Exercise 6.1

Q.1 Find the H.C.F. of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times 13x^7y^3z$

$91x^5y^6z^7 = 13\times 7x^5y^6z^7$

H.C.F= $13x^5y^3z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times 3\times 17xy^2z$

$85x^2yz = 5\times 17x^2yz$

$187xyz^2 = 11\times 17xyz^2$

H.C.F. = $17xyz$

Q.2 Find the H.C.F. of the following expressions by factorization.

(i) $x^2+5x+6,\quad x^2-4x-12$

Solution:

$$
\begin{aligned}
x^2+5x+6 &= x^2+3x+2x+6\\
&= x(x+3)+2(x+3)\\
&= (x+3)(x+2)\\
x^2-4x-12 &= x^2-6x+2x-12
\end{aligned}
$$

$$
\begin{aligned}
&= x(x-6)+2(x-6)\\
&= (x-6)(x+2)
\end{aligned}
$$

H.C.F. = $x+2$

(ii) $x^3-27, x^2+6x-27, 2x^2-18$

Solution:

$x^3-27 = (x)^3-(3)^3 = (x-3)(x^2+3x+9)$

$$
\begin{aligned}
x^2+6x-27 &= x^2+9x-3x-27\\
&= x(x+9)-3(x+9)\\
&= (x+9)(x-3)
\end{aligned}
$$

$$
\begin{aligned}
2x^2-18 &= 2(x^2-9) = 2(x^2-3^2)\\
&= 2(x+3)(x-3)
\end{aligned}
$$

H.C.F. = $x-3$

(iii) $x^3-2x^2+x,\ x^2+2x-3,\ x^2+3x-4$

Solution:

$$x^3-2x^2+x=x\left(x^2-2x+1\right)=x(x-1)^2$$

$$=x(x-1)(x-1)$$

$$x^2+2x-3=x^2+3x-x-3$$

$$=x(x+3)-1(x+3)$$

$$=(x+3)(x-1)$$

$$x^2+3x-4=x^2+4x-x-4$$

$$=x(x+4)-1(x+4)$$

$$=(x+4)(x-1)$$

H.C.F. = $x-1$

(iv) $18(x^3-9x^2+8x),\ 24\left(x^2-3x+2\right)$

Solution:

$$18(x^3-9x^2+8x)=18\left(x^2-9x+8\right)$$

$$=18x\left(x^2-8x-x+8\right)$$

$$=18x\left[x(x-8)-1(x-8)\right]$$

$$=2\times3^2\times(x-2)(x-1)$$

H.C.F. $=2\times3(x-1)=6(x-1)$

(v) $36\left(3x^4+5x^3-2x^2\right),\ 54\left(27x^4-x\right)$

Solution:

$$36\left(3x^4+5x^3-2x^2\right)=4\times9x^2\left(3x^2+5x-2\right)$$

$$=4\times9x^2\left(3x^2+6x-x-2\right)$$

$$=2^2.3^2x^2\left[3x(x+2)-1(x+2)\right]$$

$$=2^2.3^2x^2(x+2)(3x-1)$$

$$54\left(27x^4-2\right)=2\times27x\left(27x^3-1\right)$$

$$=2\times3^3x[(3x)^3-(1)]$$

$$=2\times3^3x(3x-1)(9x^2+3x+1)$$

H.C.F $=2\times3^2x(3x-1)=18x(3x-1)$

Q.3 Find the HCF of the following by division method.

(i) $x^3+3x^2-16x+12,\quad x^3+x^2-10x+8$

Solution:

$$\begin{array}{r|l} & \quad 1 \\ \hline x^3+x^2-10x+8 & x^3+3x^2-16x+12 \\ & \pm x^3 \pm x^2 \mp 10x \pm 8 \\ \hline & 2x^2-6x+4 \\ & 2(x^2-3x+2) \end{array}$$

By Ignoring 2

$$\begin{array}{r|l} \hline x^2-3x+2 & x^3+x^2-10x+8 \\ & \pm x^3 \mp 3x^2 \pm 2x \\ \hline & 4x^2-12x+8 \\ & \pm 4x^2 \mp 12x+8 \\ \hline & 0 \end{array}$$

H.C.F. $=x^2-3x+2$

(ii) $x^4+x^3-2x^2-x-3,\ 5x^3+3x^2-17x+6$

Solution:

$$\begin{array}{r|l} & \quad x+2 \\ \hline 5x^3+3x^2-17x+6 & x^4+x^3-2x^2-x-3 \\ & \times 5 \\ \hline & 5x^4+5x^3-10x^2-5x-15 \\ & \pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x \\ \hline & 2x^3+7x^2-x-15 \\ & \times 5 \\ \hline & 10x^3+35x^2-5x-75 \\ & \pm 10x^3 \pm 6x^2 \mp 34x \pm 12 \\ \hline & 29x^2+29x-87 \\ & 29(x^2+x-3) \end{array}$$

By Ignoring 29

$$\begin{array}{r|l} & \quad 5x-2 \\ \hline x^2+x-3 & 5x^3+3x^2-17x+6 \\ & \pm5x^3\pm5x^2\mp15x \\ \hline & -2x^2-2x+6 \\ & \mp4x^2\mp2x\pm6 \\ \hline \end{array}$$

$$\begin{array}{r|l} \hline & \quad 0 \end{array}$$

$H.C.F.=x^2+x-3$

(iii) $2x^5-4x^4-6x,\quad x^5+x^4-3x^3-3x^2$

Solution:

$2x^5-4x^4-6x=2x\left(x^4-2x^3-3\right)$

$x^5+x^4-3x^3-3x^2=x^2\left(x^3+x^2-3x-3\right)$

In this case H.C.F of $2x\ and\ x^2\ is\ x$

Now we find H.C.F. of $x^4-2x^3\ and\ x^3+x^2-3x-3$

$$\begin{array}{r|l} & \quad x-3 \\ \hline x^3+x^2-3x-3 & x^4-2x^3-x^2+x-3 \\ & \pm x^4\pm x^3\mp3x^3\mp3x \\ \hline & -3x^3+3x^2+3x-3 \\ & \mp3x^3\mp3x^2\pm3x\pm3 \\ \hline & 6x^2-6x-12 \\ & 6\left(x^2-x-2\right) \end{array}$$

