Exercise 4.1

Polynomials

A polynomial in the variable x is an algebraic expression of the form,

$p(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots\ldots+a_1x+a_0, a_n\neq 0$

Where n, the highest power of x, is a non-negative integer called the degree of the polynomial and each coefficient a_n is a real number.

Q1. Identify whether the following algebraic expressions are polynomials (yes or no)

(i) $3x^2+\frac{1}{x}-5$ **(ii)** $3x^3-4x^2-x\sqrt{x}+3$

(iii) $x^2-3x+\sqrt{2}$ **(iv)** $\frac{3x}{2x-1}+8$

Solution:

(i) No **(ii)** No **(iii)** Yes **(iv)** No

Q2. State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x+5}}$ **(ii)** $\frac{x^3-2x^2+\sqrt{3}}{2+3x-x^2}$

(iii) $\frac{x^2+6x+9}{x^2-9}$ **(iv)** $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Solution:

(i) No **(ii)** Yes **(iii)** Yes **(iv)** No

Q3. Reduce the following rational expressions to the lowest forms.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$ **(ii)** $\frac{8a(x+1)}{2(x^2-1)}$

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$ **(iv)** $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

(v) $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$ **(vi)** $\frac{x^2-4x+4}{2x^2-8}$

(vii) $\frac{64x^5-64x}{(8x^2+8)(2x+2)}$ **(viii)** $\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$

Solution:

1. $\frac{120x^2y^3z^5}{30x^3yz^2}=\frac{30\times 4y^{3-1}xz^{5-2}}{30x^{3-2}}=\boxed{\frac{4y^2z^3}{x}}$

2. $\frac{8a(x+1)}{2(x^2-1)}=\frac{2(4a)(x+1)}{2(x+1)(x-1)}=\boxed{\frac{4a}{x-1}}$

3. $\frac{(x+y)^2-4xy}{(x-y)^2}$

$=\frac{x^2+y^2+2xy-4xy}{(x-y)^2}$

$=\frac{x^2+y^2-2xy}{(x-y)^2}$

$=\frac{(x-y)^2}{(x-y)^2}$

$=\boxed{1}$

4. $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

$=\frac{(x^3-y^3)(x-y)^2}{(x-y)(x^2+xy+y^2)}$

$=\frac{(x^3-y^3)(x-y)^2}{(x-y)(x^2+xy+y^2)}$

$=\frac{(x-y)(x^2+xy+y^2)(x-y)}{(x^2+xy+y^2)}$

$=\boxed{(x-y)^2}$

5. $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$=\frac{(x+2)(x+1)(x-1)}{(x+1)(x+2)(x-2)}$

$=\boxed{\frac{x-1}{x-2}}$

6. $\frac{x^2-4x+4}{2x^2-8}$

$=\frac{x^2-4x+2^2}{2(x^2-4)}$

$=\frac{(x-2)^2}{2(x+2)(x-2)}$

$=\boxed{\frac{x-2}{2(x+2)}}$

7. $\frac{64x^5-64x}{(8x^2+8)(2x+2)}$

$=\frac{64x(x^4-1)}{8(x^2+1)2(x+1)}$

$=\frac{64x(x^2+1)(x^2-1)}{16(x^2+1)(x+1)}$

$=\frac{4x(x^2-1)}{x+1}$

$=\frac{4x(x+1)(x-1)}{x+1}$

$=\boxed{4x(x-1)}$

8. $\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$

$=\frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$

$=\frac{[3x+(x^2-4)][3x-(x^2-4)]}{4+3x-x^2}$

$=\frac{(x^2+3x-4)(4+3x-x^2)}{4+3x-x^2}$

$=\boxed{x^2+3x-4}$

Q4. Evaluate

(a) $\frac{x^3y-2z}{xz}$ **for**

i. $x=3, y=-1, z=-2$

ii. $x=-1, y=-9, z=4$

(b) $\frac{x^2y^3-5z^4}{xyz}$ **for** $x=4, y=-2, z=-1$

Solution:

(a) $\frac{x^3y-2z}{xz}$

(i) Putting $x=3, y=-1, z=-2$ in the above equation, we get

$\frac{x^3y-2z}{xz}$

$=\frac{(3)^3(-1)-2(-2)}{3(-2)}$

$=\frac{-27+4}{-6}$

$=\frac{-23}{-6}$

$=\frac{23}{6}$

$=\boxed{3\frac{5}{6}}$

(ii) Putting $x=-1, y=-9, z=4$ **in the above equation, we get**

$\frac{x^3y-2z}{xz}$

$=\frac{(-1)^3(-9)-2(4)}{(-1)(4)}$

$=\frac{9-8}{-4}$

$=\boxed{-\frac{1}{4}}$

(b) $\frac{x^2y^3-5z^4}{xyz}$

Putting $x=4, y=-2, z=-1$ **in the above equation, we get**

$=\frac{(4)^2(-2)^3-5(-1)^4}{4(-2)(-1)}$

$=\frac{16(-8)-5(1)}{8}=\frac{-128-5}{8}=-\frac{133}{8}$

$=\boxed{-16\frac{5}{8}}$

Q5. Perform the indicated operation and simplify.

