Exercise 5.1

Q1. Factorize

(i) $2abc-4abx+4abd$

$=2ab(c-2x+d)$

(ii) $9xy-12x^2y+18y^2$

$=3y(3x-4x^2+6y)$

(iii) $-3x^2y-3x+9xy^2$

$=-3x(xy+1-3y^2)$

(iv) $5ab^2c^3-10a^2b^3c-20a^3bc^2$

$=5abc(bc^2-2b^2-4a^2c)$

(v) $3x^3y(x-3y)-(7x^2y^2(x-3y))$

$=(x-3y)(3x^3y-7x^2y^2)$

$=(x-3y)x^2y(3x-7y)$

$=x^2y(x-3y)x^2y(3x-7y)$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

$=(x^2+5)(2xy^3+8xy^2)$

$=(x^2+5)(2xy^2)(y+4)$

$=2xy^2(x^2+5)(y+4)$

Q.2

(i) $5ax-3ay-5bx+3by$

$=5ax-5bx-3ay+3by$

$=5x(a-b)(5x-3y)$

(ii) $3xy+2y-12x-8$

$=3xy-12x+2y-8$
$=3x(y-4)+2(y-4)$
$=(y-4)(3x+2)$

(iii) $x^3+3xy^2-2x^2-6y^3$

$=x(x^2+3y^2)-2y(x^2+3y^2)$
$=(x^2+3y^2)(x-2y)$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

$=x^2z-y^2z+y^2x-z^2x$
$=x^2z-z^2x+y^2x-y^2z$
$=xz(x-z)+y^2x=y^2z$

$=(x-z)(xz+y^2)$

Q.3

(i) $144a^2+24a+1$

$=144a^2+12a+12a+1$
$=12a(12a+1)+1(12a+1)$
$=(12a+1)(12a+1)=(12a+1)^2$

(ii) $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

$=\left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)\frac{b}{a}+\left(\frac{b}{a}\right)^2$
$=\left(\frac{a}{b}-\frac{b}{a}\right)^2$

(iii) $(x+y)^2-14z(x+y)+49z^2$

$=(x+y)^2-2(x+y)(7z)+(7z)^2$
$=(x+y-7z)^2$

(iv) $12x^2-36x+27$

$=3(4x^2-12x+9)$
$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$
$=3(2x-3)^3$

Q4.

(i) $3x^2-75y^2$

$=3(x^2-25y^2)$
$=3\left[(x^2)-(5y)\right]$
$=3(x+5y)(x-5y)$

(ii) $x(x-1)-y(y-1)$

$=x^2-x-y^2+y$
$=x^2-y^2-x+y$
$=(x+y)(x-y)-1(x-y)$
$=(x-y)(x+y-1)$

(iii) $128am^2-242an^2$

$=2a(64m^2-121n^2)$
$=2a\{(8m^2)-(11n^2)\}$
$=21(8m+11n)(8m-11n)$

(iv) $3x-243x^3$

$=3x(1-81x^2)$
$=3x((1)^2-(9x^2))$
$=3x(1+9x)(1-9x)$

Q.5

(i) x^2-y^2-6y-9

$=x^2-(y^2+6y+9)$
$=x^2-((y^2)+2(y)(3)+(3)^2)$
$=x^2-(y+3)^2$
$=(x+(y+3))(x-(y+3))$
$=(x+y+3)(x-y-3)$

(ii) x^2-a^2+2a-1

$=x^2-(a^2-2a+1)$
$=x^2-((a)^2-2(a)(1)+(1)^2)$
$=x^2-(a-1)^2$
$=(x)^2-(a-1)^2$
$=(x+(a-1))(x-(a+1))$
$=(x+a-1)(x-a+1)$

(iii) $4x^2-y^2-4x-2y+3$

$=4x^2-(y^2+2y+1)$
$=(2x)^2-(y+1)^2$
$=[2x+(y+1)][2x-(y+1)]$
$=(2x+y-1)(2x-y-1)$

(iv) $x^2-y^2-4x-2y+3$

$=x^2-4x-y^2-2y+3$
$=x^2-4x-y^2-2y+3$
$=x^2-4x+4-y^2-2y-1$

$=x^2-4x+4-(y+1)^2$
$=((x-2)+(y+1))(x-2-y-1)$
$=(x+y-1)(x-y-3)$

(v) $25x^2-10x+1-36z^2$

$=(5x)^2-2(5x)(1)+(1)^2-36z^2$
$=(5x-1)^2-(6z)^2$
$=((5x-1)^2+6z)((5x-1)-6z)$
$=(5x-1+6z)(5x-1-6z)$

(vi) $x^2-y^2-4xz+4z^2$

$=x^2-4xz+4z^2-y^2$
$=(x)^2-2(x)(2z)+(2z)^2-y^2$
$=(x-2z)^2-(y)^2$
$=((x-2z)^2-y)((x-2z)y)$
$=(x-2z+y)(x-y-2z)$
$=(x+y-2y)(x-y-2z)$