By Ignoring 6

$$\begin{array}{r|l} & \quad x+2 \\ \hline x^2-x-2 & x^3+x^2-3x-3 \end{array}$$

$\pm x^3 \mp x^2 \mp 2x$

$2x^2 - x - 3$

$\pm 2x^2 \mp 2x \mp 4$

$x+1$

Then $x-2$

$x+1$ $x^2 - x - 2$

$\pm x^2 \pm x$

$-2x - 2$

$\mp 2x \mp 2$

0

$H.C.F = x+1$

Hence the H.C.F. of the given expression is

$x \times (x+1) = x^2 + x$

Q.4 Find the LCM of the following expressions.

(i) $39x^7y^3z \text{ and } 91x^5y^6z^7$

Solution:

$39x^7y^3z = 3 \times 13x^7y^3z$

$91x^5y^6z^7 = 13 \times 7x^5y^6z^7$

L.C.M. $= 3 \times 7 \times 13x^7y^6z^7$

$= 273x^7y^6z^7$

(ii) $102xy^2z,\ 85\,x^2yz \text{ and } 187\,xyz^2$

Solution:

$102xy^2z = 2 \times 3 \times 17xy^2z$

$85x^2yz = 5 \times 17x^2yz$

$187xyz^2 = 11 \times 17xyz^2$

L.C.M. $= 2 \times 3 \times 5 \times 11 \times 17x^2y^2z^2$

$= 5610x^2y^2z^2$

Q.5 Find the LCM of the following expressions by factorization.

(i) $x^2 + 25x + 100 \text{ and } x^2 - x - 20$

Solution:

$x^2 + 25x + 100 = x^2 - 20x - 5x + 100$

$= x(x-20) - 5(x-20)$

$= (x-20)(x-5)$

$$x^2-x-20 \quad = x^2-5x+4x-20$$

$$= x(x-5)+4(x-5)$$

$$= (x-5)(x+4)$$

L.C.M. $= (x-5)(x-20)(x+4)$

(ii) $x^2+4x+4,\ x^2-4,\ 2x^2+x-6$

Solution:

$$x^2+4x+4=(x+2)^2$$

$$x^2-4=(x+2)(x-2)$$

$$2x^2+x-6=2x^2+4x-3x-6$$

$$=2x(x+2)-3(x+2)$$

$$=(x+2)(2x-3)$$

$$L.C.M.=(x+2)^2(x-2)(2x-3)$$

(iii) $2\left(x^4-y^4\right),3\left(x^3+2x^2y-xy^2-2y^3\right)$

Solution:

$$2\left(x^4-y^4\right)=2\left(x^2-y^2\right)\left(x^2+y^2\right)$$

$$=2(x-y)(x+y)$$

$$=2\left(x^2-y^2\right)\left(x^2+y^2\right)\left(x^2+y^2\right)$$

$$3\left(x^3+2x^2y-xy^2-2y^3\right)=3\left[x^2(x+2y)-y^2(x+2y)\right]$$

$$=3(x+2y)\left(x^2+y^2\right)$$

$$=3(x+y)(x-y)(x+2y)$$

$$L.C.M.=2.3(x-y)(x+y)\left(x^2+y^2\right)(x+2y)$$

$$=6\left(x^2-y^2\right)\left(x^2+y^2\right)(x+2y)$$

$$=6\left(x^4-y^4\right)(x+2y)$$

(iv) $4\left(x^4-1\right),\ 6\left(x^3-x^2-x+1\right)$

Solution:

$$4\left(x^4-1\right)=4\left(x^2-1\right)\left(x^2+1\right)$$

$$=2^2\left(x-1\right)\left(x+1\right)\left(x^2+1\right)$$

$$6\left(x^3-x^2-x+1\right)=6\left[x^2\left(x-1\right)-1\left(x-1\right)\right]$$

$$=2.3\left(x-1\right)\left(x^2+1\right)$$

$$=2.3\left(x-1\right)\left(x-1\right)\left(x+1\right)$$

$$=2.3\left(x-1\right)^2\left(x+1\right)$$

$$L.C.M.=2^2.3\left(x-1\right)^2\left(x+1\right)\left(x^2+1\right)$$

$$=12\left(x-1\right)^2\left(x+1\right)\left(x^2+1\right)$$

$$=12\left(x-1\right)\left(x-1\right)\left(x+1\right)\left(x^2+1\right)$$

$$=12\left(x-1\right)\left(x^2-1\right)\left(x^2+1\right)$$

$$=12\left(x-1\right)\left(x^4-1\right)$$

Q.6 For what value of k. is $(x+4)$ the H.C.F of $(x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution:

Let $p(x)=x^2+x-(2k+2)$

And $q(x)=2x^2+kx-12$

As $x+4$ is H.C.F of $p(x)$ and $q(x)$. So $p(x)$ is exactly divisible by $x+4$ and thus $p(-4)=0$

i.e $(-4)^2+(-4)-(2k+2)=0$

$=16-4-2k-2=0$

$10-2k=0$

$2k=10$

$k=5$

Q.7 If $(x+3)(x-2)$ is the H.C.F of $p(x)=(x+3)\left(2x^2-3x+k\right)$ and

$q(x)=(x-2)\left(3x^2=7x-1\right)$, find k and l.