i. $\frac{15}{2x-3y}-\frac{4}{3y-2x}$

ii. $\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$

iii. $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

iv. $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

v. $\frac{x-2}{x^2+6x+9}-\frac{x}{2x^2-18}$

vi. $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$

Solution:

i. $\frac{15}{2x-3y}-\frac{4}{3y-2x}$

$=\frac{15}{2x-3y}-\frac{4}{-2x+3y}$

$=\frac{15}{2x-3y}-\frac{4}{-(2x-3y)}$

$=\frac{15}{2x-3y}+\frac{4}{2x-3y}$

$=\frac{15+4}{2x-3y}$

$=\frac{19}{2x-3y}$

ii. $\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$

$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$

$=\frac{1+4x+4x^2-(1-4x+4x^2)}{(1-2x)(1+2x)}$

$=\frac{1+4x+4x^2-1+4x-4x^2}{(1-2x)(1+2x)}$

$=\frac{8x}{1-4x^2}$

iii. $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

$=\left(\frac{(x^2-5)}{(x+6)(x-6)}\right)\left(\frac{(x-5)(x-6)}{(x-6)(x+6)}\right)$

$=\left(\frac{(x^2-5)-(x+5)(x-6)}{(x-6)(x+6)}\right)$

$=\left(\frac{(x^2-25)-x^2+6x-5x+30}{(x+6)(x-6)}\right)$

$=\frac{x+5}{(x+6)(x-6)}$

$=\frac{x+5}{x^2-36}$

iv. $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

$\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{(x+y)(x-y)}$

$=\frac{x(x+y)-y(x-y)-2xy}{(x+y)(x-y)}$

$=\frac{x^2+xy-xy+y^2-2xy}{(x+y)(x-y)}$

$=\frac{x^2+y^2-2xy}{(x+y)(x-y)}$

$=\frac{(x-y)^2}{(x+y)(x-y)}$

$=\frac{x-y}{x+y}$

v. $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x^2-9)}$

$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x+3)(x-3)}$

$=\frac{(x-2)2(x-3)-(x+2)(x+3)}{2(x+3)^2(x-3)}$

$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)^2(x-3)}$

$=\frac{2x^2-10x+12-x^2-5x-6}{2(x+3)^2(x-3)}$

$=\frac{x^2-15x+6}{2(x+3)^2(x-3)}$

vi. $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$

$=\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{(x^2+1)(x+1)(x-1)}$

$=\frac{(x^2+1)(x+1)-(x^2+1)(x-1)-2(x+1)(x-1)-4}{(x^2+1)(x+1)(x-1)}$

$=\frac{x^3+x^2+x+1-(x^3-x^2+x-1)-2(x^2-1)-4}{x^4-1}$

$=\frac{x^3+x^2+x+1-x^3+x^2-x+1-2x^2+2-4}{x^4-1}$

$=\frac{2}{x^4-1}$

Q6 Perform the indicated operation and simplify.

(i) $(x^2-49).\frac{5x+2}{x+7}$ **(ii)** $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}$

(iii) $\frac{x^6y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$ **(iv)** $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

(v) $\frac{x^2+xy}{y(x+y)}.\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$

Solution:

(i) $(x^2-49).\frac{5x+2}{x+7}$

$=\frac{(x+7)(x-7)}{1}\times\frac{(5x+2)}{x+7}$

$=\frac{(x+7)(x-7)(5x+2)}{x+7}$

$=(x-7)(5x+2)$

$=5x^2+2x-35x-14$

$=5x^2-33x-14$

(ii) $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}$

$=\frac{4x-12}{x^2-9}\times\frac{x^2+6x+9}{18-2x^2}$

$=\frac{4(x-3)}{(x+3)(x-3)}\times\frac{(x+3)^2}{2(9-x^2)}$

$=\frac{2(x-3)\times(x+3)\times(x+3)}{(x+3)(x-3)(3-x)(3+x)}$

$=\frac{2}{3-x}$

(iii) $\frac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

$=\frac{x^6-y^6}{x^2-y^2}\times\frac{1}{(x^4+x^2y^2+y^4)}$

$=\frac{(x^3+y^3)(x^3-y^3)}{(x+y)(x-y)}\times\frac{1}{x^4+2x^2y^2+y^4-x^2y^2}$

$=\frac{(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}{(x+y)(x-y)\{(x^2+y^2)^2-(xy)^2\}}$

