Exercise 7.1

Q1. Solve the following equations.

(i) $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Solution:

Multiplying both sides by 6 we get

$4x-3x=6x+1$

$x=6x+1$

$-5x=-\frac{1}{5}$

Solution set $=\left\{-\frac{1}{5}\right\}$

(ii) $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Solution:

Multiplying both sides by 6 we get

$2(x-3)-3(x-2)=-6$

$2x-6-3x+6=-6$

$-x=-6$

$x=6$

Solution set $=6$

(iii) $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Solution:

$\frac{1}{2}\left(\frac{6x-1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1-6x}{2}\right)$

Or $\frac{6x-1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1-6x}{6}$

Multiplying both sides by 12

$6x-1+8=10+2-12x$

$6x+12x=10+2+1-8$

$18x=5$

$x=\frac{5}{18}$

Solution set $=\frac{5}{18}$

(iv) $x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$

Solution:

$\frac{3x+1}{3}=2\left(\frac{3x-2}{3}\right)-6x$

$3x+1=2(3x-2)-18x$

$3x+1=6x-4-18x$

$3x-6x+18x=-4-1$

$15x=-5$

$x=\frac{-5}{15}=-\frac{1}{3}$

Solution set $=\left\{\frac{-1}{3}\right\}$

(v) $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Solution:

Multiplying both sides by 18 we get

$15(x-3)-18x=18-2x$

Or $15x-45-18x=18-2x$

$15x-18x+2x=18+45$

$-x=63$

Solution set $=\{-63\}$

(vi) $\frac{x}{3x-6}=2-\frac{2x}{x-2}\quad x\neq 2$

Solution:

Multiplying both sides by

$3x-6=3(x-2)$, we get

$x=2(3x-6)-3(2x)$

$x=6x-12-6x$

$x=-12$

Solution set $=-12$

(vii) $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}\quad x\neq-\frac{5}{2}$

Solution:

$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{2(2x+5)}$

Multiplying both sides by $6(2x+5)$ we get

$6(2x)=4(2x+5)-15$

Or $12x=8x+20-15$

$x=\frac{5}{4}$

So, solution set $=\frac{5}{4}$

(viii) $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}\quad x\neq 1$

Solution:

Multiplying both sides by $6(x-1)$ we get

$6(2x)+2(x-1)=5(x-1)+2(6)$

Or $12x+12-2=5x-5+12$

$14x-5x=-5+12+2$

$9x=9$

$x=1$

But it is given that $x\neq 1$

So the equation has no solution = { }

(ix) $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1},\quad x\neq\pm 1$

Solution:

Multiplying both sides by x^2-1

$2-(x-1)=x-1$

$2-x+1=x-1$

$-x+x=-1-2-1$

$-2x=-4$

$x=2$

So, solution set $=\{2\}$

(x) $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2(x+2)}$

Multiplying both sides by $6(x+2)$ we get

$2(2)=(x+2)-3$

$4=x+2-3$

$4=x-1$

$x=5$

Solution set $=\{5\}$

Q2. Solve each equation and check for extraneous solution, if any.

Extraneous solution:

When raising each side of the equation to a certain power may produce a nonequivalent equation to a certain power may produce a nonequivalent equation that has more solutions than the original equation. These additional solutions are called extraneous solutions. We must check our answer(s) for such solutions when working with radical equations.

(i) $\sqrt{3x+4}=2$

Solution:

Taking square of both sides

$3x+4=4$

$3x=4-4$

$3x=0$

Or $x=0$

$\therefore$ solution set $=\{0\}$

(ii) $\sqrt[3]{2x-4}-2=0$

Taking cube of both sides

$2x-4=2^3=8$

Or $2x=8+4$

$2x=12$

Or $x=6$

$\therefore$ solution set = {6}

(iii) $\sqrt{x-3}-7=0$

Solution:

Taking square of both sides

$x-3=49$

Or $x=49+3$

$x=52$

$\therefore$ solution set = {52}

(iv) $2\sqrt{t+4}=5$

Solution:

Taking square of both sides

$4(t+4)=25$

$4t+16=25$

$4t=25-16=9$

$t=\left\{\frac{9}{4}\right\}$

$\therefore$ solution set = $\left\{\frac{9}{4}\right\}$

(v) $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$

Solution:

Taking cube of both sides

$2x+3=x-2$

$2x-x=-2-3$

$x=-5$

$\therefore$ solution set = {-5}

(vi) $\sqrt[3]{2-t}=\sqrt[3]{2t-28}$

Solution:

Taking cube of both sides

$2-t=2t-28$

$-t-2t=-28-2$

$-3t=-30$

$t=10$

$\therefore$ solution set = {10}

(vii) $\sqrt{2t+6}-\sqrt{2t-5}=0$

Solution:

$\therefore\sqrt{2t+6}=\sqrt{2t-5}$

Taking square of both sides

$2t+6=2t-5$

$11=0$ Which is not possible

$\therefore$ Solution Set= { } or ∅

(viii) $\sqrt{\frac{x+1}{2x+5}}=2\qquad x\neq\frac{5}{2}$

Solution:

Squaring both sides

$\frac{x+1}{2x+5}=4$

$x+1=4(2x+5)$

$x-1=8x+20$

$x-8x=20-1$

$-7x=19$

$\therefore$ Solution Set $=\left\{-\frac{19}{7}\right\}$

Exercise 7.2

Q1. Identify the following statements as True of False;

i. $|x|=0$ has only one solution.

ii. All absolute value equations have two solutions.

iii. The equation $|x|=2$ is equivalent to $x=2$ or $x=-2$.

iv. The equation $|x-4|=-4$ has no solution.

v. The equation $|2x-3|=5$ is equivalent to $2x-3=5$ or $2x+3=5$

Answers:

i. **T**	ii. **F**	iii. **T**	iv. **T**	v. **F**

Q2. Solve for x:

(i) $|3x-5|=4$

Solution:

The equation is equivalent to

$3x-5=4$ or $3x-5=-4$

$3x=9$ or $3x=1$

$x=3$ or $x=\frac{1}{3}$

Solution set= $\left\{3, \frac{1}{3}\right\}$

(ii) $\frac{1}{2}|3x+2|-4=11$

Solution:

$\frac{1}{2}|3x+2|-4=11$

$|3x+2|=30$

The equation is equivalent to

$3x+2=30$ or $3x+2=-30$

$3x=30-2$ or $3x=-30-2$

$3x=28$ or $x=-32$

$x=\frac{28}{3}$ or $x=-\frac{32}{3}$

(iii) $|2x+5|=11$

Solution:

The equation is equivalent to

$2x+5=11$ or $2x+5=-11$

$2x=6$ or $2x=-16$

$x=3$ or $x=-8$

(iv) $|3+2x|=|6x-7|$

Solution:

The given equation is equivalent to

$3+2x=\pm(6x-7)$

i.e. $3+2x=6x-7$ $3+2x=-(6x-7)$

i.e. $2x-6x=-7-3$ or $2x+6x=7-3$

i.e. $-4x=-10$ or $8x=4$

i.e. $x=\frac{5}{2}$ or $x=\frac{1}{2}$

(v) $|x+2|-3=5-|x+2|$

Solution:

$|x+2|+|x+2|=5+3$

$2|x+2|=8$

$|x+2|=4$

The given equation is equivalent to

$x+2=4$ or $x+2=-4$

$x=2$ or $x=-6$

(vi) $\frac{1}{2}|x+3|+21=9$

Solution:

$\frac{1}{2}|x+3|=9-21=-12$

$|x+3|=-6$ which is not possible

Since the absolute value of non-zero integer is always positive. So,

Solution set= $\{\ \}$

(vii) $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

Solution:

$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}=1$

The given equation is equivalent to

$\left|\frac{3-5x}{4}\right|=\pm 1$

$3-5x=\pm 4$.

i.e. $3-5x=4$ or $3-5x=-4$

$-5x=4$ or $-5x=-4-3$

$-5x=1$ or $-5x=-7$

$x=-\frac{1}{5}$ or $x=\frac{7}{5}$

Solution set= $\left\{-\frac{1}{5}, \frac{7}{5}\right\}$

(viii) $\left|\frac{x+5}{2-x}\right|=6$

Solution:

The given equation is equivalent to

$\frac{x+5}{2-x}=\pm 6$

$x+5=\pm 6\,(2-x)$

i.e. $x+5=6(2-x)$ or $x+5=-6(2-x)$

$x+5=12-6x$ or $x+5=-12+6x$

$7x=7$ or $-5x=-17$

$x=1$ or $x=\frac{17}{5}$

Exercise 7.3

Q1. Solve the following inequalities.