Solution:

$p(x)=(x+3)\left(2x^2-3x+k\right)$

$q(x)=(x-2)\left(3x^2=7x-1\right)$

$H.C.F.\ of\ p(x)\ and\ q(x)=(x+3)(x-2)$

$(x+3)(x-2)$ is a factor of

$(x+3)\left(2x^2-3x+k\right)$

Hence $x-2$ is a factor of $2x^2-3x+k$

$\therefore$ $2(2)^2-3(2)+k=0$

$$8-6+k=0$$

$$2+k=0$$

$$k=-2$$

Similarly, $(x+3)(x-2)$ is a factor of $(x-2)\left(3x^2+7x-l\right)$

$x+3$ is a factor of $\left(3x^2+7x-l\right)$

$$3(-3)^2+7(-3)-l=0$$

$$27-21-l=0$$

$$6-l=0$$

$$l=6$$

Q.8 The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $2\left(x^4-1\right)$ and $(x+1)\left(x^2+1\right)$respectively. If $p(x)=x^3+x^2+x+1$, find $q(x)$.

Solution:

$$L.C.M=2\left(x^4-1\right)$$

$$H.C.F=(x+1)\left(x^2+1\right)$$

$$p(x)=x^3+x^2+x+1.$$

$$q(x)=\frac{(L.C.M)\times(H.C.F)}{P(X)}$$

$$=\frac{2\left(x^4-1\right)(x+1)\left(x^2+1\right)}{x^3+x^2+x+1}$$

$$=\frac{2\left(x^4-1\right)x^3+x+x^2+1}{x^3+x^2+x+1}$$

$$=\frac{2\left(x^4-1\right)\left(x^3+x^2+x+1\right)}{x^3+x^2+x+1}$$

$$=2\left(x^4-1\right)$$

Q.9 Let $p(x)=10\left(x^2-9\right)\left(x^2-3x+2\right)$ and $q(x)=10x(x+3)(x-1)^2$. If the H.C.F of $p(x),q(x)$ is $10(x+3)(x-1)$, find their L.C.M.

Solution:

$$p(x)=10\left(x^2-9\right)\left(x^2-3x+2\right)$$

$$q(x)=10x(x+3)(x-1)^2$$

$$H.C.F=10(x+3)(x-1)$$

$$L.C.M.=\frac{p(x)\times q(x)}{H.C.F}$$

$$=\frac{10(x^2-9)(x^2-3x+2)10x(x+3)(x-1)^2}{10(x+3)(x-1)}$$

$$=10(x^2-9)(x^2-3x+2).x(x-1)$$

$$=10(x^2-9)\left[(x^2-2x-x+2).x(x-1)\right]$$

$$=10(x^2-9)[x(x-2)-1(x-2].x.(x-1)$$

$$=10(x^2-9)(x-2)(x-1).x.(x-1)$$

$$=10(x^2-9)(x-2).x.(x-1)^2$$

$$=10x(x-2)(x-1)^2(x^2-9)$$

Q.10 Let the product of L.C.M and H.C.F. two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$. Find the value of k.

Solution:

$$(L.C.M.)(H.C.F.)=(x+3)^2(x-2)(x+5)$$

$$p(x)=(x+3)(x-2)$$

$$q(x)=x^2+kx+15$$

$$p(x).q(x)=(L.C.M.)\times(H.C.F.)$$

$$(x+3)(x-2)(x^2+kx+15)=(x+3)^2(x-2)(x+5)$$

$$x^2+kx+15=(x+3)(x+5)$$

$$x^2+kx+15=x^2+8x+15$$

$$k=8$$

Q.11 Waqas wishes to distribute 128 bananas and also176 apples equally among a certain number of children. Find the highest number of children who can get the fruit in this way.

Solution:

Required number of children = H.C.F. of 128 and 176.

```
       1
128 | 176
      128   2
      ----------
       48 | 128
             96   1
            ---------
             32 | 48
                  32   2
                 ---------
                  16 | 32
                       32
                      ----
                        0
```

H.C.F. = 16

Hence the highest number of children = 16

Exercise 6.2

Simplify each of the following as rational expression.

Q1. $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

Solution:

$$=\frac{(x^2-3x+2x-6)}{(x^2-3^2)}+\frac{(x^2+6x-4x-24)}{(x^2-4x+3x-12)}$$

$$=\frac{x(x-3)+2(x-3)}{(x-3)(x-3)}+\frac{x(x+6)-4(x-6)}{x(x-4)+3(x-4)}$$

$$=\frac{(x-3)(x+2)}{(x-3)(x-3)}+\frac{(x+6)(x-4)}{(x-4)(x+3)}$$

$$=\frac{x+2}{x+3}+\frac{x+6}{x+3}$$

$$=\frac{x+2+x+6}{x+3}$$

$$=\frac{2x+8}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q2. $\left[\frac{(x+1)}{(x-1)}-\frac{(x-1)}{(x+1)}-\frac{4x}{(x^4+1)}\right]+\frac{4x}{(x^4-1)}$

Solution:

$$=\frac{(x+1)^2(x^2+1)-(x-1)^2(x^2+1)-4x(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}+\frac{4x}{x^4-1}$$

$$=\frac{(x^2+2x+1)(x^2+1)-(x^2-2x+1)(x^2+1)-4x(x^2-1)}{(x^2-1)(x^2+1)}+\frac{4x}{x^4-1}$$

$$=\frac{x^4+x^2+2x^3+2x+1-(x^4+x^2-2x^3-2x+x^2+1)}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{x^4+2x^3+2x^2+1-x^4+2x^3-2x^2+2x-1-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{4x^3+4x-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q3. $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

Solution:

$$\frac{1}{x^2-5x-3x+15}+\frac{1}{x^2-3x-x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-5)-3(x-5)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-5)(x-3)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{x-1+x-5-2(x-3)}{(x-5)(x-3)(x-1)}$$

$$=\frac{2x-6-2x+6}{(x-5)(x-3)(x-1)}$$

$$=\frac{0}{(x-5)(x-3)(x-1)}$$

$$=0$$

Q4. $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$=\frac{(x+2)(x+3)}{x^2-3^2}+\frac{(x+2)2(x^2-16)}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)(x+3)}{(x+3)(x-3)}+\frac{2(x+2)(x+4)(x-4)}{(x-4)[x(x-3)+2(x-3)]}$$

$$=\frac{x+2}{x-3}+\frac{2(x+2)(x+4)}{(x-3)(x+2)}$$

$$=\frac{x+2}{x-3}+\frac{2(x+4)}{(x-3)}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q5. $\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$