$=\frac{(x^2-xy+y^2)(x^2+xy+y^2)}{\{(x^2+y^2)+(xy)\}\{(x^2+y^2)-(xy)\}}$

$=\frac{(x^2-xy+y^2)(x^2+xy+y^2)}{(x^2-xy+y^2)(x^2+xy+y^2)}$

$=\boxed{1}$

(iv) $\frac{x^2-1}{x^2+2x+1}\times\frac{x+5}{1-x}$

$=\frac{(x+1)(x-1)}{(x+1)^2}\times\frac{x+5}{1-x}$

$=\frac{-(x+1)(1-x)(x+5)}{(x+1)(x+1)(1-x)}$

$=\boxed{\frac{-(x+5)}{x+1}}$

(v) $\frac{x^2+xy}{y(x+y)}\times\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$

$=\frac{x^2+xy}{y(x+y)}\times\frac{x^2+xy}{y(x+y)}\times\frac{xy-2y}{x^2-x}$

$=\frac{x(x+y)}{y(x+y)}\times\frac{x(x+y)}{y(x+y)}\times\frac{y(x-2)}{x(x-1)}$

$=\boxed{\frac{x(x-2)}{y(x-1)}}$

Exercise 4.2

Q1 (i) If $a+b=10$ and $a-b=6$ then find the value of (a^2+b^2).

(ii) If $a+b=5$ and $a-b=\sqrt{17}$, then find the value of ab.

Solution:

(i) a+b=10, a-b=6

$(a+b)^2+(a-b)^2=2(a^2+b^2)$

$(10)^2+(6)^2=2(a^2+b^2)$

$100+36=2(a^2+b^2)$

$2(a^2+b^2)=136$

$a^2+b^2=68$

(ii) a+b=5, a-b=$\sqrt{17}$

$(a+b)^2-(a-b)^2=4ab$

$(5)^2-(\sqrt{17})^2=4ab$

$25-17=4ab$

or

$4ab=8$

$ab=2$

Q2. If $a^2+b^2+c^2=45$ and $a+b+c=-1$, find the value of $ab+be+ca$.

Solution:

$a^2+b^2+c^2=45$, $a+b+c=-1$

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$(-1)^2=45+2(ab+bc+ca)$

$1=45+2(ab+bc+ca)$

$1-45=2(ab+bc+ca)$

$2(ab+bc+ca)=-44$

$\Rightarrow ab+bc+ca=-22$

Q3. If m+n+p=10 and mn+np+mp=27, find the value of $m^2+n^2+p^2$.

Solution:

$(m+n+p)^2 \dots m^2+n^2+p^2+2(mn+np+mp)$

$(10)^2=m^2+n^2+p^2+2(27)$

$100-m^2+n^2+p^2+54$

So

$m^2+n^2+p^2=100-54=46$

Q.4 If x+y+z=78, and xy+yz+zx=59, find the value of x+y+z.

Solution:

$x^2+y^2+z^2=78,\ xy+yz+zx=59$

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$=78+2(59)$

$(x+y+z)^2=78+118=196$

$x+y+z=\pm\sqrt{196}=\pm14$

Q.5 If x+y+z=12 and $x^2+y^2+z^2=64$ find the value of xy+yz+zx.

Solution:

$x+y+z=12$

$x^2+y^2+z^2=64$

$(x+y+z)^2=x^2+y^2+z^2+2(x+y+z)$

$(12)^2=64+2(xy+yz+zx)$

$xy+yz+zx=40$

Q6. If x+y+z=7 and xy=12, then the value of $x^3+y^3+z^3$

Solution:

$x+y+z=7$

$xy=12$

$(x+y)^3=x^3+y^3+3xy(x+y)$

$(7)^3=x^3+y^3+3(12)(7)$

$343=x^3+y^3+252$

or

$x^3+y^3=343-353=91$

Q7. If 3x+4y=11 and xy=12, then find the value of $27x^3+64y^3$

Solution:

$3x+4y=11, xy=12(3x+4y)^3=(3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$

$(11)^3=27x^3+64y^3+36xy(11)$

$1331=27x^3+64y^3+4752$

or

$27x^3+64y^3=1331-4752$

$27x^3+64y^3=-3421$

Q8. If x-y=4 and xy=21, the find the value of x^3-y^3.

