \log 6 - \log 2$

$= \log 5 \times 6 - \log 2$

$= \log \frac{5 \times 6}{2}$

Q4. Calculate the following:

1) $\log_3 2 \times \log_2 81$

$= \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$

$= \frac{\log 81}{\log 3}$

$= \frac{\log 3^4}{\log 3}$

$= \frac{4\log 3}{\log 3}$

$= 4$

2) $\log_5 3 \times \log_3 25$

$= \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$

$= \frac{\log 25}{\log 5}$

$= \frac{\log 5^2}{\log 5}$

$= \frac{2\log 5}{\log 5}$

$= 2$

Q5. If $\log 2 = 0.3010,\ \log 3 = 0.4771,\ \log 5 = 0.6990$**, then find the values of the following.**

(i) $\log 32$

$\log 32 = \log 2^5$

$= 5\log 2$

$= 5(0.3010)$

$= 1.5050$

(ii) $\log 24$

$= \log 3 \times 8$

$= \log 3 + \log 8$

$= \log 3 + \log 2^3$

$= \log 3 + 3\log 2$

$= 0.4771 + 3(0.301)$

$= 0.4771 + 0.9030$

$= 1.3801$

(iii) $\log \sqrt{3\frac{1}{3}}$

$= \log \sqrt{\frac{10}{3}}$

$= \log\left(\frac{10}{3}\right)^{\frac{1}{2}}$

$= \frac{1}{2}(\log 10 - \log 3)$

$= \frac{1}{2}(\log 5 \times 2 - \log 3)$

$= \frac{1}{2}(0.6990 + 0.3010 - 0.4771)$

$= 0.2615$

(iv) $\log \frac{8}{3}$

$= \log 8 - \log 3$

$= \log 2^3 - \log 3$

$= 3\log 2 - \log 3$

$= 3(0.3010) - 0.4771$

$= 0.9030 - 0.4771$

$= 0.4259$

(v) $\log 30$

$= \log 2 \times 3 \times 5$

$= \log 2 + \log 3 + \log 5$

$= 0.3010 + 0.4771 + 0.6990$

$= 1.4771$

Exercise 3.4

Q1 Use log tables to find the value of the following

(i) 0.8176 x 13.64 **(ii)** $(789.5)^{\frac{1}{8}}$

(iii) $\dfrac{0.678\times 9.01}{0.0234}$ **(iv)** $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

(v) $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$ **(vi)** $\sqrt[3]{\dfrac{0.7214\times 20.37}{60.8}}$

(vii) $\dfrac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$ **(viii)** $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Solution

i. 0.8176 x 13.64

Let $x = 0.8176\times 13.64$

Taking log on both sides

$\log x = \log(0.8176\times 13.64)$
$\log x = \log 0.8176 + \log 13.64$
$\log x = \bar{1}.9125 + 1.1348$
$\log x = -1 + 0.9125 + 1 + 0.1348$
$\log x = 0.9125 + 0.1348$
$\log x = 1.0473$

Taking Antilog on both sides

Antilog $(\log x) =$ Antilog(1.0473)

Characteristic=1
Reference point=1.115

$x = 11.15$

ii. $(789.5)^{\frac{1}{8}}$

Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$\log x = \log(789.5)^{1/8}$
$\log x = \frac{1}{8}\log 789.5$
$\log x = \frac{1}{8}(2.8974)$
$\log x = 0.3622$

Taking Antilog on both sides

Antilog$(\log x) =$ Antilog(0.3622)

Characteristic=0
Reference Point: 2.302

$x = 2.302$

iii. $\dfrac{0.678\times 9.01}{0.0234}$

Let $x = \dfrac{0.678\times 9.01}{0.0234}$

Taking log on both sides

$\log x = \log\dfrac{0.678\times 9.01}{0.0234}$
$\log x = \log 0.678 + \log 9.01 - \log 0.0234$
$\log x = \bar{1}.8312 + 0.9547 - \bar{2}.3693$
$\log x = -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$
$\log x = -1 + 2 + 1.7859 - 0.3692$
$\log x = 1 + 1.4167$
$\log x = 2.4167$

Taking Antilog on both sides

Antilog$(\log x) =$ Antilog(2.4167)

Characteristic = 2
Reference Point = 2.610

$x = 261.0$

iv. $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

$x = \sqrt[5]{2.709}\times\sqrt[7]{1.239}$

$\log x = \log\left[(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}\right]$
$\log x = \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$
$\log x = \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$
$\log x = \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$
$\log x = 0.08656 + 0.0133$
$\log x = 0.0999$

Taking Antilog on both sides

Antilog$(\log x) =$ Antilog(0.0999)