Solution:

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-3^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$=\frac{x+3}{(x+3)(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$=\frac{1}{(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$=\frac{2(2x-3)+2x+3-(4x)2}{2(2x+3)(2x-3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x+3)(2x-3)}$$

$$=\frac{-2x-3}{2(2x+3)(2x-3)}$$

$$=\frac{-(2x+3)}{2(2x+3)(2x-3)}$$

$$=\frac{-1}{2(2x-3)}$$

$$=\frac{1}{2(3-2x)}$$

Q6. $A-\frac{1}{A}$ **where** $A=\frac{a+1}{a-1}$

Solution:

$$A-\frac{1}{A}=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$=\frac{a^2+2a+1-(a^2-2a+1)}{a^2-1}$$

$$=\frac{a^2+2a+1-a^2+2a-1}{a^2-1}$$

$$=\frac{4a}{a^2-1}$$

Q7. $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

Solution:

$$=\left[\frac{x-1}{x-2}+\frac{2}{-(x-2)}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-(x^2-4)}\right]$$

$$=\left[\frac{x-1}{x-2}+\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{x^2-4}\right]$$

$$=\frac{x-1-2}{x-2}-\frac{(x+1)(x-2)-4}{x^2-4}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-2-4}{x^2-4}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{x^2-4}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{x^2-2^2}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)+2(x-3)}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)(x+2)}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x-3}{x-2}$$

$$=0$$

Q8. What rational expression should be subtracted from $\frac{2x^2+2x-7}{x^2+x-6}$ to get $\frac{x-1}{x-2}$?

Solution:

Required expression

$$=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7}{(x+3)(x-2)} - \frac{x-1}{x-2}$$

$$= \frac{2x^2+2x-7-(x+3)(x-1)}{(x+3)(x-2)}$$

$$= \frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$= \frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$= \frac{x^2-4}{(x+3)(x-2)}$$

$$= \frac{(x+2)(x-2)}{(x+3)(x-2)}$$

$$= \frac{x+2}{x-3}$$

Q9. Perform the indicated operations and simplify to the lowest form $\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-4}{x^2-9}$.

Solution:

$$= \frac{x^2+3x-2x-6}{x^2-3x+2x-6} \times \frac{x^2-2^2}{x^2-3^2}$$

$$= \frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x+3)(x-2)}{(x-3)(x+2)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x+3)(x-2)}{(x-3)(x+2)}$$

$$= \frac{(x-2)^2}{(x-3)^2}$$

Q10. $\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1}$

Solution:

$$= \frac{x^3-2^3}{x^2-2^2} \times \frac{x^2+4x+2x+8}{(x-1)^2}$$

$$= \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)} \times \frac{x(x+4)+2(x+4)}{(x-1)(x-1)}$$

$$= \frac{x^2+2x+4}{x+2} \times \frac{(x+4)(x+2)}{(x-1)(x-1)}$$

$$= \frac{(x+4)(x^2+2x+4)}{(x-1)^2}$$

Q11. $\mathbf{\frac{x^4-8x}{2x^2+5x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x^2-2x}}$

Solution:

$$= \frac{x^4-8x}{2x^2+5x-3} \times \frac{2x-1}{x^2+2x+4} \frac{x+3}{x^2-2x}$$

$$= \frac{x(x^3-8)}{2x^2+6x-x-3} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{2x(x+3)-1(x+3)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2+2x+4)}{(x+3)(2x-1)} \times \frac{2x-1}{x^2+2x+4} \times \frac{x+3}{x(x-2)}$$

$$= 1$$

Q12. $\mathbf{\frac{2y^2+7y-4}{3y^2-13y+4} \div \frac{4y^2-1}{6y^2+y-1}}$

Solution:

$$= \frac{2y^2+7y-4}{3y^2-13y+4} \times \frac{6y^2+y-1}{4y^2-1}$$

$$= \frac{2y^2+8y-y-4}{3y^2-12y-y+4} \times \frac{6y^2+3y-2y-1}{(2y)^2-1^2}$$

$$= \frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)} \times \frac{3y(2y+1)-1(2y+1)}{(2y+1)(2y-1)}$$

$$= \frac{(y+4)(2y-1)}{(y-4)(3y-1)} \times \frac{(2y+1)(3y-1)}{(2y+1)(2y-1)}$$

$$= \frac{y+4}{y-4}$$

Q13. $\mathbf{\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]}$

Solution:

$$= \frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)} \div \frac{(x+y)^2-(x-y)^2}{(x+y)(x-y)}.$$

$$= \frac{x^4+2x^2y^2+y^4-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-(x^2-2xy+y^2)}{(x+y)(x-y)}.$$

$$= \frac{x^4+2x^2y^2+y^4-x^4+2x^2y^2-y^4}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-x^2+2xy-y^2}{(x+y)(x-y)}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{4xy}{x^2-y^2}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy}$$

$$= \frac{xy}{x^2+y^2}$$

Exercise 6.3

Use factorization to find the square root of following equations.

(i) $4x^2-12xy+9y^2$

Solution:

$= (2x)^2-2(2x)(3y)+(3y)^2$

$= (2x-3y)^2$

$= \sqrt{(2x-3y)^2}$

Required square root is $\pm(2x-3y)$

(ii) $x^2-1+\frac{1}{4x^2} \qquad (x\neq 0)$

Solution

$= (x)^2-2(x)\left(\frac{1}{2x}\right)+\left(\frac{1}{2x}\right)^2$

$= \left(x-\frac{1}{2x}\right)^2$

Required square root is $\pm\left(x-\frac{1}{2x}\right)^2$

(iii) $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

Solution:

$= \left(\frac{1}{16}x\right)^2-2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right)+\left(\frac{1}{6}y\right)^2$