Solution:

$x-y=4,$

$xy=21$

$(x-y)^3=x^3-y^3-3xy(x-y)$

$(4)^3=x^3-y^3 3(21)(4)$

$64=x^3-y^3-252$

$x^3-y^3=64+252=316$

Q9. If 5x-6=13 and xy=6, then find the value if $125x^3-216y^3$

Solution:

$5x-6y=13$

$xy=6$

$(5x-6y)^3=(5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$

$(13)^3=125x^3-216y^3-1170(6)$

$2197=125x^3-216y^3-7020$

$125x^3-216y^3=2197+7020=9217$

Q10. If $x+\frac{1}{x}=3$, then find the value of $x^3+\frac{1}{x^3}$

Solution:

$x+\frac{1}{x}=3\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3x\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)(3)^3=x^3+\frac{1}{x^3}+9$

$27=x^3+\frac{1}{x^3}+9$

$x^3+\frac{1}{x^3}=27-9=18$

Q11. If $x-\frac{1}{x}$=7 then find the value of $x^3-\frac{1}{x^3}$

Solution:

$x-\frac{1}{x}=7$

$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

$(7)^3=x^3-\frac{1}{x^3}-21$

$343=x^3-\frac{1}{x^3}-21$

$x^3-\frac{1}{x^3}=343+21=364$

Q12. If $\left(3x+\frac{1}{3x}\right)$ then find the value of $\left(27x^3-\frac{1}{27x^3}\right)$

Solution:

$\left(3x+\frac{1}{3x}\right)^3=(3x)^3+\frac{1}{(3x)^3}+3(3x)\left(\frac{1}{3x}\right)\left(3x+\frac{1}{3x}\right)$

$(5)^3=27^3+\frac{1}{27x^3}+3(5)$

$125=27x^3+\frac{1}{27x^3}+15$

So

$27x^3+\frac{1}{27x^3}=125-15=110$

Q13. If $\left(5x-\frac{1}{5x}\right)=6$, then find the value of $\left(125x^3-\frac{1}{125x^3}\right)$

Solution:

$\left(5x-\frac{1}{5x}\right)^3=(5x)^3-\left(\frac{1}{5x}\right)^3-3(5x)\left(\frac{1}{5x}\right)\left(5x-\frac{1}{5x}\right)$

$(6)^3=125x^3-\frac{1}{125x^3}-3(6)$

$216=125x^3-\frac{1}{125x^3}=216+18=234$

Q14. Factorize

(i) x^3-y^3-x+y

$=(x-y)(x^2+xy+y^2)-1(x-y)$

$=(x-y)(x^2+y^2-1)$

(ii) $8x^3-\frac{1}{27y^3}$

$=(2x)^3-\left(\frac{1}{3y}\right)^3$

$=\left(2x-\frac{1}{3y}\right)\left((2x)^2+2x.\frac{1}{3y}+\left(\frac{1}{3y}\right)^2\right)$

$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\left(\frac{1}{3y}\right)^2\right)$

$=\left(2x-\frac{1}{3y}\right)\left(4x^2\frac{2x}{3y}+\frac{1}{9y^2}\right)$

Q15. Find the product, using formulas.

(i) $(x^2+y^2)(x^4-x^2y^2+y\ )$

$=(x^2+y^2)\left((x^2)^2-x^2.y^2+(y^2)^2\right)$

$=(x^2)^3+(y^2)^3=x^6+y^6$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$

$=(x^2+y^2)\left((x^2)^2-x^2.y^2+(y^2)^2\right)$

$=(x^3)^3-(y^3)^3=x^9-y^9$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

$=\left((x-y)(x^2+xy+y^2)\right)\left((x+y)(x^2-xy+y^2)\right)\left((x^2+y^2)(x^4-x^2y^2+y^2)\right)$

$=(x^3-y^3)(x^3+y^3)\left((x^2)^3+(y^2)^3\right)$

$=\left((x^3)^2-(y^3)^2\right)(x^6+y^6)$

$=(x^6-y^6)(x^6+y^6)$

$=(x^6)^2-(y^6)^2$

$=x^{12}-y^{12}$

(iv) $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)$

$=\left((2x^2-1)(4x^4+2x^2+1)\right)\left((2x^2+1)(4x^4-2-1)\right)$

$=(2x^2-1)\left((2x^2)^2+2x^2.1+(1)^2\right)(2x^2+1)(4x^4-2x^2+1)$

$=(2x^2-1)\left((2x^2)^2+2x^2.1+(1)^2\right)(2x^2+1)(2x^2+1)^2\left((2x^2)^2-(2x^2)(1)+(1)^2\right)$

$=\left(\left((2x^2)^3-(1)^3\right)\right)\left((2x^2)^3+(1)^3\right)$

$=(8x^6-1)(8x^6+1)$

$=(8x^6)^2-(1)^2$

$=64x^{12}-1$

Exercise 4.3

Q1. Express each of the following surds in the simplest form.

