Exercise 1.1

1. Write the following quadratic equations in the standard form and point out pure quadratic equations.

i. **(x+7) (x-3) = -7**

Solution:

$$(x+7)(x-3)=-7$$
$$x(x-3)+7(x-3)=-7$$
$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The above equation is a quadratic equation.

ii. $\frac{x^2+4}{3}-\frac{x}{7}=1$

Solution:

$$\frac{x^2+4}{3}-\frac{x}{7}=1$$

Multiply both sides by 21, we get

$$21\times\frac{x^2+4}{3}-21\times\frac{x}{7}=1\times 21$$
$$7(x^2+4)-3x=21$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The above equation is a quadratic equation.

iii. $\frac{x}{x+1}+\frac{x+1}{x}=6$

Solution:

$$\frac{x}{x+1}+\frac{x+1}{x}=6$$
$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+2x+1=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$2x^2-6x^2+2x-6x+1=0$$
$$-4x^2-4x+1=0$$
$$-(4x^2+4x-1)=0$$
$$\Rightarrow 4x^2+4x-1=0$$

The above equation is a quadratic equation.

iv $\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$

Solution:

$$\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$$

$$\frac{x(x+4)-(x-2)^2+4x(x-2)}{x(x-2)}=0$$

$$\left(x^2+4x\right)-\left(x^2-4x+4\right)+4\left(x^2-2x\right)=0$$

$$x^2+4x-x^2+4x-4+4x^2-8x=0$$

$$x^2-x^2+4x^2+4x+4x-8x-4=0$$

$$4x^2+8x-8x-4=0$$

$$4x^2-4=0$$

$$4\left(x^2-4\right)=0$$

$$x^2-1=0$$

The above equation is a pure quadratic equation.

v. $\frac{x+3}{x+4}-\frac{x-5}{x}=1$

Solution:

$$\frac{x+3}{x+4}-\frac{x-5}{x}=1$$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$

$$\left(x^2+3x\right)-x(x-5)-4(x-5)=x(x+4)$$

$$x^2+3x-x^2+5x-4x+20=x^2+4x$$

$$x^2-x^2+3x+5x-4x+20=x^2+4x$$

$$4x+20=x^2+4x$$

$$-x^2+4x-4x+20=0$$

$$-x^2-20=0$$

$$-\left(x^2-20\right)=0$$

$$x^2-20=0$$

The above equation is a pure quadratic equation.

vi. $\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$

$$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{x(x+3)+1(x+3)+\left(x^2+4x+4\right)}{x(x+3)+2(x+3)}=\frac{25}{12}$$

$$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$
$$25\left(x^2+5x+6\right)=12\left(2x^2+8x+7\right)$$
$$25x^2+125x+150=24x^2+96x+84$$
$$x^2+29x+66=0$$

The above equation is a quadratic equation.

Q2. Solve by factorization:

i. $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$
$$x^2-5x+4x-20=0$$
$$x(x-5)+4(x-5)=0$$
$$(x+4)(x-5)=0$$

$$\textit{Either}\quad x+4=0 \quad or \quad x-5=0$$
$$x=-4 \qquad x=5$$

$$\text{Thus, solution set}=\{-4,5\}$$

ii. $3y^2=y(y-5)$

Solution:

$$3y^2=y(y-5)$$
$$3y^2=y^2-5y$$
$$3y^2-y^2+5y=0$$
$$2y^2+5y=0$$
$$y(2y+5)=0$$
$$\textit{Either}\quad y=0 \text{ or } 2y+5=0$$

$$2y=-5$$
$$y=\frac{-5}{2}$$

$$\text{Thus, solution set}=\left\{0,-\frac{5}{2}\right\}$$

iii. $4-32x=17x^2$

Solution:

$$4-32x=17x^2$$
$$or \quad 17x^2+32x-4=0$$
$$17x^2+34x-2x-4=0$$
$$17x(x+2)-2(x+2)=0$$
$$(17x-2)(x+2)=0$$

$$\textit{Either}\quad 17x-2=0 \quad or \quad x+2=($$
$$17x=2 \qquad x=-2$$
$$x=\frac{2}{17}$$

$$\text{Thus, solution set}=\left\{\frac{2}{17},-2\right\}$$

iv. $x^2-11x=152$

Solution:

$x^2-11x=152$
$x^2-11x-152=0$
$x^2-19x+8x-152=0$
$x(x-19)+8(x-19)=0$
$(x+8)(x-19)=0$

Either $(x+8)=0$ *or* $(x-19)=0$
$x=-8 \qquad x=19$

Thus, solution set = {-8,19}

v. $\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$

Solution:

$\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$
$\dfrac{(x+1)^2+x^2}{x(x+1)}=\dfrac{25}{12}$
$\dfrac{x^2+2x+1+x^2}{x^2+x}=\dfrac{25}{12}$
$\dfrac{2x^2+2x+1}{x^2+x}=\dfrac{25}{12}$
$25(x^2+x)=12(2x^2+2x+1)$
$25x^2+25x=24x^2+24x+12$
$25x^2-24x^2+25x-24x-12=0$
$x^2+x-12=0$
$x^2+4x-3x-12=0$
$x(x+4)-3(x+4)=0$
$(x-3)(x+4)=0$

Either $x-3=0$ *or* $x+4=0$
$x=3 \qquad x=-4$

Thus, solution set = $\{3,-4\}$

vi. $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

Solution:

$\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$
$\dfrac{2}{x-9}=\dfrac{(x-4)-(x-3)}{(x-3)(x-4)}$
$\dfrac{2}{x-9}=\dfrac{x-4-x+3}{x^2-7x+12}$
$\dfrac{2}{x-9}=\dfrac{-1}{x^2-7x+12}$
$2(x^2-7x+12)=-1(x-9)$
$2x^2-14x+24=-x+9$
$2x^2-14x+x+24-9=0$
$2x^2-13x+15=0$
$2x^2-10x-3x+15=0$
$2x(x-5)-3(x-5)=0$

$(2x-3)(x-5)=0$

$$\begin{aligned} &\textit{Either } 2x-3=0 \textit{ or } x-5=0 \\ &\qquad 2x=3 \qquad\quad x=5 \\ &\qquad x=\frac{3}{2} \qquad\quad x=5 \end{aligned}$$

Thus solution set $=\{5,\frac{3}{2}\}$

Q3. Solve the following equations by completing square:

i. $7x^2+2x-1=0$

Solution:

$$\begin{aligned} &7x^2+2x-1=0 \\ &7x^2+2x=1 \\ &\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7} \\ &x^2+\frac{2x}{7}=\frac{1}{7} \\ &(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2 \\ &\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49} \\ &\left(x+\frac{1}{7}\right)^2=\frac{8}{49} \end{aligned}$$

Takin square root on both sides, we get

$$\begin{aligned} &x+\frac{1}{7}=\pm\sqrt{\frac{8}{49}} \\ &x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7} \\ &x=-\frac{1}{7}\pm\frac{2\sqrt{2}}{7} \\ &x=\frac{-1\pm2\sqrt{2}}{7} \end{aligned}$$

Thus, solution set $=\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

ii. $ax^2+4x-a=0$, $a\neq0$

Solution:

$ax^2+4x=a$

$$\frac{ax^2}{a}+\frac{4x}{a}=\frac{a}{a}$$

$$x^2+\frac{4x}{a}=1$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root on both sides, we get

$$x+\frac{2}{a}=\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x=-\frac{2}{a}\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