$= \left(\frac{1}{4}x-\frac{1}{6}y\right)^2$

Required square root is $\pm\left(\frac{1}{4}x-\frac{1}{6}y\right)^2$

Or $\pm\left(\frac{x}{4}-\frac{y}{6}\right)$

(iv) $4(a+b)^2-12(a^2-b^2)+9(a-b)^2$

Solution:

$= [2(a+b)^2-2[2(a+b)][3(a-b)]+[3(a-b)]^2$

$= [2(a+b)-3(a-b)]^2$

$= (2a+2b-3a+3b)^2$

$= (5b-a)^2$

Required square root is $\pm(5b-a)^2$

(v) $\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution:

$$=\frac{\left(2x^3\right)^3-2\left(2x^3\right)\left(3y^3\right)+\left(3Y^3\right)^2}{\left(3x^2\right)^2-2\left(3x^2\right)\left(4y^2\right)+\left(4y^2\right)^2}$$

$$=\frac{\left(2x^3-3y^3\right)^2}{\left(3x^2+4y^2\right)^2}$$

$$=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Required square root is $\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$

(vi) $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right) \quad (x\neq 0)$

Solution:

$$=x^2+2+\frac{1}{x^2}-4\left(x-\frac{1}{x}\right)$$

$$=x^2+2+\frac{1}{x^2}-4\left(x-\frac{1}{x}\right)+4$$

$$=\left(x-\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)+4$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right).2+(2)^2$$

$$=\left[\left(x-\frac{1}{x}\right)-2\right]^2$$

Required square root is $\pm\left[\left(x-\frac{1}{x}\right)-2\right]$

(vii) $\left(x^2-\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12 \quad (x\neq 0)$

Solution:

$$=x^4+2+\frac{1}{x^4}-4\left(x^2+2+\frac{1}{x^2}\right)+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$= \left(x^2+\frac{1}{x^2}\right)^2 - 2\left(x^2+\frac{1}{x^2}\right).2+(2)^2$$

$$= \left[\left(x^2+\frac{1}{x^2}\right)-2\right]^2$$

Required square root is $\pm\left[\left(x^2+\frac{1}{x^2}\right)-2\right]$

(viii) $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

Solution:

$$= \left(x^2+2x+x+2\right)\left(x^2+3x+x+3\right)\left(x^2+3x+2x+6\right)$$

$$= \left[x(x+2)+1(x+2)\right]\left[x(x+3)+1(x+3)\right]$$

$$\left[x(x+3)+2(x+3)\right]$$

$$= (x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$$

$$= (x+1)^2(x+2)^2(x+3)^2$$

$$= \left[(x+1)(x+2)(x+3)\right]^2$$

Required square root is $\pm\left[(x+1)(x+2)(x+3)\right]$

(ix) $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

Solution:

$$= \left(x^2+7x+x+7\right)\left(2x^2-3x+2x-3\right)$$

$$\left(2x^2+14x-3x-21\right)$$

$$= \left[x(x+7)+1(x+7)\right]\left[x(2x-3)+1(2x-3)\right.$$

$$= \left[2x(x+7)-3(x+7)\right]$$

$$= (x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$$

$$= (x+7)^2(x+1)^2(2x-3)^2$$

$$= \left[(x+7)(x+1)(2x-3)\right]^2$$

So, the required square root is $\pm\left[(x+7)(x+1)(2x-3)\right]$

Q2. Use division method to find the square root of following equations.

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution:

	$2x$	$+3y$	$+4$			
$2x$	$4x^2$	$+12xy^2$	$+9y^2$	$+6x$	$+24y$	$+16$
	$\pm 4x^2$					
$4x$ $+3y$	$12xy$	$+16x$	$+9y^2$	$+24y$	$+16$	
	$\pm 12xy$		$\pm 9y^2$			
$4x+$ $6y+$ 4		$16x$		$+24y$	$+16$	
		$\pm 16x$		$\pm 24y$	± 16	
		0				

The square root is $\pm(2x+3y+4)$

(ii) $x^4-10x^3+37x^2-60x+36$

Solution:

	x^2	$-5x$	$+6$		
x^2	x^4	$-10x^3$	$+37y^2$	$-60x$	$+36$
	$\pm x^4$				
$2x^2-$ $5x$		$-10x^3$	$+37x^2$		
		$\pm 10x^3$	$\pm 25y^2$		
$2x^2-$ $10x+6$			$12x^2$	$-60x$	$+36$
			$\pm 12x^2$	$\pm 60x$	± 36
			0		

The square root is $\pm(x^2+5x+6)$

(iii) $9x^4-6x^3+7x^2-2x+1$

Solution:

$$
\begin{array}{r|ccccc}
 & 3x & x & +1 & & \\ \hline
3x^2 & 9x^4 & +6x^3 & +7x^2 & -2x & +1 \\
 & \pm 9x^4 & & & & \\ \hline
6x^2-x & & -6x^3 & +7x^2 & & \\
 & & & \pm x^2 & & \\
 & & \pm 6x^3 & & & \\ \hline
6x^2-2x & & & 6x^2 & +2x & +1 \\
+1 & & & & & \\
 & & & \pm 6x^2 & \pm 2x & \pm 1 \\ \hline
 & & & 0 & &
\end{array}
$$

The square root is $\pm\left(3x^2-x+1\right)$

(i) $4+25x^2-12x-24x^3+16x^4$

Solution:

$$
\begin{array}{r|ccccc}
 & 4x^2 & -3x & +2 & & \\ \hline
4x^2 & 16x^4 & -24x^3 & +25x^2 & -12x & +4
\end{array}
$$

$$
\begin{array}{r|ccccc}
 & \pm 16x^4 & & & & \\ \hline
8x^2-3x & & -24x^3 & +25x^2 & & \\
 & & & \pm 9x^2 & & \\
 & & \pm 24x^3 & & & \\ \hline
8x^2-6z+ & & & 16x^2 & +12x & +4 \\
+2 & & & & & \\
 & & & \pm 16x^2 & \pm 12x & \pm 4 \\ \hline
 & & & 0 & &
\end{array}
$$