Solution:

(i) $\sqrt{180}$

$= \sqrt{9.4.5}$
$= \sqrt{9} \times \sqrt{4} \times \sqrt{5}$
$= 3.2.\sqrt{5}$
$= 6\sqrt{5}$

(ii) $3\sqrt{162}$

$= 3\sqrt{81.2}$
$= 3.\sqrt{81}.\sqrt{2}$
$= 3.9.\sqrt{2}$
$= 27\sqrt{2}$

(iii) $\frac{3}{4}\sqrt[3]{128}$

$= \frac{3}{4}\sqrt[3]{64.2}$
$= \frac{3}{4}.\sqrt[3]{64}.\sqrt[3]{2}$
$= \frac{3}{4}.4.\sqrt[3]{2}$
$= 3.\sqrt[3]{2}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

$= \sqrt[5]{32.3.x^5.x.y^5.y^2.z^5.z^3}$
$= \sqrt[5]{(2xyz)^5}.\sqrt{3xy^2z^3}$

$= 2xyz\sqrt[5]{3xy^2z^3}$

Q2. Simplify

Solution:

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$= \frac{\sqrt{18}}{\sqrt{6}}$
$= \sqrt{\frac{18}{6}}$
$= \sqrt{3}$

(ii) $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$= \frac{\sqrt{21 \times 9}}{\sqrt{63}}$
$= \frac{\sqrt{189}}{\sqrt{63}}$
$= \sqrt{\frac{189}{63}}$
$= \sqrt{3}$

(iii) $\sqrt[5]{243x^5y^{10}z^{15}}$

$= \sqrt[5]{3^5}.\sqrt[5]{x^5}.\sqrt[5]{y^{10}}.\sqrt[5]{z^{15}}$
$= 3xy^2z^3$

(iv) $\frac{4}{5}\sqrt[3]{125}$

$= \frac{4}{5}\sqrt[3]{5^3}$
$= \frac{4}{5}.5$
$= 4$

(v) $\sqrt{21} \times \sqrt{7} \times \sqrt{3}$

$= \sqrt{21} \times \sqrt{21}$
$= \left(\sqrt{21}\right)^2$
$= 21$

Q3. Simplify by combining similar terms.

Solution:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

$= \sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 4\sqrt{5}$
$= \sqrt{9}.\sqrt{5} - 3.\sqrt{4}.\sqrt{5} + 4.\sqrt{5}$
$= 3\sqrt{5} - 3 \times 2\sqrt{5} + 4\sqrt{5}$
$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$
$= (3 - 6 + 4)\sqrt{5}$
$= \sqrt{5}$

(ii) $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300}$

$= 4\sqrt{4 \times 3} + 5\sqrt{9 \times 3} - 3\sqrt{25 \times 3} + \sqrt{100 \times 3}$
$= 4 \times 2 \times \sqrt{3} + 5 \times 3 \times \sqrt{3} - 3 \times 5 \times \sqrt{3} + 10 \times \sqrt{3}$
$= (8 + 15 - 15 + 10)\sqrt{3}$
$= 18\sqrt{3}$

(iii) $\sqrt{3}\left(2\sqrt{3} + 3\sqrt{3}\right)$

$= \sqrt{3}.\left[(2+3)\sqrt{3}\right]$
$= \left(\sqrt{3}\right)^2(5)$
$= 3 \times 5$
$= 15$

(iv) $2\left(6\sqrt{5} - 3\sqrt{5}\right)$

$= 2.\left[(6-3)\sqrt{5}\right]$
$= 2.3\sqrt{5}$
$= 6\sqrt{5}$

Q4. Simplify

Solution:

(i) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

$= (3)^2 - \left(\sqrt{3}\right)^2$
$= 9 - 3$
$= 6$

(ii) $\left(\sqrt{5}+\sqrt{3}\right)^2$

$= \left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 + 2\sqrt{5}.\sqrt{3}$
$= 5 + 3 + 2\sqrt{15}$
$= 8 + 2\sqrt{15}$

(iii) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$= \left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2$
$= 5 - 3$
$= 2$

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

$= \left(\sqrt{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= 2 - \frac{1}{3}$

$= \frac{6-1}{3}$

$= \frac{5}{3}$

(v) $(\sqrt{x}+\sqrt{y})\left(\sqrt{x}-\sqrt{y}\right)\left(x+y\right)\left(x^2+y^2\right)$

$= \left(\left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2\right)(x+y)\left(x^2+y^2\right)$
$= (x-y)(x+y)\left(x^2+y^2\right)$
$= \left(x^2-y^2\right)\left(x^2+y^2\right)$
$= \left(x^2\right)^2 - \left(y^2\right)^2$
$= x^4 - y^4$

Exercise 4.4

Q1. Rationalize the denominator of the following.

i) $\frac{3}{4\sqrt{3}}$

$= \frac{3}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{4.3}$

$= \frac{\sqrt{3}}{4}$

ii) $\frac{14}{\sqrt{98}}$

$= \frac{14}{\sqrt{98}} \cdot \frac{\sqrt{98}}{\sqrt{98}}$

$= \frac{14\sqrt{98}}{98}$

$= \frac{1}{7}\sqrt{49 \times 2}$

$= \frac{1}{7}.7.\sqrt{2}$

$= \sqrt{2}$

iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

$= \frac{6}{\sqrt{8}\sqrt{27}} \cdot \frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}}$