Exercise 5.2

Factorize

Q.1 (i) $x^4+\frac{1}{x^4}-3$

$=x^4+\frac{1}{x^4}-2-1$
$=x^4-2+\frac{1}{x^4}-1$
$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$
$=\left(\left(x^2-\frac{1}{x^2}\right)+1\right)\left(\left(x^2-\frac{1}{x^2}\right)-1\right)$
$=\left(x^2-\frac{1}{x^2}+1\right)\left(x^2-\frac{1}{x^2}-1\right)$

(ii) $3x^4+12y^4$

$=3\left(x^4+4y^4\right)$
$=3\left(x^4+4x^2y^2+4y^4-4x^2y^2\right)$
$=3\left(x^2+2y^2\right)^2-4x^2y^2$
$=3\left((x^2+2y^2)^2+(2xy)^2\right)$
$=3\left(\left(x^2+2y^2\right)+2xy\right)\left(\left(x^2+2y^2\right)-2xy\right)$
$=3\left(x^2+2xy+2y^2\right)\left(x^2-2xy+2y^2\right)$

(iii) $a^4+3a^2b^2+ab^4$
$=a^4+4a^2b^2-a^2b^2+4b^2$
$=a^4+4a^2b^2+4b^2-a^2b^2$
$=\left(a^2+2b^2\right)^2-(ab)^2$
$=\left(a^2+2b^2+ab\right)\left(a^2+2b^2-ab\right)$
$=\left(a^2+ab+2b^2\right)\left(a^2-ab+2b^2\right)$

(iv) $4x^4+81$
$=\left(2x^2\right)^2+(9)^2+36x^2-36x^2$
$=\left(2x^2+9\right)^2-(6x)^2$
$=\left(2x^2+9+6x\right)\left(2x^2+9-6x\right)$
$=\left(2x^2+6x+9\right)\left(2x^2-6x+9\right)$

(v) x^4+x^2+25
$=x^4+10x^2+25-9x^2$
$=\left(x^2\right)^2+2\left(x^2\right)5-9x^2$
$=\left(x^2+5\right)^2-(3x)^2$
$=\left(x^2+5+3x\right)\left(x^2+5-3x\right)$
$=\left(x^2+2x+4\right)\left(x^2-2x+4\right)$

(vi) x^4+4x^2+16
$=x^4+8x^2+16-4x^2$
$=\left(x^2+4\right)^2-(2x)^2$
$=\left(x^2+4+2x\right)\left(x^2+4-2x\right)$
$=\left(x^2+2x+4\right)\left(x^2+2x-4\right)$

Q.2 (i) $x^2+14x+48$

$=x^2+8x+6x+48$
$=x(x+8)+6(x+8)$
$=(x+8)(x+6)$

(ii) $x^2-21x+108$

$=x^2-12x-9x+108$
$=x(x-12)-9(x-12)$
$=(x-12)(x-9)$

(iii) $x^2-11x-42$

$=x^2-14x+3x-42$
$=x(x-14)+3(x-14)$
$=(x-14)(x+3)$

(iv) $x^2+x-132$

$=x^2+12x-11x-132$
$=x(x+12)-11(x+12)$
$=(x+12)(x-11)$

Q.3 (i) $4x^2+12x+5$

$=4x^2+10x+2x+5$
$=2x(2x+5)+1(2x+5)$
$=(2x+5)(2x+1)$

(ii) $30x^2+7x-15$

$=30x^2+25x-18x-15$
$=5x(6x+5)-3(6x+5)$
$=(6x+5)(5x-3)$

(iii) $24x^2-65x+21$

$=24x^2-56x-9x+21$
$=8x(3x-7)-3(3x-7)$
$=(3x-7)(8x-3)$

(iv) $5x^2-16x-21$

$=5x^2-21x+5x-21$

$=x(5x-21)+1(5x-21)$
$=(5x-21)(x+1)$

(v) $4x^2-17xy+4y^2$

$=4x^2-16xy-xy+4y^2$
$=4x(x-4y)-y(x-4y)$
$=(x-4y)(4x-y)$

(vi) $3x^2-38xy-13y^2$

$=3x^2-39xy+xy-13y^2$
$=3x(x-13y)+y(x-13y)$
$=(x-13y)(3x+y)$

(vii) $5x^2+33xy-14y^2$

$=5x^2+35xy-2xy-14y^2$
$=5x(x+7y)+y(x+7y)$
$=(x+7y)(5x+y)$

(viii) $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$

let $5x-\frac{1}{x}=y$
$=y^2+4y+4$
$(y+2)^2=(y+2)(y+2)$
by putting value of $y=5x-\frac{1}{x}$
$=\left(5x-\frac{1}{x}+2\right)\left(5x-\frac{1}{x}+2\right)$

Q.4 (i) $\left(x^2+5x+4\right)\left(x^2+5x+6\right)-3$

let $x^2+5x=y$
$(y+4)(y+6)-3$
$=y^2+6y+4y+24-3$
$=y^2+10y+21$
$=y^2+7y+3y+21$
$=y(y+7)+3(y+7)$
$=(y+7)(y+3)$
by putting value of $y=x^2+5x$
$=\left(x^2+5x+7\right)\left(x^2+5x+3\right)$