The square root is $\pm\left(4x^2-3x+2\right)$

(ii) $\frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{x}{y}+\frac{x^2}{y^2}$ $\quad (x\neq 0)(y\neq 0)$

Solution:

$$\begin{array}{c|ccccc}
 & & \frac{x}{y} & -5 & \frac{y}{x} & \\
\hline
\frac{x}{y} & \frac{x^2}{y^2} & -10\frac{x}{y} & +27 & -10\frac{x}{y} & +\frac{x^2}{y^2} \\
 & \pm\frac{x^2}{y^2} & & & & \\
\hline
\end{array}$$

$$\begin{array}{c|ccccc}
2\frac{x}{y}-5 & & -10\frac{x}{y} & +27 & & \\
 & & & \pm 25 & & \\
 & & \pm 10\frac{x}{y} & & & \\
\hline
 & & & 2 & -10\frac{x}{y} & +\frac{x^2}{y^2} \\
2\frac{x}{y}-10 & & & & & \\
+\frac{y}{x} & & & & & \\
 & & & \pm 2 & \pm 10\frac{x}{y} & \pm\frac{x^2}{y^2} \\
\hline
 & & & 0 & & \\
\end{array}$$

The square root is $\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$

Q3. Find the value of k for which these following expressions will become perfect square.

(i) $4x^4-12x^3+37x^2-42x+k$

Solution:

$$\begin{array}{c|ccccc}
 & & 2x^2 & -3x & +7 & \\
\hline
2x^2 & 4x^4 & -12x^3 & +37x^2 & -42x & +k \\
\end{array}$$

$$\begin{array}{r|lllll}
 & \pm 4x^4 & & & & \\
\hline
84x^2-3 & & -12x^3 & +37x^2 & & \\
 & & \pm 12x^3 & \pm 9x^2 & & \\
\hline
4x^2-6x & & & 28x^2 & +42x & +k \\
+7 & & & & & \\
 & & & \pm 28x^2 & \pm 42x & \pm 49 \\
\hline
 & & & k-49 & &
\end{array}$$

The given expression will be perfect square when reminder = 0

If $k-49=0$

i.e. $k=49$

(iii) $x^4-4x^3+10x^2-kx+9$

Solution:

$$\begin{array}{r|lllll}
 & & x^2 & -2x & +3 & \\
\hline
x^2 & x^4 & -4x^3 & +10x^2 & -kx & +9 \\
 & \pm x^4 & & & & \\
\hline
2x^2-2x & & -4x^3 & +10x^2 & -kx & +9
\end{array}$$

$$\begin{array}{r|lllll}
 & & & \pm 4x^2 & & \\
 & & \pm 4x^3 & & & \\
\hline
2x^2-4x & & & 6x^2 & -kx & +9 \\
+3 & & & & & \\
 & & & \pm 6x^2 & \pm 12x & \pm 9 \\
\hline
 & & & -kx & +12x &
\end{array}$$

The given expression will be perfect square when reminder = 0

If $-kx+12x=0$

$-k+12=0$

$-k=-12$

i.e. $k=12$

Q4. Find the value of L and m for which the following expression will become perfect square.

(i) $x^4+4x^3+16x^2-lm+m$

Solution:

$$\begin{array}{r|lllll}
 & & x^2 & +2x & +6 & \\
\hline
x^2 & x^4 & +4x^3 & +16x^2 & +lx & +m \\
 & \pm x^4 & & & & \\
\hline
\end{array}$$

$$\begin{array}{r|llll}
\hline
2x^2+2x & 4x^3 & +16x^2 & +lx & +m \\
 & \pm 4x^3 & \pm 4x^2 & & \\
\hline
2x^2+4x & & 12x^2 & +lx & +m \\
+6 & & & & \\
 & & \pm 12x^2 & \pm 24x & \pm 36 \\
\hline
 & (l-24) & x- & (m & -36)
\end{array}$$

The given expression will be perfect square when reminder = 0

If $\qquad l-24=0$

And $\qquad m-36=0$

$\therefore \qquad l=24 \qquad m=36.$

(ii) $49x^4-70x^3+109x^2+lx-m$

Solution:

$$\begin{array}{r|lllll}
 & & 7x^2 & -5x & +6 & \\
\hline
7x^2 & 49x^4 & -70x^3 & +109x^2 & +lx & -m \\
 & \pm 49x^4 & & & & \\
\hline
14x^2+5x & & -70x^3 & +109x^2 & & \\
\end{array}$$

	$\pm 70x^3$	$\pm 29x^2$		
$2x^2+4x$ $+6$		$84x^2$	$+lx$	$-m$
		$\pm 84x^2$	$\pm 60x$	± 36
	$(l+60)$	$x-$	m	-36

The given expression will be perfect square

If $l+60=0$ And $-m-36=0$

$\therefore$ $l=-60$; $m=-36.$

Q5. To make the expression $9x^4-12x^3+22x^2-13x+12,$ a perfect square

(i) What should be added to it?
(ii) What should be subtracted from it?
(iii) What should be the value of x?

Solution:

		$3x^2$	$-2x$	$+3$	
$3x^2$	$9x^4$	$-12x^3$	$+22x^2$	$-13x.$	$+12$
	$\pm 9x^4$				
$6x^2-$ $2x$		$-12x^3$	$+22x^2$		

	$\pm 12x^3$	$\pm 4x^2$		
$6x^2-$ $4x+3$		$18x^2$	$+13x$	$+12$
		$\pm 18x^2$	$\pm 60x$	± 9
		$-x$	$+3$	

(i) To make the expression a perfect square we should add $x-3$
(ii) To make the expression a perfect square we should subtract $-x+3$
(iii) To make the expression perfect square reminder = 0

$-x-3$ Or $x=3$.

Review Exercise 6

Q.1 Choose the correct answer.

(i) H.C.F. of $p^3q - pq^3$ and $p^5q^2 - p^2q^5$ is……

(a) $pq(p^2 - q^2)$ (b) $pq(p - q)$

(c) $p^2q^2(p - q)$ (d) $pq(p^3 - q^3)$

(ii) H.C.F of $5x^2y^2$ and $20x^3y^3$ is……..