Thus, solution set $=\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

iii. $11x^2-34x+3=0$

Solution:

$$11x^2-34x+3=0$$

$$11x^2-34x=-3$$

$$\frac{11x^2}{11}-\frac{34x}{11}=-\frac{3}{11}$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=-\frac{3}{11}+\left(\frac{34}{22}\right)^2$$

$$\left(x-\frac{34}{22}\right)^2=-\frac{3}{11}+\frac{1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{-132+1156}{484}$$

$$\left(x-\frac{34}{22}\right)^2=\frac{1024}{484}$$

Taking square root on both sides we get

$$\left(x-\frac{34}{22}\right)=\pm\sqrt{\frac{1024}{484}}$$

$$x-\frac{34}{22}=\pm\frac{32}{22}$$

$$x=\frac{34}{22}\pm\frac{32}{22}$$

$$x=\frac{34\pm 32}{22}$$

$$x=\frac{34+32}{22},\qquad x=\frac{34-32}{22}$$

$$=\frac{66}{22}\qquad\qquad =\frac{2}{22}$$

$$=3\qquad\qquad =\frac{1}{11}$$

Thus solution set $\left\{3,\frac{1}{11}\right\}$

iv. $lx^2+mx+n=0\ ,\ l\neq 0$

Solution:

$$lx^2+mx+n=0$$

$$lx^2+mx=-n$$

$$\frac{lx^2}{l}+\frac{mx}{l}=-\frac{n}{l}$$
$$x^2+\frac{mx}{l}=-\frac{n}{l}$$
$$(x)^2+2(x)\left(\frac{m}{2l}\right)+\left(\frac{m}{2l}\right)^2=-\frac{n}{l}+\left(\frac{m}{2l}\right)^2$$

$$\left(x+\frac{m}{2l}\right)^2=-\frac{n}{l}+\frac{m^2}{4l^2}$$
$$\left(x+\frac{m}{2l}\right)^2=\frac{-4l\,\mathrm{n}+m^2}{4l^2}$$
$$\left(x+\frac{m}{2l}\right)^2=\frac{m^2-4l\,\mathrm{n}}{4l^2}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{m}{2l}\right)^2}=\pm\sqrt{\frac{m^2-4\,\mathrm{ln}}{4l^2}}$$

$$x+\frac{m}{2l}=\pm\sqrt{\frac{m^2-4l\,\mathrm{n}}{4l^2}}$$
$$x=-\frac{m}{2l}\pm\sqrt{\frac{m^2-4l\,\mathrm{n}}{4l^2}}$$
$$x=\frac{-m\pm\sqrt{m^2-4l\,\mathrm{n}}}{2l}$$

Thus solution set $=\left\{\frac{-m\pm\sqrt{m^2-4l\,\mathrm{n}}}{2l}\right\}$

v. $3x^2+7x=0$

Solution:

$$3x^2+7x=0$$
$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$
$$x^2+\frac{7}{3}x=0$$
$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$
$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\left(\frac{7}{6}\right)^2}$$
$$x+\frac{7}{6}=\pm\frac{7}{6}$$

$$x=-\frac{7}{6}\pm\frac{7}{6}$$
$$x=-\frac{7}{6}+\frac{7}{6}\quad or\quad x=-\frac{7}{6}-\frac{7}{6}$$
$$x=0\qquad\qquad x=-\frac{14}{6}$$
$$x=-\frac{7}{3}$$

Thus solution set $=\left\{0,-\frac{7}{3}\right\}$

vi. $x^2-2x-195=0$

Solution:

$$x^2-2x-195=0$$
$$x^2-2x=195$$
$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$
$$(x-1)^2=195+1$$
$$(x-1)^2=196$$

Taking square root on both sides

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$
$$x-1=\pm14$$
$$x=1\pm14$$
$$x=1+14 \quad or \quad x=1-14$$
$$=15 \qquad\qquad =-13$$

Thus, solution set $=\{-13,15\}$

vii. $-x^2+\frac{15}{2}=\frac{7}{2}x$

Solution:

$$-x^2-\frac{7}{2}x=-\frac{15}{2}$$
$$-\left(x^2+\frac{7}{2}x\right)=-\frac{15}{2}$$
$$x^2+\frac{7}{2}x=\frac{15}{2}$$
$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$
$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$
$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$
$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$
$$x+\frac{7}{4}=\pm\frac{13}{4}$$
$$x=-\frac{7}{4}\pm\frac{13}{4}$$
$$x=-\frac{7}{4}+\frac{13}{4} \quad or \quad x=-\frac{7}{4}-\frac{13}{4}$$
$$x=\frac{3}{2} \qquad\qquad x=-5$$