1. $3x+1< 5x-4$

$3x-5x<-4-1$

Solution: $-2x<5$

$x>5/2$

2. $4x-10.3\le 21x-1.8$

Solution: $4x-21x-1.8+10.3$

$x\ge -0.5$

3. $4-\frac{1}{2}x\ge -7+\frac{1}{4}x$

Solution:

$\frac{-3}{4}x\ge -11$

$x\le \frac{44}{3}$

4. $x-2(5-2x)\ge 6x-3\frac{1}{2}$

Solution:

$=x-0+4x\ge 10-\frac{7}{2}$

$=5x-6x\ge 10-\frac{7}{2}$

$=-x\ge \frac{13}{2}$

$=x\le -6.5$

5. $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

Solution:

$=3x+2-3(2x+1)>-9$

$=3x\_6x>-9-2+3$

$=-3x>-8$

$=x<\frac{8}{3}$

6. $3(2x+1)-2(2x+5)<5(3x-2)$

Solution:

$6x+3-4x-10<15x-10$

$2x-7<15x-10$

$-13x<-3$

$x>\frac{3}{13}$

7. $3(x-1)-(x-2)>-2(x+4)$

Solution:

$3x-3-x+2>-2x-8$

$2x+2x<-8+1$

$x>-\frac{7}{4}$

8. $2\frac{2}{3}x+\frac{2}{3}(5x-4)>-\frac{1}{3}(8x+7)$

Solution:

$\frac{8}{3}x+\frac{10}{3}x-\frac{8}{3}>-\frac{8}{3}x-\frac{7}{3}$

Multiplying by 3

$8x+10x-8>-8x-7$

$18x+8x>-7+8$

$26x>1$

$x>\frac{1}{26}$

Q2. Solve the following inequalities.

(i) $-4<3x+5<8$

Solution:

The given equality represents two inequalities

$-4<3x+5$

And $3x+5<8$

The first inequality $3x+5<8$

The first inequality $-4<3x+5$ gives

$-3x<5+4$

$3x>-9$

$x>-\frac{9}{3}$

$x>-3$

$-3<x$..........(i)

The second inequality $3x+5<8$ gives

$3x<8-5$

$3x<3$

Or $x<1$ .... (ii)

Combining (i) and (ii), we have

$-3<x<1$

Solution set is $\{x|-3<x<1\}$

(ii) $-5\le \frac{4-3x}{2}<1$

Solution:

The given inequality represents two inequalities

$-5\le \frac{4-3x}{2}$

And $\frac{4-3x}{2}<1$

The first inequality gives

$-5\le \frac{4-3x}{2}$

$-10\le 4-3x$

$-10-4\le -3x$

$14\ge 3x$

Or $\frac{14}{3}\ge x$...............(i)

The second inequality gives $\frac{4-3x}{2}<1$

$\Rightarrow 4-3x<2$

$4-2<3x$

$2<3x$

$\Rightarrow \frac{2}{3}<x$..............$(ii)$

Combining (i) and (ii)

$\frac{2}{3}<x\le \frac{14}{3}$

Solution Set is $\left\{x|\frac{2}{3}<x<\frac{14}{3}\right\}$

(iii) $-6<\frac{x-2}{4}<6$

Solution:

This inequality is equivalent to two inequalities

$-6<\frac{x-2}{4}$

And $\frac{x-2}{4}<6$

The first inequality gives

$-24<x-2$

$\Rightarrow -24+2<x$

$\Rightarrow -22<x$..............(i)

The second inequality gives

$x-2<24$

$x<24+2$

$x<26$.............(ii)

Combining (i) and (ii) we have

$-22<x<26$

Solution Set is $\{x|-22<x<26\}$

(iv) $3\ge \frac{7-x}{2}\ge 1$

Solution:

This inequality represents two inequality $3\ge \frac{7-x}{2}$ and $\frac{7-x}{2}\ge 1$.