(a) $5x^2y^2$ (b) $20x^3y^3$
(c) $100x^5y^5$ (d) $5xy$

(iii) H.C.F. of $x - 2$ and $x^2 + x - 6$ is……

(a) $x^2 + x - 6$ (b) $x + 3$

(c) $x - 2$ (d) $x + 6$

(iv) H.C.F of $a^3 + b^3$ and $a^2 - ab + b^2$ is…….

(a) $a + b$ (b) $a^2 - ab + b^2$

(c) $(a - b)^2$ (d) $a^2 + b^2$

(v) H.C.F of $x^2 - 5x + 6$ and $x^2 - x - 6$ is……..

(a) $x - 3$ (b) $x + 2$

(c) $x^2 - 4$ (d) $x - 2$

(vi) H.C.F of $a^2 - b^2$ and $a^3 - b^3$ is………

(a) $a - b$ (b) $a + b$

(c) $a^2 + ab + b^2$ (d) $a^2 - ab + b^2$

(vii) H.C.F of $x^2 + 3x + 2, x^2 + 4x + 3$ and $x^2 + 5x + 4$ is……

(a) $x + 1$ (b) $(x + 1)(x + 2)$

(c) $x + 3$ (d) $(x + 4)(x + 1)$

(viii) L.C.M. of $15x^2, 45xy$ and $30xyz$ is ……..

(a) $90xyz$ (b) $90x^2yz$

(c) $15xyz$ (d) $15x^2yz$

(ix) L.C.M. of $a^2 + b^2$ and $a^4 - b^4$ is……..

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $a^4 - b^4$ (d) $a - b$

(x) The product of algebraic expressions is equal to the…….. of their H.C.F. and L.C.M.

(a) Sum (b) Difference

(c) Product **(d)** Quotient

(xi) **Simplify** $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}=\cdots$

(a) $\frac{4a}{9a^2-b^2}$ **(b)** $\frac{4a-b}{9a^2-b^2}$

(c) $\frac{4a+b}{9a^2-b^2}$ **(d)** $\frac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=\cdots$

(a) $\frac{a+7}{a-6}$ **(b)** $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ **(d)** $\frac{a-2}{a+3}$

(xiii) **Simplify** $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2-b^2}=\cdots$

(a) $\frac{1}{a+b}$ **(b)** $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ **(d)** $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=\cdots$

(a) $\frac{x}{x+y}$ **(b)** $\frac{y}{x+y}$

(c) $\frac{y}{x}$ **(d)** $\frac{x}{y}$

(xv) **The square root of $a2-2a+1$ is……**

(a) $\pm(a+1)$ **(b)** $\pm(a-1)$

(c) $a-1$ **(d)** $a+1$

(xvi) **What should be added to complete the square of $x4+64$**

(a) $8x^2$ **(b)** $-8x^2$

(c) $16x^2$ **(d)** $4x^2$

(xvii) **The square root of $x4+1x4+2$ is…….**

(a) $\pm(x+\frac{1}{x})$ **(b)** $(x^2+\frac{1}{x^2})$

(c) $\pm(x-\frac{1}{x})$ **(d)** $\pm(x^2-\frac{1}{x^2})$

Answers:

(i) b	(ii) a	(iii) c	(iv) b	(v) a
(vi) a	(vii) a	(viii) b	(ix) c	(x) c
(xi) c	(xii) a	(xiii) a	(xiv) d	(xv) b
(xvi) c	(xvii) b			

Q2. Find the H.C.F of the following by factorization.

$\mathbf{8x^4 - 128}$, $\mathbf{12x^3 - 96}$

Solution:

$$8x^4 - 128 = 8(x^4 - 16) = 8(x^4 - 4^2)$$

$$= 8(x^2 - 2^2)(x^2 + 2^2)$$

H.C.F. $= 2^3(X - 2)(x + 2)(x^2 + 4)$

$$12x^3 - 96 = 12(x^3 - 8) = 2^2 \times 3(x^3 - 2^3)$$

$$= 2^2 \times 3(x - 2)(x^2 + 2x + 4)$$

H.C.F. $= 2^2(x - 2)$

$= 4(x-2)$

Q3. Find the H.C.F. of the following by division method.

$\mathbf{y^3 + 3y^2 - 3y - 9,\ y^3 + 3y^2 - 8y - 24}$

Solution:

$$\begin{array}{r|l} & \quad 1 \\ \hline y^3 + 3y^2 - 3x - 9 & y^3 - 3y^2 - 8y + 24 \\ & \pm y^3 \pm x3y^2 \mp 3y \pm 9 \\ \hline & \quad -5y - 15 \\ & \quad -5(y + 3) \end{array}$$

By Ignoring -5

$$\begin{array}{r|l} & \quad y^2 - 3 \\ \hline y + 3 & y^3 + 3y^2 - 3y - 9 \\ & \pm y^3 \mp 3y^2 \\ \hline & \quad -3y - 9 \\ & \quad \mp 3y \mp 9 \\ \hline & 0 \end{array}$$

So, H.C.F. is $y + 3$

Q4. Find the L.C.M. OF the following by factorization.

$$\mathbf{12x^2-75, 6x^2-13x-5, 4x^2-20x+25}$$

Solution:

$$12x^2-75=3(4x^2-25)=3[(2x)^2-5^2)$$

$$=3(2x-5)(2x+5)$$

$$6x^2-13x-5=6x^2-15x+2x-5$$

$$=3x(2x-5)(3x+1)$$

$$=(2x-5)(3x+1)$$

$$4x^2-20x+5=(2x)^2-2(2x)(5)+25$$

$$=3x(2x-5)(3x+1)$$

$$=(2x-5)^2$$

So, L.C.M. $=3(2x+5)(3x+1)(2x-5)^2$

Q5. If H.C.F. of $x^4+3x^3+5x^2+26x+56$ and $x^4+2x^3-4x^2-x+28$ is x^2+5x+7. Find their L.C.M.