Thus, solution set $=\left\{\frac{3}{2},-5\right\}$

viii. $x^2+17x+\frac{33}{4}=0$

Solution:

$$x^2+17x=-\frac{33}{4}$$
$$(x)^2+2(x)\left(\frac{17}{2}\right)+\left(\frac{17}{2}\right)^2=-\frac{33}{4}+\left(\frac{17}{2}\right)^2$$
$$\left(x+\frac{17}{2}\right)^2=-\frac{33}{4}+\frac{289}{4}$$
$$\left(x+\frac{17}{2}\right)^2=\frac{256}{4}$$

Taking square root on both sides,

$$\sqrt{\left(x+\frac{17}{2}\right)^2}=\pm\sqrt{\frac{256}{4}}$$

$$x+\frac{17}{2}=\pm\frac{16}{2}$$
$$x=-\frac{17}{2}\pm\frac{16}{2}$$
$$x=-\frac{17}{2}+\frac{16}{2} \quad or \quad x=-\frac{17}{2}-\frac{16}{2}$$
$$x=-\frac{1}{2} \qquad x=-\frac{33}{2}$$

Thus solution set $=\left\{-\frac{1}{2},-\frac{33}{2}\right\}$

ix. $4-\dfrac{8}{3x+1}=\dfrac{3x^2+5}{3x+1}$

Solution:

$$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{4(3x+1)-8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{12x+4-8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{12x-4}{3x+1}=\frac{3x^2+5}{3x+1}$$

Multiplying both sides by $(3x+1)$, we get

$$12x-4=3x^2+5$$
$$3x^2+5-12x+4=0$$
$$3x^2-12x+9=0$$
$$3\left(x^2-4x+3\right)=0$$
$$\Rightarrow x^2-4x+3=0$$
$$x^2-4x=-3$$
$$(x)^2-2(x)(2)+(2)^2=-3+(2)^2$$
$$(x-2)^2=1$$

Taking square root on both sides,

$$\sqrt{(x-2)^2}=\pm\sqrt{1}$$
$$x-2=\pm1$$
$$x=2\pm1$$
$$x=2+1 \quad or \quad x=2-1$$
$$x=3 \qquad x=1$$

Thus solution set $=\{1,3\}$

x. $7(x+2a)^2+3a^2=5a(7x+23a)$

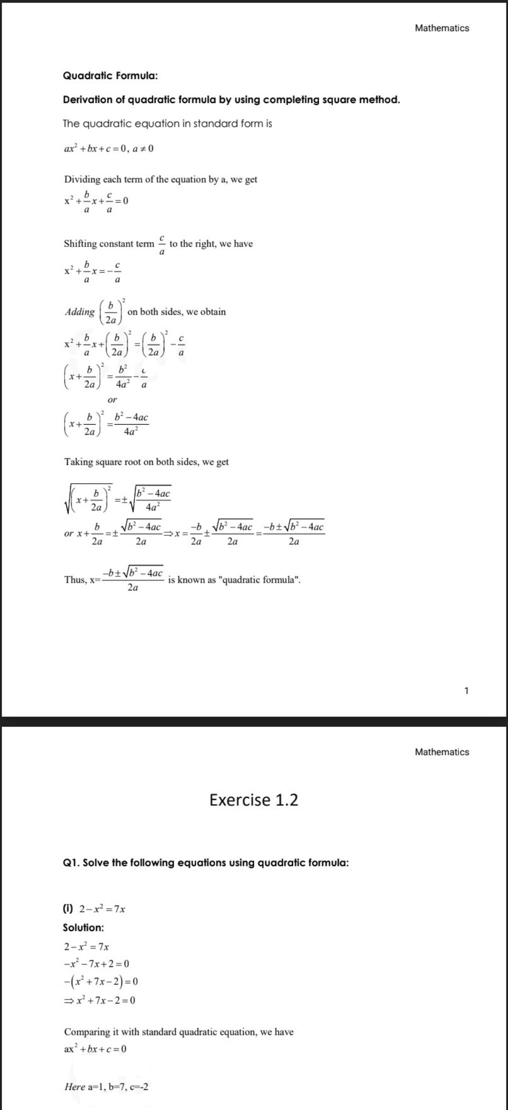
Solution:

$$7(x+2a)^2+3a^2=5a(7x+23a)$$
$$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$$
$$7x^2+28ax+28a^2+3a^2=35ax+115a^2$$
$$7x^2+28ax-35ax+28a^2+3a^2-115a^2=0$$
$$7x^2-7ax-84a^2=0$$
$$7(x^2-ax-12a^2)=0$$
$$x^2-ax-12a^2=0$$
$$x^2-ax=12a^2$$
$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root on both sides, we get

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a}{2}+\frac{7a}{2} \quad or \quad x=\frac{a}{2}-\frac{7a}{2}$$
$$=\frac{8a}{2} \qquad =-\frac{6a}{2}$$
$$=4a \qquad =-3a$$

Thus solution set = $\{-3a,4a\}$



Quadratic Formula:

Derivation of quadratic formula by using completing square method.

The quadratic equation in standard form is

$ax^2 + bx + c = 0,\ a \neq 0$

Dividing each term of the equation by a, we get

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Shifting constant term $\frac{c}{a}$ to the right, we have

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides, we obtain

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

or

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking square root on both sides, we get

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$or\ x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is known as "quadratic formula".

Exercise 1.2

Q1. Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution:

$2 - x^2 = 7x$

$-x^2 - 7x + 2 = 0$

$-\left(x^2 + 7x - 2\right) = 0$

$\Rightarrow x^2 + 7x - 2 = 0$

Comparing it with standard quadratic equation, we have

$ax^2 + bx + c = 0$

Here a=1, b=7, c=-2

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{-7\pm\sqrt{49+8}}{2}$$

$$x=\frac{-7\pm\sqrt{57}}{2}$$

Thus, solution set $=\left\{\dfrac{-7\pm\sqrt{57}}{2}\right\}$

(ii) $5x^2+8x+1=0$

Solution:

$5x^2+8x+1=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=5, b=8, c=1$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-8\pm\sqrt{(8)^2-4(5)(1)}}{2(5)}$$

$$x=\frac{-8\pm\sqrt{64-20}}{10}$$

$$x=\frac{-8\pm\sqrt{44}}{10}$$

$$x=\frac{-8\pm2\sqrt{11}}{10}$$

$$x=\frac{2\left(-4\pm\sqrt{11}\right)}{10}$$

$$x=\frac{-4\pm\sqrt{11}}{5}$$

Thus solution set $=\left\{\dfrac{-4\pm\sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2+x=4\sqrt{3}$

Solution:

$\sqrt{3}x^2+x=4\sqrt{3}$

$\sqrt{3}x^2+x-4\sqrt{3}=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=\sqrt{3}, b=1, c=-4\sqrt{3}$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-1\pm\sqrt{(1)^2-4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\left(\sqrt{3}\right)}$$

$$x=\frac{-1\pm\sqrt{1+48}}{2\left(\sqrt{3}\right)}$$
$$x=\frac{-1\pm\sqrt{49}}{2\sqrt{3}}$$
$$x=\frac{-1\pm 7}{2\sqrt{3}}$$

$$x=\frac{-1+7}{2\sqrt{3}} \quad or \quad x=\frac{-1-7}{2\sqrt{3}}$$

$$x=\frac{6}{2\sqrt{3}} \qquad x=\frac{-8}{2\sqrt{3}}$$
$$x=\frac{3}{\sqrt{3}} \qquad x=-\frac{4}{\sqrt{3}}$$
$$x=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$x=\frac{3\sqrt{3}}{\left(\sqrt{3}\right)^2}$$
$$x=\frac{3\sqrt{3}}{3}$$
$$x=\sqrt{3}$$