Characteristic = 0
Reference Point = 1.259

$x = 1.259$

v. $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

Let $x = \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

$\log x = \log\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$
$\log x = \log[(1.23)(0.6975)] - \log[(0.0075)(1278)]$
$\log x = \log(1.23) + \log(0.6975) - \log(0.0075) - \log(1278)$
$\log x = 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$
$\log x = -1 + 3 - 3 + 0.9334 - 0.9816$
$\log x = -2 + 0.9518$
$\log x = \bar{2}.9518$

Taking Antilog on both sides, we get

Antilog $x =$ Antilog $(\bar{2}.9518)$

Characteristic = $\bar{2}$
Reference = 8950

$x = 0.08950$

vi. $\sqrt[3]{\dfrac{0.7214\times 20.37}{60.8}}$

Let $x = \sqrt[3]{\dfrac{0.7214\times 20.37}{60.8}}$

$\log x = \log\left(\dfrac{0.7214\times 20.37}{60.8}\right)^{\frac{1}{3}}$
$\log x = \frac{1}{3}\log\left(\dfrac{0.7214\times 20.37}{60.8}\right)$
$\log x = \frac{1}{3}\log(0.7214\times 20.37) - \log(60.8)$
$\log x = \frac{1}{3}[\log(0.7214) + \log(20.37) - \log(60.8)]$

$\log x = \frac{1}{3}[\bar{1}.8582 + 1.3089 - 1.7839]$
$\log x = \frac{1}{3}[-1 + 0.8582 + 1.3089 - 1.7839]$
$\log x = \frac{1}{3}(-0.6168)$
$\log x = -0.2056$

$\log x$ is negative, so

$\log x = -1 + 1 - 0.2056$
$\log x = -1 + 79144$
$\log x = \bar{1}.7944$

Taking Antilog on both sides, we get

Antilog $x =$ Antilog $(\bar{1}.7944)$

Characteristic = $\bar{1}$
Reference = 6229

$x = 0.6229$

vii. $\dfrac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Let $x = \dfrac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Taking log on both sides, we get
$\log x = \log(83\times\sqrt[3]{92}) - \log(127\times\sqrt[5]{246})$
$\log x = \log 83 + \log(\sqrt[3]{92}) - \log 127 - \log(\sqrt[5]{246})$
$\log x = \log 83 + \log(92)^{1/3} - \log 127 - \log(246)^{1/5}$
$\log x = \log 83 + \frac{1}{3}\log 92 - \log 127 - \frac{1}{5}\log 246$

$\log x = 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.3909)$
$\log x = 1.9191 + 0.65460 - 2.1038 - 0.47818$
$\log x = -0.0083$
$\log x = -1 + 1 - 0.0083$ (Adding and subtracting 1)
$\log x = -1 + 0.9917$
$\log x = \bar{1}.9917$

Taking Antilog on both sides, we get
Antilog$(\log x) =$ Antilog$(\bar{1}.9917)$

Characteristic = $\bar{1}$
Reference Point = 9.811

$x = 0.9811$

viii. $\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Let $x = \dfrac{(438)^3\sqrt{0.056}}{(388)^4}$

Taking log on both sides
$\log x = \log\dfrac{(438)^3\sqrt{0.056}}{(388)^4}$
$\log x = \log(438)^3 + \log\sqrt{0.056} - \log(388)^4$
$\log x = \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$
$\log x = 3\log 438 + \frac{1}{2}\log 0.056 - 4\log 388$
$\log x = 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$
$\log x = 3(2.6415) + \frac{1}{2}(-2 + 0.7482) - 4(2.5888)$
$\log x = 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$
$\log x = 7.9245 - 0.6259 - 10.3552$
$\log x = -3.0566$

Since log is in negative, so adding and subtracting 4
$\log x = -4 + 4 - 3.0566$
$\log x = -4 + 0.9434$
$\log x = \bar{4}.9434$

Taking Antilog on both sides, we get

Antilog$(\log x) =$ Antilog$(\bar{4}.9434)$

Characteristic = $\bar{4}$
Reference Point = 8778

$x = 0.0008778$

Q2. A gas is expanding according to the law $pv^n = C$. Find C when $p = 80,\ v = 3.1$ and $n = \frac{5}{4}$

Solution:

Given $pv^n = C$ ...(i)

Substituting $p = 80,\ v = 3.1$ and $n = \frac{5}{4}$ in (i)

$C = 80(3.1)^{\frac{5}{4}}$

Taking log on both sides
$\log C = \log 80(3.1)^{\frac{5}{4}}$
$\log C = \log 80 + \frac{5}{4}\log 3.1$
$\log C = 1.9031 + \frac{5}{4}(0.4914)$
$\log C = 1.9031 + \frac{2.570}{4}$
$\log C = 1.9031 + 0.6143$
$\log C = 2.5174$

Taking Antilog on both sides, we get

Antilog$(\log C) =$ Antilog(2.5174)

$C = 329.2$

Q3. The formula $p = 90(5)^{\frac{-q}{10}}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00?