(ii) $\left(x^2-4x\right)\left(x^2-4x-1\right)-20$

let $x^2-4x=y$
$=y(y-1)-20$
$=y^2-y-20$
$=y^2-5y+4y-20$
$=y(y-5)+4(y-5)$
$=(y-5)(y+5)$
by putting value of $y=x^2-4x$
$=\left(x^2-4x-5\right)\left(x^2-4x+4\right)$
$=\left(x^2-5x+x-5\right)\left(x^2-2x-2x+4\right)$
$=\left(\left(x(x-5)+1(x-5)\right)\right)\left(x(x-2)-2(x-2)\right)$
$=\left((x-5)(x+1)\right)\left((x-2)(x-2)\right)$
$=(x-5)(x+1)(x-2)^2$

(iii) $(x+2)(x+3)(x+4)(x+5)-15$

By using commutative property of addition
As $2+5=3+4$
$=(x^2+7x+10)\left(x^2+7x+12\right)-15$
let $x^2+7x=y$
$=(y+10)(y+12)-15$

$=y^2+22y+120-15$
$=y^2+22y+105$
$=y^2+15y+7y+105$
$=y(y+15)+7(y+15)$
$=(y+15)(y+7)$
By putting value of $y=x^2+7x$
$=\left(x^2+7x+15\right)\left(x^2+7x+7\right)$

(iv) $(x+4)(x-5)(x+6)(x-7)-504$

By using commutative property of subtraction
As $4-5=6-7$
$=\left(x^2-x-20\right)\left(x^2-x-42\right)-504$
let $x^2-x=y$
$=(y-20)(y-42)-504$
$=y^2-42y-20y+840-504$
$=y^2-62y+336$
$=y(y-56)(y-6)$
by putting value of $y=x^2-x$
$=\left(x^2-x-56\right)\left(x^2-x-6\right)$
$=\left(x^2-8x+7x-56\right)\left(x^2-3x+2x-6\right)$
$=\left(x(x-8)+7(x-8)\right)\left(x(x-3)+2(x-3)\right)$
$=(x-8)(x+7)(x-3)(x+2)$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

By using commutative property of multiplication
As $(1)(6)=(2)(3)$
$=\left(x^2+7x+6\right)\left(x^2+5x+6\right)-3x^2$
$=\left(x^2+6+7x\right)\left(x^2+6+5x\right)-3x^2$
let $x^2+6=y$
$=(y+7x)(y+5x)-3x^2$
$=y^2+5xy+7xy+35x^2-3x^2$
$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$
$=y(y+8x)+4x(y+8x)$
$=(y+8x)(y+4x)$
By putting value of $y=x^2+6$
$=\left(x^2+6+8x\right)\left(x^2+6+4x\right)$
$=x\left(x+8+\frac{6}{x}\right).x\left(x+4+\frac{6}{x}\right)$
$=x^2\left(x+\frac{6}{x}+8\right)\left(x+\frac{6}{x}+4\right)$

Q.5 (i) $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$
$=x^3-3.x^2.4+3.x.4^2-4^3$
$=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$
$=(2x)^3+3.(2x)^2.5+3.(2x).5^2+5^3$
$=(2x-5y)^3$

(iii) $x^3-18x^2+108x-216$
$=x^3-3x^2.6+3.x.6^2-6^3$
$=(x-6)^3$

(iv) $8x^3-125y^3-60x^2y+150xy^2$
$=8x^3-60x^2y+150xy^2-125y^3$
$=(2x)^3-3.(2x)^2.5y+3.(2x).(5y)^2-(5y)^3$
$=(2x-5y)^3$

Q.6 (i) $27+8x^3$

$=(3)^2+(2x)^3$
$=(3+2x)\left(3^2-3.2x+(2x)^2\right)$
$=(3+2x)\left(9-6x+4x^2\right)$

(ii) $125x^3-216y^3$
$=(5x)^3-(6y)^3$
$=(5x-6y)\left((5x)^2+5x.6y+(6y)^2\right)$
$=(5x-6y)\left(25x^2+30xy+36y^2\right)$

(iii) $64x^3+27y^3$
$=(4x)^3+(3y)^3$
$=(4x+3y)\left((4x)^2-4x.3y+(3y)^2\right)$
$=(4x+3y)\left(16x^2-12xy+9y^2\right)$

(iv) $8x^3+125y^3$
$=(2x)^3+(5y)^3$
$=(2x+5y)\left((4x)^2-2x.5y+(5y)^2\right)$
$=(2x+5y)\left(4x^2-10xy+25y^2\right)$