$= \frac{6}{8 \times 27}.\sqrt{8}\sqrt{27}$

$= \frac{1}{36}.\sqrt{4.2}.\sqrt{9.3}$

$= \frac{1}{4 \times 9} \times 2 \times 3.\sqrt{2}\sqrt{3}$

$= \frac{1}{6}\sqrt{6}$

$= \frac{\sqrt{6}}{6}$

iv) $\frac{1}{3+2\sqrt{5}}$

$= \frac{1}{3+2\sqrt{5}} \cdot \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}$

$= \frac{3-2\sqrt{5}}{9-(4 \times 5)}$

$= \frac{3-2\sqrt{5}}{-11}$

$= -\frac{1}{11}(3-2\sqrt{5})$

v) $\frac{15}{\sqrt{31}-4}$

$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$

$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2-(4)^2}$

$= \frac{15(\sqrt{31}+4)}{31-16}$

$= \frac{15(\sqrt{31}+4)}{15}$

$= \sqrt{31}+4$

vi) $\frac{2}{\sqrt{5}-\sqrt{3}}$

$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= 2\frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= 2\frac{\sqrt{5}+\sqrt{3}}{5-3}$

$= \frac{2(\sqrt{5}+\sqrt{3})}{2}$

$= \sqrt{5}+\sqrt{3}$

vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$

$= \frac{(\sqrt{3})^2-2\sqrt{3}+(1)^2}{3-1}$

$= \frac{3-2\sqrt{3}+1}{2}$

$= \frac{4-2\sqrt{3}}{2}$

$= \frac{2(2-\sqrt{3})}{2}$

$= 2-\sqrt{3}$

viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{(\sqrt{5})^2+2\sqrt{15}+(\sqrt{3})^2}{5-3}$

$= \frac{5+3+2\sqrt{15}}{2}$

$= \frac{8+2\sqrt{15}}{2}$

$= \frac{2(4+\sqrt{15})}{2}$

$= 4+\sqrt{15}$

Q2. Find the conjugate of $x+\sqrt{y}$

i) $3+\sqrt{7}$ **iv)** $2+\sqrt{5}$

ii) $4-\sqrt{5}$ **v)** $5+\sqrt{7}$

iii) $2+\sqrt{3}$ **vi)** $4-\sqrt{15}$

vii) $7-\sqrt{6}$ **viii)** $9-\sqrt{2}$

Solution:

i) Conjugate of $3+\sqrt{7}$ is $3-\sqrt{7}$
ii) Conjugate of $4-\sqrt{5}$ is $4+\sqrt{5}$
iii) Conjugate of $2+\sqrt{3}$ is $2-\sqrt{3}$
iv) Conjugate of $2+\sqrt{5}$ is $2-\sqrt{5}$
v) Conjugate of $5+\sqrt{7}$ is $5-\sqrt{7}$
vi) Conjugate of $4-\sqrt{15}$ is $4+\sqrt{15}$
vii) Conjugate of $7-\sqrt{6}$ is $7+\sqrt{6}$
viii) Conjugate of $9-\sqrt{2}$ is $9+\sqrt{2}$

Q3.

(i) If $x=2-\sqrt{3}$, **find** $\frac{1}{x}$

(ii) If $x=4-\sqrt{17}$, **find** $\frac{1}{x}$

(iii) If $x=\sqrt{3}+2$, **find** $x+\frac{1}{x}$

Solution:

i) $x=2-\sqrt{3}$

$\frac{1}{x}=\frac{1}{2-\sqrt{3}}$

$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2} = \frac{2+\sqrt{3}}{4-3}$

$= 2+\sqrt{3}$

ii) $x=4-\sqrt{17}$

$\frac{1}{x}=\frac{1}{4-\sqrt{17}}$

$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$

$= \frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2}$

$= \frac{4+\sqrt{17}}{16-17}$

$= -(4+\sqrt{17})$

$= -4-\sqrt{17}$

iii) $x=\sqrt{3}+2$

$\frac{1}{x}=\frac{1}{\sqrt{3}+2}$

$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$= \frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$

$= \frac{\sqrt{3}-2}{3-4} = \frac{\sqrt{3}-2}{-1}$

$= -(\sqrt{3}-2)$

$= -\sqrt{3}+2$

$= 2-\sqrt{3}$

$\therefore\ x+\frac{1}{x} = \sqrt{3}+2+2-\sqrt{3} = \boxed{4}$

Q4. Simplify:

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

(ii) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

Solution:

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{2\sqrt{5}-2\sqrt{6}}{5-3}$

$= \frac{2(\sqrt{5}-\sqrt{6})}{2}$

$= \sqrt{5}-\sqrt{6}$

(ii) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

$= \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}} \cdot \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}}$