The first inequality gives

$6\ge 7-x$

$\Rightarrow 6-7\ge -x$

$\Rightarrow -1\ge -x$

Or $1\le x$..........(i)

The second inequality gives

$7-x\ge 2$

$-x\ge 2-7$

$\Rightarrow -x\ge -5$

Or $x\le 5$..........(ii)

Combining (i) and (ii) we have

$1\le x\le 5$

The Solution Set is $\{x|1\le x\le 5\}$

(v) $3x-10\le 5\le x+3$

Solution:

The inequality is equivalent to

$3x-10\le 5$

And $5\le x+3$

The first inequality gives

$3x-10\le 5$

$\Rightarrow 3x\le 5+10$

Or $3x\le 15$

Or $x\le 5$..........(i)

The second inequality gives

$5<x+3$

Or $5-3<x$

Or $2<x$..........(ii)

Combining (i) and (ii) we have

$2<x\le 5$

The Solution Set is $\{x|2<x\le 5\}$

(vi) $-3\le \frac{x-4}{-5}<4$

Solution:

The equation is equivalent to

$-3\le \frac{x-4}{-5}$

And $\frac{x-4}{-5}<4$

The first inequality gives

$-3\le \frac{x-4}{-5}$

$\Rightarrow 15\ge x-4$

Or $15+4\ge x$

$\Rightarrow 19\ge x$

Or $x\le 19$..........(i)

This second inequality gives

$\frac{x-4}{-5}<4$

Or $x-4>-20$

$\Rightarrow x>-20+4$

Or $x>-16$

Or $-16<x$..........(ii)

Combining (i) and (ii) we have

$-16<x\le 19$

The Solution Set is $\{x|-16<x\le 19\}$

(vii) $1-2x<5-x\le 25-6x$

Solution:

This inequality is equivalent to

$1-2x<5-x$

And $5-x\le 25-6x$

The first inequality gives

$1-2x<5-x$

Or $-2x+x<5-1$

Or -x < 4 or x > -4

Or $-4<x$..........(i)

The second inequality gives

$5-x\le 25-6x$

$\Rightarrow -x+6x\le 25-5$

Or $5x\le 20$

$\Rightarrow x\le 4$..........(ii)

Combining (i) and (ii) we have

$-4<x\le 4$

The Solution Set is $\{x|-4<x\le 4\}$

(viii) $3x-2<2x+1<4x+17$

Solution:

This is equivalent to

$3x-2<2x+1$

And $2x+1<4x+17$

The first inequality gives

$3x-2<2x+1$

Or $3x-2x<1+2$

Or $x<3$

The second inequality gives

$2x+1<4x+17$

$2x-4x<17-1$

Or $-2x>16$

Or $2x>-16$

Or $x>-8$

Or $-8<x$..........(ii)

Combining (i) and (ii) we have

$-8<x<3$

Solution Set is $\{x|-8<x<3\}$

REVIEW EXERCISE 7

Q1. Choose the correct answer.

(1) Which of the following is the solution of the inequality $-4x \le 11$?

(a) -8 **(b)** -2
(c) -4 **(d)** none of these

(2) A statement involving any of the symbols $<, >, \le or \ge$ **is called……**

(a) equation **(b)** identity
(c) inequality **(d)** linear equation

(3) X=…….. is a solution of the inequality $-2 < x < \frac{3}{2}$

(a)-5 **(b)** 3
(c) 0 **(d)** $\frac{3}{2}$

(4) If x is no larger than 10, then ………

(a) $x > 8$ **(b)** $x < 8$
(c) $x < 10$ **(d)** $x > 10$

(5) If the capacity c of an elevator is at most 1600 pound, then …..

(a) $c \ge 1600$ **(b)** $c > 1600$
(c) $c < 1600$ **(d)** $c \le 1600$

(6) $x = 0$ **is a solution of the inequality …….**

(a) $x > 0$ **(b)** $3x+5<0$
(c) $x+2<0$ **(d)** $x-2<0$

ANSWERS:
(1) b **(2)** c **(3)** c **(4)** b **(5)** c **(6)** d

Q2. Identify the following statements as true or false.

(1) The equation $3x-5=7-x$ is a linear equation.
(2) The equation $x-0.3x=0.7x$ is an identity.
(3) The equation $-2x+3 = 8$ is equivalent to $-2x = 11$.
(4) To eliminate fraction, we multiply each side of an equation by the LCM of denominators.
(5) $4(x+3) = x+3$ is a conditional equation.
(6) The equation $2(3x+5) = 6x+12$ is an inconsistent equation.
(7) To solve $\frac{2}{3}x = 1.2$, we should multiply each side by $\frac{2}{3}$
(8) Equation having exactly the same solution are called equivalent equation.
(9) A solution that does not satisfy the original equation is called extraneous solution.