Thus solution set $=\left\{\sqrt{3},-\frac{4}{\sqrt{3}}\right\}$

(iv) $4x^2-14=3x$

Solution:

$$4x^2-14=3x$$
$$4x^2-3x-14=0$$

Comparing it with standard quadratic equation, we have
$ax^2+bx+c=0$

Here $a=4, b=-3, c=-14$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-14)}}{2(4)}$$
$$x=\frac{3\pm\sqrt{9+224}}{8}$$
$$x=\frac{3\pm\sqrt{233}}{8}$$
$$x=\frac{3\pm\sqrt{233}}{8}$$

Thus solution set $=\left\{\frac{3\pm\sqrt{233}}{8}\right\}$

(v) $6x^2-3-7x=0$

Solution:

Comparing it with standard quadratic equation, we have
$ax^2+bx+c=0$

Here $a=6, b=-7, c=-3$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-3)}}{2(6)}$$

$$x=\frac{7\pm\sqrt{49+72}}{12}$$

$$x=\frac{7\pm\sqrt{121}}{12}$$

$$x=\frac{7\pm 11}{12}$$

$$x=\frac{7+11}{12}\quad or\quad x=\frac{7-11}{12}$$

$$x=\frac{18}{12}\qquad x=-\frac{4}{12}$$

$$x=\frac{3}{2}\qquad x=-\frac{1}{3}$$

Thus solution set $=\left\{-\frac{1}{3},\frac{3}{2}\right\}$

(vi) $3x^2+8x+2=0$

Solution:

$3x^2+8x+2=0$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=3, b=8, c=2$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-8\pm\sqrt{(8)^2-4(3)(2)}}{2(3)}$$

$$x=\frac{-8\pm\sqrt{64-24}}{6}$$

$$x=\frac{-8\pm\sqrt{40}}{6}$$

$$x=\frac{-8\pm 2\sqrt{10}}{6}$$

$$x=\frac{2\left(-4\pm\sqrt{10}\right)}{6}$$

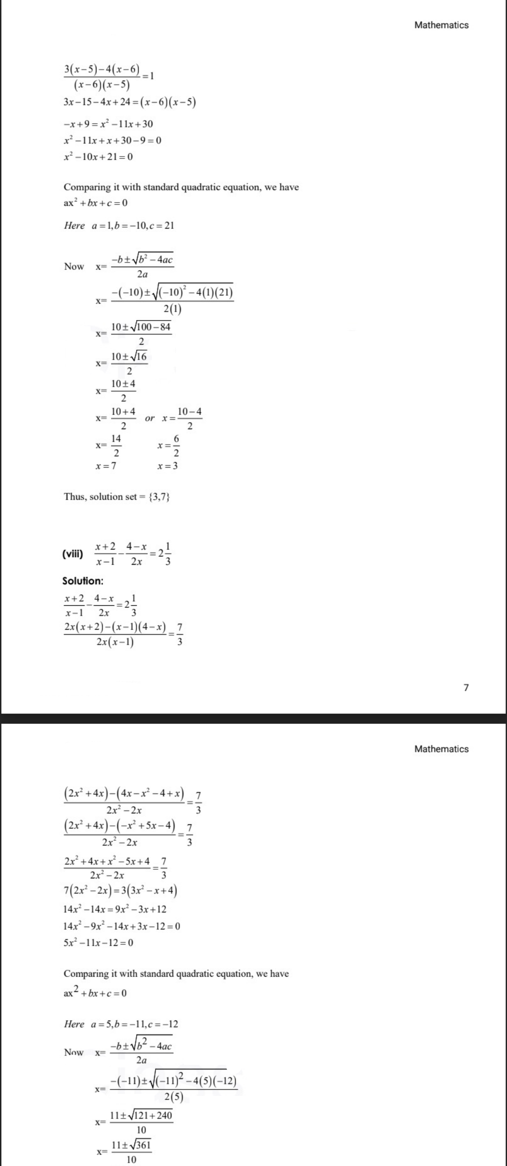
$$x=\frac{-4\pm\sqrt{10}}{3}$$

Thus solution set $=\left\{\frac{-4\pm\sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$