Solution:

$p = 90(5)^{-q/10}$

Taking log on both sides
$\log p = \log[90(5)^{-q/10}]$
$\log p = \log 90 + \log(5)^{-q/10}$
$\log p = \log 90 - \frac{q}{10}\log 5$

Putting $p = 18$ in the above equation, we get
$\log 18 = \log 90 - \frac{q}{10}\log 5$
$1.2553 = 1.9542 - \frac{q}{10}(0.6990)$
$1.2553 - 1.9542 = -\frac{q}{10}(0.6990)$
$-0.6989 = -\frac{q}{10}(0.6990)$
$\cancel{-}0.6989\times 10 = \cancel{-}q(0.6990)$
$6.989 = q(0.6990)$
$\frac{6.989}{0.6990} = q$

$q = 10$ approximately

Hence, 10 units will be demanded

Q4. If $A = \pi r^2$ Find A, when $\pi = \frac{22}{7}$ and $r = 15$.

Solution:

$A = \pi r^2$

Taking log on both sides
$\log A = \log \pi r^2$
$\log A = \log \pi + \log r^2$
$\log A = \log \pi + 2\log r$
$\log A = \log\frac{22}{7} + 2\log 15$
$\log A = \log 22 - \log 7 + 2\log 15$
$\log A = 1.342 - 0.8451 + 2(1.1761)$
$\log A = 2.8495$

Taking Antilog on both sides, we get

Antilog$(\log A) =$ Antilog(2.8495)

$A = 707.1$

Q5. If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$

Solution:

$V = \frac{1}{3}\pi r^2 h$

Taking log on both sides
$\log V = \log\frac{1}{3}\pi r^2 h$
$\log V = \log\frac{1}{3} + \log \pi r^2 h$
$\log V = \log 1 - \log 3 + \log \pi r^2 + \log h$
$\log V = 0 - 0.4771 + \log \pi + \log r^2 + \log h$
$\log V = -0.4771 + \log \pi + 2\log r + \log h$
$\left(\pi = \frac{22}{7}, r = 2.5, \text{ and } h = 4.2\right)$

$\log V = -0.4771 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$
$\log V = -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$
$\log V = 1.4394$

Taking Antilog on both sides, we get

Antilog$(\log V) =$ Antilog(1.4394)

$V = 27.50$

Review Exercise 3

Q1. Multiple choice questions. Choose the correct answer.

(i) If $a^x = n,$, then......

(a) $a = \log_x n$ (c) $x = \log_a n$
(b) $x = \log_n a$ (d) $a = \log_n x$

(ii) The relation $y = \log_z x$ implies

(a) $x^y = z$ (c) $x^z = y$
(b) $z^y = x$ (d) $y^z = x$

(iii) The logarithm of unity to any base is.....

(a) 1 (c) e
(b) 10 (d) 0

(iv) The logarithm of any number to itself as base is......

(a) 1 (c) -1
(b) 0 (d) 10

(v) log e = ..., where e ≈ 2.718

(a) 0 (c) ∞
(b) 0.4343 (d) 1

(vi) The value of $\log\left(\frac{p}{q}\right)$ is........

(a) $\log p - \log q$ (c) $\log p + \log q$
(b) $\frac{\log p}{\log q}$ (d) $\log q - \log p$

(vii) $\log p - \log q$ is same as......

(a) $\log\left(\frac{q}{p}\right)$ (c) $\left(\frac{\log p}{\log q}\right)$
(b) $\log(p-q)$ (d) $\log\left(\frac{p}{q}\right)$

(viii) $\log(m^n)$ can be written as......

(a) $(\log m)^n$ (c) $n \log m$
(b) $m \log n$ (d) $\log(mn)$

(ix) $\log_b a \times \log_c b$ can be written as......

(a) $\log_a c$ (c) $\log_a b$
(b) $\log_c a$ (d) $\log_b c$

(x) $\log_y x$ will be equal to......

(a) $\frac{\log_z x}{\log_y z}$ (c) $\frac{\log_z x}{\log_z y}$
(b) $\frac{\log_x z}{\log_y z}$ (d) $\frac{\log_z y}{\log_z x}$

Solution:

(i) c	(ii) b	(iii) d	(iv) a	(v) b
(vi) a	(vii) d	(viii) c	(ix) b	(x) c

Q2. Complete the following.

i) For common logarithms, the base is ....
ii) The integral part of the common logarithm of a number is called the ....
iii) The decimal part of the common logarithm of a number is called the ....
iv) If $x = \log y$, then y is called ...... of x.
v) If the characteristic of the logarithm of a number is $\bar{2}$, that number will have .... zero(s) immediately after the decimal point.
vi) If the characteristic of the logarithm of a number is 1, that number will have .... digits in its integer part.