Exercise 5.3

Q.1 Use Remainder theorem to find the remainder when

(i) $3x^3-10x^2+13x-6$ **is divided by** $(x-2)$

Solution

Let $p(x)=3x^3-10x^2+13x-6$

When $p(x)$is divided by $x-2$

The remainder $R=p(2)$

$p(2)=3(2)^3-10(2)^2+13(2)-6$
$p(2)=24-40+26-9=4$

Therefore remainder = 4

(ii) $4x^3-4x+3$ **is divided by** $(2x-1)$

Solution

Let $p(x)=4x^3-4x+3$

When $p(x)$ is divided by $2x-1$

The remainder $R=p\frac{1}{2}$

$p\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3-4\left(\frac{1}{2}\right)+3$
$p\left(\frac{1}{2}\right)=4\left(\frac{1}{8}\right)-4\left(\frac{1}{2}\right)+3$
$p\left(\frac{1}{2}\right)=\frac{1}{2}-\frac{4}{2}+3$
$p\left(\frac{1}{2}\right)=\frac{1-4+6}{2}=\frac{3}{2}$

Therefore remainder = $\frac{3}{2}$

(iii) $6x+2x^3-x+2$ **divided by** $(x+2)$

Solution:

Let $p(x)=6x^4+2x^3-x+2$

When $p(x)$ is divided by $x+2$

The remainder $R=p(-2)$

$p(-2)=6(-2)^4+2(-2)^3-2+2$
$p(-2)=96-16+2+2=84$

Therefore remainder = 84

(iv) $(2x-1)^3+6(3+4x)^2-10$ **divided by** $(2x+1)$

Solution:

Let $p(x)=(2x-1)^3+6(3+4x)^2-10$

When $p(x)$ is divided by $2x+1$

The remainder $R=p\left[\frac{-1}{2}\right]$

$p\left(\frac{-1}{2}\right)=\left[2\left(\frac{-1}{2}\right)-1\right]^3+6\left[3+4\left(\frac{-1}{2}\right)\right]^2-10$
$p\left(\frac{-1}{2}\right)=(-1-1)^3+6(3-2)^2-10$
$p\left(\frac{-1}{2}\right)=(-2)^3+6(1)^2-10$
$p\left(\frac{-1}{2}\right)=-8+6-10=-12$

Therefore remainder = -12

(v) $x^3-3x^2+4x-14$ **divided by** $(x+2)$

Solution

Let $p(x)=x^3-3x^2+4x-14$

When $p(x)$ is divided by $x+2$

The remainder $R=p(-2)$

$p(-2)=(-2)^3-3(-2)^2+4(-2)-14$
$p(-2)=-8-12-8-14=-42$

Therefore remainder = -42

Q2. (i) If $(x+2)$ **is a factor of** $3x^2-4kx-4k^2$ **then find the value(s) of** k**.**

Solution:

Let $p(x)=3x^2-4kx-4k^2$

As $x+2=x-(-2)$ is a factor of $p(x)$

So $p(-2)=0$

$3(-2)^2-4k(-2)-4k^2=0$
$12+8k-4k^2=0$
$or 3+2k-k^2=0$
$3+3k-k-k^2=0$
$3(1+k)-k(1+k)=0$
$(1+k)(3-k)=0$
$1+k=0:3-k=0$
$k=-1:k=3$
$\Rightarrow k=-1,3$

(ii) If $(x-1)$ **is a factor of** $x^3-kx^2+11x-6$**, then find the value(s) of** k**.**

Solution:

Let $p(x)=x^3-kx^2+11x-6$

As $x-1$ is a factor of $p(x)$ we have $p(1)=0$

$(1)^3-k(1)^2+11(1)-6=0$
$1-k+11-6=0$
$-k+6=0$
$\Rightarrow k=6$

Q3. Without actual long division determine whether

(i) $(x-2)$ **and** $(x-3)$ **are factors of** $p(x)=x^3-12x^2+44x-48$

Solution:

The remainder for $x-2$ is

$p(2)=(2)^3-12(2)^2+44(2)-48$
$p(2)=8-48+88-48$
$p(2)=0$

Since remainder =0 therefore $x-2$ is a factor of $p(x)$

The remainder for $x-3$ is

$p(3)=(3)^3+2(3)^2+44(3)-48$
$p(3)=3$

Since remainder is not equal to zero therefore $x-3$ is not a factor.

(ii) $(x-2),(x+3)$ and $(x-4)$ are factors of $q(x)=x^3+2x^2-5x-6$

Solution:

The remainder for $x-2$ **is**

$p(2)=(2)^3+2(2)^2-5(2)-6$
$p(2)=8+8-10-6$
$p(2)=0$

Since remainder=0 therefore $x-2$ is a factor of $q(x)$.

The remainder for $x+3$ is

$p(-3)=(-3)^3+2(-3)^2-5(-3)-6$
$p(-3)=-27+18+15-6$
$p(-3)=0$

Since remainder=0 therefore $x+3$ is a factor of $q(x)$.

The remainder for $x-4$ is

$p(4)=(4)^3+2(4)^2-5(4)-6$
$p(4)=64+32-20-6$
$p(4)=70$

Not a factor as remainder is not equal to zero.