$= \frac{2-\sqrt{3}}{4-3}+\frac{2(\sqrt{5}+\sqrt{3})}{5-3}+\frac{2-\sqrt{5}}{4-5}$

$= \frac{2-\sqrt{3}}{1}+\frac{2(\sqrt{5}+\sqrt{3})}{2}+\frac{2-\sqrt{5}}{-1}$

$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$

$= 2\sqrt{5}$

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

$= \frac{2}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}-\frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{2(\sqrt{5}-\sqrt{3})}{5-3}+\frac{(\sqrt{3}-\sqrt{2})}{3-2}-\frac{3(\sqrt{5}-\sqrt{2})}{5-2}$

$= \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-(\sqrt{5}-\sqrt{2})$

$= \sqrt{5}-\sqrt{2}-\sqrt{5}+\sqrt{2} = \boxed{0}$

Q5. i) **If** $x=2+\sqrt{3}$, **find the value of** $x-\frac{1}{x}$ **and** $\left(x-\frac{1}{x}\right)^2$

ii) $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, **find the value of** $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ **and** $x^3+\frac{1}{x^3}$

$\left[\begin{array}{c} \textbf{Hint}: a^2+b^2=(a+b)^2-2ab \\ \textbf{and} \\ a^3+b^3=(a+b)^3-3ab(a+b) \end{array}\right]$

Solution:

i) $x=2+\sqrt{3}$

$\frac{1}{x}=\frac{1}{2+\sqrt{3}}$

$= \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2-\sqrt{3}}{4-3} = \boxed{2-\sqrt{3}}$

$x-\frac{1}{x} = 2+\sqrt{3}-2+\sqrt{3} = \boxed{2\sqrt{3}}$

and

$\left(x-\frac{1}{x}\right)^2 = (2\sqrt{3})^2 = 4 \times 3 = \boxed{12}$

ii) $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$\frac{1}{x}=\boxed{\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}}$

$x+\frac{1}{x}=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\left(\sqrt{5}-\sqrt{2}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2}{(\sqrt{5})^2+(2)^2}$

$= \frac{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2}{5-2}$

$= \frac{5+2+5+2}{3} = \frac{14}{3}$

$\left(x+\frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$

$x^2+\frac{1}{x^2}+2 = \frac{196}{9}$

$= \frac{196}{9}-2$

$= \frac{196-8}{9} = \frac{178}{9}$

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$

$= \left(\frac{14}{3}\right)^3-3\left(\frac{14}{3}\right)$

$= \frac{2744-378}{27} = \boxed{\frac{2366}{27}}$

Q6. Determine the rational number a and b if

$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1} = a+b\sqrt{3}$

Solution:

$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1} = a+b\sqrt{3}$

$\Rightarrow \frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = a+b\sqrt{3}$

$\Rightarrow \frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{3-1} = a+b\sqrt{3}$

$\frac{8}{2} = a+b\sqrt{3}$

$4 = a+b\sqrt{3}$

$4+0 = a+b\sqrt{3}$

By comparing both sides of the equation

$\Rightarrow a=4, \quad b=0$

REVIEW EXERCISE 4

Q1. Multiple Choice Questions. Choose the correct answers.

(i) $4x+3y-2$ **is an algebraic...............**

(a) expression **(b)** sentence
(c) equation **(d)** in equation

(ii) The degree of polynomial $4x^4+2x^{2y}$ **is ....**

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

(iii) a^3+b^3 **is equal to..............**

a) ....$(a-b)(a^2+ab+b^2)$
b) ....$(a+b)(a^2-ab+b^2)$
c) ....$(a-b)(a^2-ab+b^2)$
d) ....$(a-b)(a^2+ab-b^2)$

(iv) Conjugate of surd $a+\sqrt{b}$ **is...............**

(a) $-a+\sqrt{b}$ **(b)** $a-\sqrt{b}$
(c) $a+\sqrt{b}$ **(d)** $-a-\sqrt{b}$

(v) $(3+\sqrt{2})(3-\sqrt{2})$ **is equal to..............**

(a) 7 **(b)** -7
(c) 1 **(d)** -1

(vi) $\frac{1}{a-b}-\frac{1}{a+b}$ **is equal to.......................**

(a) $\frac{2a}{a^2+b^2}$ **(b)** $\frac{2b}{a^2-b^2}$
(c) $\frac{-2a}{a^2-b^2}$ **(d)** $\frac{-2b}{a^2+b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ **is equal to....................**

(a) $(a-b)^2$ **(b)** $(a+b)^2$
(c) $a+b$ **(d)** $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ **is equal to...................**

(a) a^2+b^2 **(b)** a^2-b^2
(c) $a-b$ **(d)** $a+b$

Answer:

(i)	c	**(ii)**	d	**(iii)**	b	**(iv)**	b
(v)	a	**(vi)**	b	**(vii)**	d	**(viii)**	c

Q2. Fill in the blanks:

i. **the degree of polynomial** $x^2y^2+3xy+y^3$ **is ....**
ii. $x^2-4=$ ....
iii. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)$(..........)
iv. $2(a^2+b^2)=(a+b)^2+(........)^2$.
v. $\left(x-\frac{1}{x}\right)^2=$..........
vi. **Order of surd** $\sqrt[3]{x}$ **is ....**
vii. $\frac{1}{2-\sqrt{3}}=$...........