ANSWERS:

(1) T **(2)** T **(3)** F **(4)** T **(5)** T **(6)** T **(7)** F
(8) T **(9)** T

Q3. Answer the following short question.

(1) Define linear inequality in one variable.
Solution:
A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and is of the form

$Ax + b < 0 \ , a \ne 0$

Where a and b are equal real numbers.

(2) State the trichotomy and transitive properties of inequalities.
Solution:
LAW OF TRICHOTOMY
For any $a, b \in R$, one and only of the statements is true.

$A < b$ or $a = b$ or $a > b$

LAW OF TRANSITY

Let $a, b, c \in R$

(a) If $a > b$ and $b > c$ then $a > c$
(b) If $a < b$ and $b < c$ then $a < c$

(3) The formula relating degrees Fahrenheit to degrees Celsius is $F = \frac{9}{5}C + 32$**, for what value of C is F<0?**
Solution:

$$F = \frac{9}{5}c + 32$$
$$F < 0$$
$$\Rightarrow \frac{9}{5}C + 32$$
$$9c + 160 < 0$$
$$9c < -160$$
$$c < \frac{-160}{9}$$

(4) Seven times, the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship.
Solution:
Let x be the integer. so according to the question

$$50 \le 7(x+12) \le 60$$

This is equivalent to two inequalities

$$50 \le 7(x+2)$$
$$7(x+12) \le 60$$

The first inequality gives

$$50 \le 7(x+12)$$
$$50 \le 7x+84$$
$$50-84 \le 7x$$
$$-34 \le 7x$$
$$-\frac{34}{7} \le x$$

The second inequality gives

$$7(x+12) \le 60$$
$$7x+84 \le 60$$
$$7x \le 60-84$$
$$x \le -\frac{24}{7}$$

From (1) and (2) we get

$$-\frac{34}{7} \le x \le -\frac{24}{7}$$

Q4. Solve each of the following and check for extraneous solution if any:

(1) $\sqrt{2t+4} = \sqrt{t-1}$

Solution:
Squaring both sides

$$2t+4 = t-1$$
$$2t-t = -1-4$$
$$t = -5$$

On checking

$$\sqrt{-10+4} = \sqrt{-5-1}$$
$$\sqrt{-6} = \sqrt{-6}$$

Since $\sqrt{-6}$ = imaginary

(2) $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

Solution:

$$\sqrt{3x-1} - 2\sqrt{8-2x} = 0$$

Squaring both sides

$$3x-1 = 4(8-2x)$$
$$3x-1 = 32-8x$$
$$11x = 33$$
$$x = 3$$

Q5. Solve for x.

(1) $|3x+14| - 2 = 5x$
Solution:

$$|3x+14| = 5x+2$$

This is equivalent to

$$3x+14 = \pm(5x+2)$$
$$3x+14 = 5x+2$$
$$3x-5x = 2-14$$
$$-2x = -12$$
$$x = 6$$

On checking we see that $x = 6$ satisfies the given equation but $x = -12$ does not satisfy the given equation. So, the solution set is $\{6\}$

(2) $\frac{1}{3}|x-3| = \frac{1}{2}|x+2|$
Solution:
Multiplying both sides by 6 we get

$$2|x-3| = \frac{1}{2}|x+2|$$

Which is equivalent to

$$2(x-3) = 3(x+2)$$
$$2x-6 = 3x+6$$
$$2x-3x = 6+6$$
$$-x = -12$$

On checking $x = -12$

Q6. Solve the following inequality

(1) $-\frac{1}{3}x + 5 \le 1$

Solution:
Multiplying by 3

$$-x+15 \le 3$$
$$-x \le 3-15$$
$$-x \le 3-15$$
$$x \ge 12$$

(2) $-3 < \frac{1-2x}{5} < 1$
Solution:

$$-3 < \frac{1-2x}{5}$$
$$-15 < 1+15$$
$$2x < 1+15$$
$$2x < 16$$
$$x < 8$$
$$1-2x < 5$$
$$-2x < 5-1$$
$$-2x > 4$$
$$x > -2$$
$$8 > x > -2$$
$$-2 < x < 8$$

Solution set is $\{x | 8 > x > -2\}$