Q4. For what value of m is the polynomial $p(x)=4x^3-7x^2+6x-3m$ **exactly divisible by** $x+2$**?**

Solution:

As $p(x)$ is exactly divisible $x+2$ therefore remainder=0

$4(-2)^3-7(-2)^2+6(-2)-3m=0$
$-72-3m=0$
$-24-m=0$
$m=-24$

Q5. Determine the value of k **if** $p(x)=kx^3+4x^2+3x-4$ **and** $q(x)=x^3-4x+k$ **leaves the same remainder when divided by** $(x-3)$**.**

Solution:

$p(3)=k(3)^3+4(3)^2+3(3)-4$
$p(3)=27k+41$

Now

$q(3)=(3)^2-4(3)+k$
$q(3)=15+k$

According to given condition:

$27k+41=15+k$
$k=-1$

Q6. The reminder after dividing the polynomial $p(x)=x^3+ax^2+7$ **by** $(x+1)$ **is** $2b$**. Calculate the value of** a **and** b **if this expression leaves a remainder of** $(b+5)$ **on being divided by** $(x-2)$**.**

Solution:

$p(x)=x^3+ax^2+7$

When $p(x)$ is divided by $x+1$, then the remainder $p(-1)=0$

$p(-1)=(-1)^3+a(-1)^2+7$
$p(-1)=-1+a+7$
$p(-1)=a+6$

As given reminder = $2b$

$a+6=2b$
$a-2b=-6$ ....(i)

When $p(x)$is divided by $(x-2)$, then the remainder $p(2)=0$

$p(2)=(2)^3+a(2)^2+7$
$p(2)=4a+15$

As given reminder $=b+15$

Therefore, calculated remainder = given remainder

$4a+15=b+5$

$4a-b=-10$ ....(ii)

Multiply eq(ii) by 2 and subtract from eq(i):

$a-2b=-6$
$-8a\mp 2b=\mp 20$
$-7a=14$

Put $a=-2$ in eq(i), we get

$-2-2b=-6$

$-2b=-4$ or $b=2$

$a=-2,b=-2$

Q7. The polynomial $x^3+lx^2+mx+24$ **has factor** $(x+4)$ **and it leaves a remainder of 36 when divided by** $(x-2)$**. Find the value of** l **and** m**.**

Solution:

Let $p(x)=x^3+lx^2+mx+24$

As $x+4$ is a factor of $p(x)$

i.e. $(4)^3+l(-4)^2+m(-4)+24=0$

$4l-m=10$ ....(i)

When $p(x)$is divided by $x-2$

When remainder is $p(2)$

Then $p(2)=36$

$x^3+lx^2+mx+24=36$
$8+4l+2m+24=36$
$4l+2m=4$

$2l+3m=2$ ....(ii)

By adding eq(i) and eq(ii) we get:

$6l=12$
$l=2$

Putting $l=2$ in eq (i)

$8-m=10$
$-m=2$
$m=-2$

So, $l=2,m=-2$.

Q8. The expression lx^3+mx^2-4 **leaves remainder of -3 and 12 when divided by** $(x-1)$ **and** $(x+2)$ **respectively. Calculate the values of** l **and** m**.**

Solution:

When $p(x)$ is divided by $x-1$ the remainder

$l(1)^3+m(1)^2-4=-3$
$l+m-4=-3$

$l+m=1$ ....(i)

When $p(x)$ is divided by $x+2$ the remainder

$p(-2)=12$
$l(-2)^3+m(-2)^2-4=12$
$-8l+4m-4=12$
$-8l+4m=16$

$-2l+m=4$ ....(ii)

Subtracting eq(ii) from eq(i):

$3l=-3$
$l=-1$

Putting $l=-1$ in eq(i):

$-1+m=1$
$m=2$

So $l=-1,m=2$.

Q9. The expression $ax^3-9x^2+bx+3a$ **is exactly divisible** x^2-5x+6**. Find the values of** a **and** b**.**

Solution:

Let $p(x)=ax^3-9x^2+bx+3a$

$q(x)=x^2-5x+6$
$q(x)=x^2-3x-2x+6$
$q(x)=x(x-3)-2(x-3)$
$q(x)=(x-3)(x-2)$

As $p(x)$ is exactly divisible by $q(x)$. So, $p(x)$ is exactly divisible by $x-2$ and $x-3(x=2,x=3)$.

Hence $p(2)=0$

And $p(3)=0$

$p(2)=2(2)^3-9(2)^2+b(2)+3a=0$
$8a-36+2b+3a=0$

$11a+2b=36$ ....(i)

$p(3)=2(3)^3-9(3)^2+b(3)=3a=0$
$27a-81+3b+3a=0$
$30a+3b=81$

$10a+b=27$ ....(ii)

By multiplying eq(ii) by 2 and subtracting from eq(i):

$11a+2b=36$
$\pm 20a\pm 2b=\pm 54$
$-9a=-18$

$a=2$

Putting in eq(ii):

$20+b=27$
$b=7$

So, $a=2$ and $b=7$.