Answers:
i. 3
ii. $(x+2)(x-2)$
iii. $x^2+2+\frac{1}{x^2}$
iv. $(a+b)^2(a-b)^2$
v. $x^2-2+\frac{1}{x^2}$
vi. 3
vii. $2+\sqrt{3}$

Q3. If $x+\frac{1}{x}=3$ **find these.**

i. $x^2+\frac{1}{x^2}$ ii. $x^4+\frac{1}{x^4}$

Solution:

i. $\left(x+\frac{1}{x}\right)^2=(3)^2$

$x^2+\frac{1}{x^2}+2 = 9$

$x^2+\frac{1}{x^2}=7$

ii. $\left(x^2+\frac{1}{x^2}\right)^2=(7)^2$

$x^4+\frac{1}{x^4}=47$

Q4. If $x-\frac{1}{x}=2$ **find these:**

i. $x^2+\frac{1}{x^2}$ ii. $x^4+\frac{1}{x^4}$

Solution:

i. $\left(x-\frac{1}{x}\right)^2=(2)^2$

$x^2+\frac{1}{x^2}-2=4$

$x^2+\frac{1}{x^2}=6$

ii. $\left(x^2+\frac{1}{x^2}\right)^2=(6)^2$

$x^4+\frac{1}{x^4}+2=36$

$x^4+\frac{1}{x^4}=34$

Q5. Find the value of x^3+y^3 **and** xy **if** $x+y=5$ **and** $x-y=3$

Solution:

x+y= 5, x-y=3

4xy= $(x+y)^2-(x-y)^2$

= $(5)^2-(3)^2$

xy = 4

Now,

$x^3+y^3=(x+y)^3-3xy(x+y)$

= $(5)^3-3(4)(5)$

=125-60

=65

6. If p=2 +(3)[1\2], find:

Solution:

$\frac{1}{p}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$
$=2-\sqrt{3}$

1. P+ 1/p= $2+\sqrt{3}+2-\sqrt{3}=4$
2. $p-\frac{1}{p}=2+\sqrt{3}-(2-\sqrt{3})=2\sqrt{3}$
3. $=p^2+\frac{1}{p^2}=(p+\frac{1}{p})^2-2=(4)^2-2=16-2=14$
4. $=p^2-\frac{1}{p^2}=(p+\frac{1}{p})(p-\frac{1}{p})=4(2\sqrt{3})=8(\sqrt{3})$

Q7. Simplify

Solution:

$\frac{1}{q}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$
$=\sqrt{5}-2$

1. $q+\frac{1}{q}=\sqrt{5}+2+\sqrt{5}-2=2\sqrt{5}$
2. $q-\frac{1}{q}=\sqrt{5}+2-(\sqrt{5}-2)=4$
3. $q^2+\frac{1}{q^2}=(q+\frac{1}{q})^2-2=(2\sqrt{5})^2-2=20-2=18$
4. $q^2-\frac{1}{q^2}=(q+\frac{1}{q})(q-\frac{1}{q})=(2\sqrt{5})(4)=8\sqrt{5}$

Q8. Simplify:

Solution:

$$\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}=\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2+2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$
$$=\frac{a^2+2-2\sqrt{a^2-4}+a^2-2}{a^2+2-a^2+2}$$

1. $=\frac{2a^2-2\sqrt{a^2-4}}{4}$

$$=\frac{2(a^2-\sqrt{a^2-4)}}{4}$$
$$=\frac{a^2-\sqrt{a^2-4}}{2}$$

$$=\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}=\frac{a+\sqrt{a^2-x^2}-(a-\sqrt{a^2-x^2})}{(a-\sqrt{a^2-x^2})(a+\sqrt{a^2-x^2})}$$
$$\frac{a+\sqrt{a^2-x^2}-a+\sqrt{a^2-x^2}}{(a)^2-(\sqrt{a^2-x^2})^2}$$

2. $=\frac{2\sqrt{a^2-x^2}}{a^2-(a^2-x^2)}$

$$=\frac{2\sqrt{a^2-x^2}}{a^2-a^2+x^2}$$
$$=\frac{2\sqrt{a^2-x^2}}{x^2}$$