Solution:

Let $p(x)=x^4+3x^3+5x^2+26x+56$

$Q(x)=x^4+2x^3-4x^2-x+28$

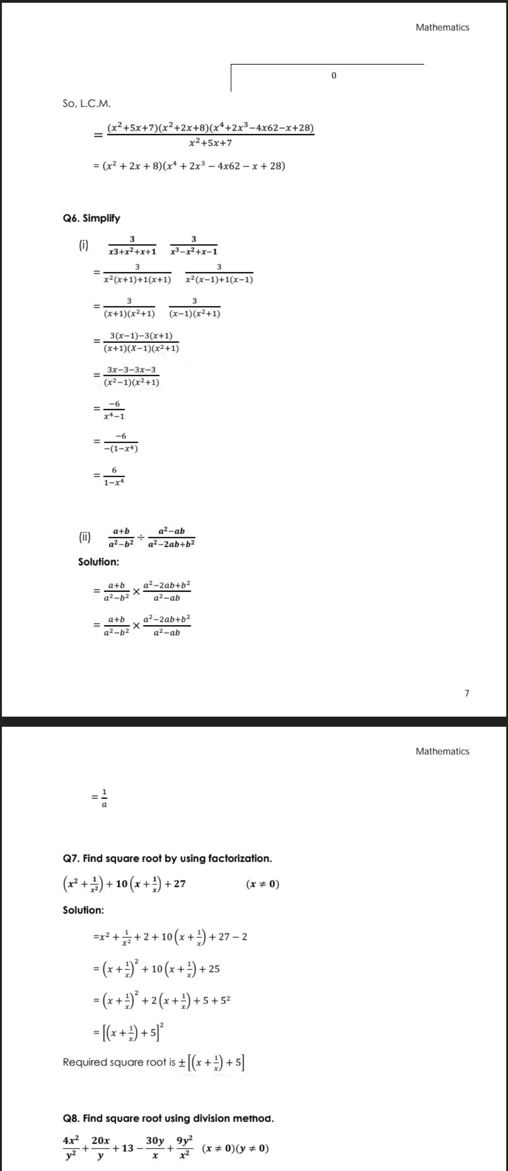
H.C.F. $=x^2+5x+7$

L.C.M.$=\frac{p(x)\times Q(X)}{H.C.F}$

$$=\frac{(x^4+3x^3+5x^2+26x+56)(x^4+2x^3-4x^2-x+28)}{x^2+5x+7}$$

Now dividing

		x²	-2x	+8	
x²-5x+7	x^4	$+3x^3$	$+5x^2$	$+26x$	$+56$
	$\pm x^4$	$\pm 5x^3$	$\pm 7^2$		
		$-2x^3$	$+2x^2$	$+26x$	
		$\pm 2x^3$	$\pm 10x^2$	$\pm 14x$	
			$8x^2$	$+40x$	$+56$
			$\pm 8x^2$	$\pm 40x$	± 56



$$\begin{array}{|c} \hline 0 \end{array}$$

So, L.C.M.

$$= \frac{(x^2+5x+7)(x^2+2x+8)(x^4+2x^3-4x62-x+28)}{x^2+5x+7}$$

$$= (x^2 + 2x + 8)(x^4 + 2x^3 - 4x62 - x + 28)$$

Q6. Simplify

(i) $\frac{3}{x3+x^2+x+1} \quad \frac{3}{x^3-x^2+x-1}$

$$= \frac{3}{x^2(x+1)+1(x+1)} \quad \frac{3}{x^2(x-1)+1(x-1)}$$

$$= \frac{3}{(x+1)(x^2+1)} \quad \frac{3}{(x-1)(x^2+1)}$$

$$= \frac{3(x-1)-3(x+1)}{(x+1)(X-1)(x^2+1)}$$

$$= \frac{3x-3-3x-3}{(x^2-1)(x^2+1)}$$

$$= \frac{-6}{x^4-1}$$

$$= \frac{-6}{-(1-x^4)}$$

$$= \frac{6}{1-x^4}$$

(ii) $\frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2}$

Solution:

$$= \frac{a+b}{a^2-b^2} \times \frac{a^2-2ab+b^2}{a^2-ab}$$

$$= \frac{a+b}{a^2-b^2} \times \frac{a^2-2ab+b^2}{a^2-ab}$$

$$= \frac{1}{a}$$

Q7. Find square root by using factorization.

$\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \qquad (x \neq 0)$

Solution:

$$=x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{x}\right)+27-2$$

$$=\left(x+\frac{1}{x}\right)^2+10\left(x+\frac{1}{x}\right)+25$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)+5+5^2$$

$$=\left[\left(x+\frac{1}{x}\right)+5\right]^2$$

Required square root is $\pm\left[\left(x+\frac{1}{x}\right)+5\right]$

Q8. Find square root using division method.

$\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2} \quad (x \neq 0)(y \neq 0)$

Solution:

$$\begin{array}{c|ccccc}
\multicolumn{1}{c}{} & & \frac{2x}{y} & +5 & \frac{3y}{x} & \\
\hline
\frac{2x}{y} & \frac{4x^2}{y^2} & +\frac{20x}{y} & +13 & -\frac{30x}{y} & +\frac{9y^2}{x^2} \\
 & \pm\frac{4x^2}{y^2} & & & & \\
\hline
\frac{4x}{y}+5 & & \frac{20x}{y} & +13 & & \\
\end{array}$$

$$\begin{array}{c|ccccc}
 & & \pm\frac{20x}{y} & \pm 25 & & \\
\hline
\frac{4x}{y}+10-\frac{3y}{x} & & & -12 & -\frac{30y}{x} & +\frac{9y^2}{x^2} \\
 & & & \pm 12 & \pm\frac{30y}{x} & \pm\frac{9y^2}{x^2} \\
\hline
 & & & 0 & & \\
\end{array}$$

So, the required root is $\pm\left(\frac{2x}{y}+5-\frac{3y}{x}\right)$