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

1. x^3-2x^2-x+2

Solution:

Let $p(x)=x^3-2x^2-x+2$

Possible factors of constant zeros of $p(x)$ are $p=\pm1,\pm2$ and possible factors of leading coefficient 1 are $q=\pm1$ Thus the expected zeros of $p(x)$ are

$\frac{p}{q}=\pm1,\pm2$

Now $p(1)=(1)^3-2(1)^2-1+2$

$p(1)=1-2-1+2$
$p(1)=0$

Hence $x=1$ is a zero of $p(x)$ therefore $x-1$ is a factor of $p(x)$

$p(-1)=(-1)^3-2(-1)^2-(-1)+2$
$p(-1)=-1-2+1+2$
$p(-1)=0$

Hence $x=-1$ is a zero of $p(x)$ therefore $x+1$ is a factor of $p(x)$

$p(2)=2^3-2(2)^2-2+2$
$p(2)=8-8-2+2$
$p(2)=0$

Hence $x=2$ is a zero of $p(x)$ and therefore $x-2$ is a factor of $p(x)$

Hence required factors are $(x-1)(x+1)(x+2)$.

2. $x^3-x^2-22x+40$

Solution:

Let $p(x)=x^3-x^2-22x+40$

Possible factors of constant term 40 are

$p=\pm1,\pm2,\pm3,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

And those of leading coefficient 1 are

Thus the possible zeros of $p(x)$

$\pm1,\pm2,\pm3,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Now $p(1)=(1)^3-(1)^2-22(1)+40$

$p(1)=1-1-22+40$
$p(1)=18\neq0$

So, $x-1$ is not a factor of $p(x)$

Now $p(-1)=(-1)^3-(-1)^2-22(-1)+40$

$p(-1)=-1-1+22+40$
$p(-1)=60\neq0$

So, $x+1$ is not a factor of $p(x)$

Now $p(-2)=(-2)^3-(-2)^2-22(2)+40$

$p(-2)=-8-4-44+40$
$p(-2)=-8\neq0$

So, $x+2$ is not a factor of $p(x)$

Now $p(2)=(2)^3-(2)^2-22(-2)+40$

$p(2)=8-4+44+40$
$p(2)=72\neq0$

So, $x-2$ is not a factor of $p(x)$

Now $p(-4)=(-4)^3-(-4)^2-22(-4)+40$

$p(-4)=-64-16+88+40$
$p(-4)=48\neq0$

So, $x+4$ is not a factor of $p(x)$

Now $p(4)=4^3-(4)^2-22(4)+40$

$p(4)=64-16-88+40$
$p(4)=0$

So, $x-4$ is a factor of $p(x)$

Now $p(5)=(5)^3-(5)^2-22(5)+40$

$p(5)=125-25-110+40$
$p(5)=30\neq0$

$x-5$ is not a factor of $p(x)$

Now $p(-5)=(-5)^3-(-5)^2-22(-5)+40$

$p(-5)=-125-25+110+40$
$p(-5)=0$

So, $x+5$ is a factor of $p(x)$

Hence required factors are $(x-4)$ and $(x+5)$.

3. Factorize $x^3-6x^2+3x+10$ cubic polynomials by factor theorem.

Solution:

Let $p(x)=x^3-6x^2+3x+10$

Possible factors of constant term 10 are

$\pm1,\pm2,\pm5,\pm10$

Now $p(-1)=(-1)^3-6(-1)^2-3(-1)+10$

$p(-1)=-1-6+3+10$
$p(-1)=6\neq0$

So, $x+1$ is not a factor

Now $p(-2)=(-2)^3-6(-2)^2+3(-2)+10$

$p(-2)=-8-24-6+10$
$p(-2)=-28\neq0$

So, $x+2$ is not a factor

Now $p(2)=(2)^3-6(2)^2+3(2)+10$

$p(2)=8-24+6+10$
$p(2)=0$

So, $x-2$ is a factor

Now $p(-4)=(-4)^3-6(-4)^2+3(-4)+10$

$p(-4)=-64-96-12+10$
$p(-4)=162\neq0$

So, $x+4$ is not a factor

Now $p(-5)=(-5)^3-6(-5)^2+3(-5)+10$

$p(-5)=-125-150-15+10$
$p(-5)=280\neq0$

So, $x+5$ is not a factor

Now $p(5)=(5)^3-6(5)^2+3(5)+10$

$p(5)=125-150+15+10$
$p(5)=0$

So, $x-5$ is a factor

Hence required factors are $(x-2)$and $(x-5)$.