Answers:
i) 10
ii) Characteristic
iii) Mantissa
iv) Antilog
v) One
vi) 2

Q3. Find the value of x in the following.

i) $\log_3 x = 5$

$3^5 = x$
$x = 3^5$
$x = 243$

ii) $\log_4 256 = x$

$4^x = 256$
$4^x = 4^4$
$x = 4$

iii) $\log_{625} 5 = \frac{1}{4}x$

$(625)^{\frac{1}{4x}} = 5$
$(5^4)^{\frac{1}{4x}} = 5$
$5^x = 5^1$
$x = 1$

iv) $\log_{64} x = \frac{-2}{3}$

$(64)^{\frac{-2}{3}} = x$
$(4^3)^{\frac{-2}{3}} = x$
$4^{-2} = x$
$x = \frac{1}{4^2}$
$x = \frac{1}{16}$

Q4. Find the value of x in the following.

i. $\log x = 2.4543$
ii. $\log x = 0.1821$
iii. $\log x = 0.0044$
iv. $\log x = \bar{1}.6238$

Solution:
i) $\log x = 2.4543$

$x = \text{antilog } 2.4543$

From the table against the row of 0.45 under 4 we have 2844 and difference under 3 is 2. Adding 2844 and 0 we get 2846.

$x = 284.6$

ii) $\log x = 0.1821$

$x = \text{antilog } 0.1821$

From the table against the row of 0.18 under 2 we have 1521 and difference under 1 is 0. Adding 1521 and 0 we get 1521.

$x = 1.521$

iii) $\log x = 0.0044$

$x = \text{antilog } 0.0044$

From the table against the row of 0.00 under 4 we have 1009 and difference under 4 is 1. Adding 1009 and 1 we get 1010.

$x = 1.010$

iv) $\log x = \bar{1}.6238$

$x = \text{antilog } \bar{1}.6238$

From the table against the row of 0.62 under 3 we have 4198 and difference under 8 is 8. Adding 4192 and 8 we get 4206.

$x = 0.04206$

Q5. If $\log 2 = 0.3010, \log 3 = 0.4771,$ and $\log 5 = 0.6990$, then find the value of the following.

Solution:

i) $\log 45$
$= \log(3 \times 3 \times 5)$
$= \log 3 + \log 3 + \log 5$
$= 0.4771 + 0.4771 + 0.6990$
$= 1.6532$

ii) $\log \frac{16}{15}$
$= \log 16 - \log 15$
$= \log 2^4 - \log 3 \times 5$
$= 4\log 2 - \log 3 - \log 5$
$= 4(0.3010) - 0.4771 - 0.6999$
$= 1.2040 - 1.1761$
$= 0.0279$

iii) $\log 0.048$
$= \log \frac{48}{1000}$
$= \log 48 - \log 1000$
$= \log 3 \times 16 - \log 10^3$
$= \log 3 + \log 16 - 3\log 10$
$= 0.4771 + \log 2^4 - 3(1)$
$= 0.4771 + 4\log 2 - 3$
$= 0.4771 + 4(0.3010) - 3$
$= 1.6811 - 3$
$= 1 + 0.6811 - 3$
$= -2 + 0.6811$
$= \bar{2}.6811$

Q6. Simplify the following.

Solution:

i) $\sqrt[3]{25.47}$

Let $x = \sqrt[3]{25.47}$

$x = (25.47)^{\frac{1}{3}}$

Taking log on both sides
$\log x = \log(25.47)^{\frac{1}{3}}$

$\log x = \frac{1}{3}\log 25.47$
$\log x = \frac{1}{3}(1.4060)$
$\log x = 0.4686$

Taking Antilog on both sides, we get

$\text{antilog}(\log x) = \text{antilog}(0.4686)$

$x = 2.942$

ii) $\sqrt[5]{342.2}$

Let $x = \sqrt[5]{342.2} = (342.2)^{\frac{1}{5}}$

Taking log on both sides
$\log x = (342.2)^{\frac{1}{5}}$
$\log x = \frac{1}{5}\log 342.2$
$\log x = \frac{1}{5}(2.5343)$
$\log x = 0.5069$

Taking Antilog on both sides, we get

$\text{antilog}(\log x) = \text{antilog}(0.5069)$

$x = 3.213$

iii) $\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Let $x = \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

$x = \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$

Taking log on both sides, we get
$\log x = \log \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$
$\log x = \log(8.97)^3 \times (3.95)^2 - \log(15.37)^{\frac{1}{3}}$
$\log x = \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$
$\log x = 3\log(8.97) + 2\log(3.95) - \frac{1}{3}\log(15.37)$
$\log x = 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$
$\log x = 2.8584 + 1.1932 - 0.3956$
$\log x = 3.6560$

Taking antilog on both sides, we get
$x = \text{antilog}(3.6560)$

$x = 4529$