Solution:

$$\frac{3}{x-6}-\frac{4}{x-5}=1$$



$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$

$$3x-15-4x+24=(x-6)(x-5)$$

$$-x+9=x^2-11x+30$$

$$x^2-11x+x+30-9=0$$

$$x^2-10x+21=0$$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=1, b=-10, c=21$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm 4}{2}$$

$$x=\frac{10+4}{2} \quad or \quad x=\frac{10-4}{2}$$

$$x=\frac{14}{2} \qquad x=\frac{6}{2}$$

$$x=7 \qquad x=3$$

Thus, solution set $=\{3,7\}$

(viii) $\dfrac{x+2}{x-1}-\dfrac{4-x}{2x}=2\dfrac{1}{3}$

Solution:

$$\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$$

$$\frac{2x(x+2)-(x-1)(4-x)}{2x(x-1)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-x^2-4+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(-x^2+5x-4)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x+x^2-5x+4}{2x^2-2x}=\frac{7}{3}$$

$$7(2x^2-2x)=3(3x^2-x+4)$$

$$14x^2-14x=9x^2-3x+12$$

$$14x^2-9x^2-14x+3x-12=0$$

$$5x^2-11x-12=0$$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=5, b=-11, c=-12$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121+240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$$x=\frac{11+19}{10} \quad or \quad x=\frac{11-19}{10}$$

$$x=\frac{30}{10} \qquad x=-\frac{8}{10}$$

$$x=3 \qquad x=-\frac{4}{5}$$

Thus, solution set $=\left\{3,-\frac{4}{5}\right\}$

(ix) $\frac{a}{x-b}+\frac{b}{x-a}=2$

Solution:

$$\frac{a}{x-b}+\frac{b}{x-a}=2$$

$$\frac{a(x-a)+b(x-b)}{(x-a)(x-b)}=2$$

$$ax-a^2+bx-b^2=2(x-a)(x-b)$$

$$ax+bx-a^2-b^2=2(x^2-ax-bx+ab)$$

$$ax+bx-a^2-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-3(a+b)x+(2ab+a^2+b^2)=0$$

$$2x^2-3(a+b)x+(a+b)^2=0$$

Comparing it with standard quadratic equation, we have

$ax^2+bx+c=0$

Here $a=2, b=-3(a+b), c=(a+b)^2$

Now $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$$x=\frac{3(a+b)+(a+b)}{4} \qquad x=\frac{3(a+b)-(a+b)}{4}$$

$$x=\frac{3a+3b+a+b}{4} \qquad x=\frac{3a+3b-a-b}{4}$$

$$x=\frac{4a+4b}{4} \qquad x=\frac{2a+2b}{4}$$

$$x=\frac{4(a+b)}{4} \qquad x=\frac{2(a+b)}{4}$$

$$x=a+b, \qquad x=\frac{1}{2}(a+b)$$

Thus, solution set $=\left\{(a+b),\frac{1}{2}(a+b)\right\}$

(x) $-(l+m)-lx^2+(2l+m)x=0$, $l\neq 0$

Solution:

$$-(l+m)-lx^2+(2l+m)x=0$$

$$-lx^2+(2l+m)x-(l+m)=0$$

$$-\left[lx^2-(2l+m)x+(l+m)\right]=0$$

$$lx^2-(2l+m)x+(l+m)=0$$

Comparing it with standard quadratic equation, we have

$\mathrm{ax}^2+bx+c=0$

Here $a=l,b=-(2l+m),c=(l+m)$

Now $\mathrm{x}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\mathrm{x}=\frac{-\left[-(2l+m)\right]\pm\sqrt{\left[-(2l+m)\right]^2-4(l)(l+m)}}{2l}$$