4. $x^3+x^2-10x+8$

Solution:

Let $p(x)=x^3+x^2-10x+8$

Possible factors of constant term 8 are

$p=\pm1,\pm2,\pm4,\pm8$

Thus the expected zeros of $p(x)$ are

$\frac{p}{q}=\pm1,\pm2,\pm4,\pm8$ $\frac{p}{q}=\pm1,\pm2,\pm4,\pm8$

$p(1)=(1)^3+(1)^2-10(1)+8$
$=1+1-10+8=0$

So, $x-1$ is a factor of $p(x)$

$p(-1)=(-1)^3+(-1)^2-10(-1)+8$
$=-1+1+10+8=18\neq0$

$x+1$ is not a factor of $p(x)$

$p(2)=(2)^3+(2)^2-10(2)+8$
$=8+4-20+8=0$

$x+2$ is not a factor of $p(x)$

$p(-4)=(-4)^3+(4)^2-10(4)+8$
$=-64+16-40+8=0$

$x-4$ is not a factor of $p(x)$

$p(-4)=(-4)^3+(4)^2-10(4)+8$
$=-64+16-40+8=0$

So, $x+4$ is a factor of $p(x)$

Hence required factors are $(x-1)(x-2)(x+4)$.

Q5. x^3-2x^2+5x+6

Solution:

Let $p(x)=x^3-2x^2+5x+6$

Possible factors of constant term 6 are

$p=\pm1,\pm2,\pm3,\pm6$

Thus the possible zeros of p(x) are

$\frac{p}{q}=\pm1,\pm2,\pm3,\pm6$

$p(1)=1^3-2(1)^2-5(1)+6$

$=-1-2+5+6=8\neq0$

So, $x-1$ is a factor of $p(x)$

$p(-1)=1^3-2(1)^2-5(1)+6$
$=-1-2+5+6=8\neq0$

$x+1$ is not a factor of $p(x)$

$p(-2)=-2^3-2(2)^2-5(-2)+6$
$=-8-8+10+6=0$

$x+2$ is a factor of $p(x)$

$p(3)=3^3-2(3)^2-5(3)+6$
$=27-18+15+6=0$

So, $x-3$ is a factor of $p(x)$

Hence required factors are $(x-1)(x-3)(x+2)$.

Q6. $x^3+5x^2-2x-24$

Solution:

Let $p(x)=x^3-x^2-22x+40$

Possible factors of constant term 40 are

$p=\pm1,\pm2,\pm3,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

And those of leading coefficient 1 are Thus the possible zeros of $p(x)$

$\pm1,\pm2,\pm3,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Now $p(1)=1,-1,-22,+44=18\neq0$

So, $x-1$ is not a factor of $p(x)$

$p(x)=-1^3-1^2-22(-1)+40$
$=8-4-44+40=0$
$=-1-1+22+40=60\neq0$

$x+1$ is not a factor of $p(x)$

$p(2)=-2^3-(-2)^2-22(2)+40$
$=8-4-44+40=0$

$x-2$ is a factor of $p(x)$

$(-2)^2-22(-2)+40$
$=8-4+44+40=72\neq0$

$x+2$ is not a factor of $p(x)$

$p(4)=4^3-(4)^2-22(4)+40$
$=64-16-88+40=48\neq0$

$x-4$ is not a factor of $p(x)$

$p(-4)=-4^3-(4)^2-22(4)+40$

$x+4$ is not a factor of $p(x)$

$p(5)=5^3-(5)^2-110+40$
$=125-25-110+40=30\neq0$

$x-5$ is not a factor of $p(x)$

$p(-5)=-5^3-(-5)^2-22(-5)+40$
$=125-25+110+40=0$

So, $x+5$ is a factor of $p(x)$

Hence required factors are $(x-2)(x-4)(x+5)$.

Q7. $3x^3-x^2-12x+4$

Solution:

Let $p(x)=3x^3-x^2-12x+4$

Possible factors of the constant term 4 are

P = $\pm1,\pm2,\pm4$

And those of the leading coefficient 3 are q $=\pm1,\pm3$

Thus the possible zeros of $p(x)$

$p(-1)=3(-1)^3-(-1)^2-12(-1)+4$
$=3-1+12+4=12\neq0$

So, $x+1$ is not a factor of $p(x)$

$p(-2)=3(-2)^3-(-2)^2-12(-2)-24$
$=-24-4+4+4=0$

So, $x+2$ is a zero of $p(x)$

$p(\frac{1}{3})=3(\frac{1}{3})^3-(\frac{1}{3})^2-12(\frac{1}{3})+4$
$=\frac{1}{9}-\frac{1}{9}-4+4=0$

So, $3x-1$ is zero of $p(x)$

Hence $(x-2),(x+2)$ and $(3x-1)$ are factors of $p(x)$

Hence required functions are $(x-2)(x+2)(3x-1)$.

Q8. $2x^3+x^2-2x-1$

Solution:

Let $p(x)=2x^3+x^2-2x-1$

Possible factors of the constant term -1 are $p=\pm1$ and those of leading coefficient 2 are $q=\pm1,\pm2$

Thus the possible zeros $p(x)$ are $\frac{p}{q}=\pm1,\pm\frac{1}{2}$

$p(1)=2(1)^3+(1)^2-2(1)-1$

$=2+1-2-1=0$

So, x-1 is a zero of $p(x)$

$p(\frac{1}{2})=2(\frac{1}{2})^3+(\frac{1}{2})^2-2(\frac{1}{2})-1$
$=\frac{1}{4}+\frac{1}{4}-11=0$

So, $x=-\frac{1}{2}$ is a zero of $p(x)$

Hence, $x+1$ and $2x+1$ are factors of $p(x)$

Hence, required factors are $(x+1)(x-1)(2x+1)$.

Review Exercise 5

Q1. Multiple choice questions. Choose the correct answers.

1. The factors of x^2-5x+6 are ....

(a) $x+1, x-6$ (b) $x-2, x-3$
(c) $x+6, x-1$ (d) $x+2, x+3$

2. Factors of $8x^3-27y^3$ are ....

(a) $(2x+3y),(4x^2+9y^2)$ (b) $(2x+3y),(4x^2-9y^2)$
(c) $(2x+3y),(4x^2-6xy+9y^2)$ (d) $(2x-3y),(4x^2+6xy+9y^2)$

3. Factors of $3x^2-x-2$ are ....

(a) $(x+1),(3x-2)$ (b) $(x+1),(3x+2)$
(c) $(x-1),(3x+2)$ (d) $(x-1),(3x-2)$

4. Factors of a^4-x-2 are ....

(a) $(a-b),(a+b),(a^2+4b^2)$ (b) $(a^2-2b^2),(a^2+2b^2)$
(c) $(a-b),(a+b),(a^2-4b^2)$ (d) $(a-2b),(a^2-2b^2)$

5. What will be added to complete the square of $9a^2-12ab$?

(a) $-16b^2$ (b) $16b^2$
(c) $4b^2$ (d) $-4b^2$

6. Find m so that x^2+4x+m is a complete square...
(a) 8 (b) -8
(c) 4 (d) 16

7. Factors of $5x^2-17xy-12y^2$ are....
(a) $(x+4y),(5x+3y)$ (b) $(x-4y),(5x-3y)$

(c) $(x-4y),\ (5x+4y)$ (d) $(x+4y),\ (5x-4y)$

8. Factors of $27x^3-\frac{1}{x^3}$

(a) $\left(3x-\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$ (b) $\left(3x+\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$

(c) $\left(3x-\frac{1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$ (d) $\left(3x+\frac{1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$

ANSWERS:

(1) b (2) c (3) d (4) b (5) c (6) c
(7) c (8) a

Q2. Complete the following items. Fill in the blanks

(1) $x^2+5x+6=$________

(2) $4a^2-16=$________

(3) $4a^2+4ab+$________ is a complete square.

(4) $\frac{x^2}{y^2}-2+\frac{x^2}{y^2}=$________

(5) $(x+y)(x^2-xy+y^2)=$________

(6) Factored form of x^4-16 is ________

(7) If $x+2$ is factor of $p(x)=x^2+2kx+8$, then $k=$________

ANSWERS:

(1) $(x+2),(x+3)$ (2) $4(a-2)(a+2)$ (3) b^2 (4) $\left(\frac{x}{y}-\frac{y}{x}\right)^2$ (5) x^3+y^3

(6) $(x-2)(x+2)(x^2+4)$ (7) -3

Q3. Factorize the following:

(1) $x^2+8x+16-4y^2$
Solution:
$=(x+4)^2-(2y)^2$
$=(x+4+2y)(x+4-2y)$
$=(x+2y+4)(x-2y+4)$

(2) $4x^2-16y^2$
Solution:
$=4(x^2-4y^2)$
$=4\left[(x)^2-(2y)^2\right]$
$=4(x+2y)(x-2y)$

(3) $9x^2+27x+8$
Solution:
$=9x^2+24x+3x+8$
$=3x(3x+8)+1(3x+8)$
$=(3x+8)(3x+1)$

(4) $1-64z^3$
Solution;
$=(1)^3-(4z)^3$
$=(1-4z)[(1)^2+1(4z)+(4z)^2]$
$=(1-4z)(1+4z+16z^2)$

(5) $8x^3-\frac{1}{27y^3}$
Solution:
$=(2x)^3-\left(\frac{1}{3y}\right)^3$
$=\left(2x-\frac{1}{3y}\right)\left[(2x)+2x\frac{1}{3y}+\left(\frac{1}{3y}\right)^2\right]$
$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$

(6) $2y^2+5y-3$
Solution:
$=2y^2+6y-y-3$
$=2y(y+3)-1(y+3)$
$=(y+3)(2y-1)$

(7) x^3+x^2-4x-4
Solution:
$=x^2(x+1)-4(x+1)$
$=(x+1)(x^2-4)$
$=(x+1)(x^2-2^2)$
$=(x+1)(x+2)(x-2)$

(8) $25m^2n^2+10mn+1$
Solution:
$=(5mn)^2+2(5mn)(1)+(1)^2$
$=(5mn+1)^2$

(9) $1-12pq+36p^2q^2$
Solution:
$=(1)^2-2(1)(6pq)+(6pq)^2$
$=(1-6pq)^2$