$$\mathrm{x}=\frac{(2l+m)\pm\sqrt{(2l+m)^2-4l(l+m)}}{2l}$$

$$\mathrm{x}=\frac{(2l+m)\pm\sqrt{4l^2+4lm+m^2-4l^2-4lm}}{2l}$$

$$\mathrm{x}=\frac{(2l+m)\pm\sqrt{m^2}}{2l}$$

$$\mathrm{x}=\frac{(2l+m)\pm m}{2l}$$

$$\mathrm{x}=\frac{(2l+m)+m}{2l}, \qquad \mathrm{x}=\frac{(2l+m)-m}{2l}$$

$$\mathrm{x}=\frac{2l+2m}{2l}, \qquad \mathrm{x}=\frac{2l+m-m}{2l}$$

$$\mathrm{x}=\frac{2(l+m)}{2l}, \qquad \mathrm{x}=\frac{2l}{2l}$$

$$\mathrm{x}=\frac{l+m}{l}, \qquad \mathrm{x}=l$$

Thus, solution set $=\left\{\frac{l+m}{l},l\right\}$

Exercise 1.3

Q1. Solve the following equations.

(1) $2x^4 - 11x^2 + 5 = 0$

Solution:

$2x^4 - 11x^2 + 5 = 0 \quad ......(i)$

$Let\, x^2 = y \;\; then\;\; x^4 = y^2$

So eq.(i) becomes

$$\begin{aligned}
&2y^2 - 11y + 5 = 0\\
&2y^2 - 10y - y + 5 = 0\\
&2y(y-5) - 1(y-5) = 0\\
&(2y-1)(y-5) = 0
\end{aligned}$$

$Either\;(2y-1)=0 \;\; or \;\; (y-5)=0$

$2y = 1 \qquad y = 5$

$Put \quad y = \frac{1}{2}$ in $x^2 = y$, we get $\qquad Put\;\; y = 5 \;\; in\;\; x^2 = y, we\; get$

$$x^2 = \frac{1}{2} \qquad\qquad x^2 = 5$$

$$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \qquad\qquad \sqrt{x^2} = \pm\sqrt{5}$$

$$x = \pm\frac{1}{\sqrt{2}} \qquad\qquad x = \pm\sqrt{5}$$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

(2) $2x^4 = 9x^2 - 4$

Solution:

$2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0 \quad ......(i)$

$Let\, x^2 = y \;\; then\;\; x^4 = y^2$

So eq.(i) becomes

$$\begin{aligned}
&2y^2 - 9y + 4 = 0\\
&2y^2 - 8y - y + 4 = 0\\
&2y(y-4) - 1(y-4) = 0\\
&(2y-1)(y-4) = 0
\end{aligned}$$

$Either\;(2y-1)=0 \;\; or \;\; (y-4)=0$

$2y = 1 \qquad y = 4$

$Put \quad y = \frac{1}{2}$ in $x^2 = y$, we get $\qquad Put\;\; y = 4 \;\; in\;\; x^2 = y, we\; get$

$$x^2 = \frac{1}{2} \qquad\qquad x^2 = 4$$

$$\sqrt{x^2}=\pm\sqrt{\frac{1}{2}} \qquad\qquad \sqrt{x^2}=\pm\sqrt{4}$$

$$x=\pm\frac{1}{\sqrt{2}} \qquad\qquad x=\pm 2$$

Thus, solution set = $\left\{\pm\frac{1}{\sqrt{2}},\pm 2\right\}$

(3) $5x^{\frac{1}{2}}=7x^{\frac{1}{4}}-2$

Solution:

$$5x^{\frac{1}{2}}=7x^{\frac{1}{4}}-2$$

$$5x^{\frac{1}{2}}-7x^{\frac{1}{4}}+2=0 \qquad \ldots\ldots(i)$$

Let $x^{\frac{1}{4}}=y$ *then* $x^{\frac{1}{2}}=y^2$

So eq.(i) becomes

$$5y^2-7y+2=0$$
$$5y^2-5y-2y+2=0$$
$$5y(y-1)-2(y-1)=0$$
$$(5y-2)(y-1)=0$$

Either $(5y-2)=0$ *or* $(y-1)=0$

$$5y=2 \qquad\qquad y=1$$

$$y=\frac{2}{5} \qquad\qquad y=1$$

Put $y=\frac{2}{5}$ in $x^{\frac{1}{4}}=y$, we get *Put* $y=1$ *in* $x^{\frac{1}{4}}=y$, *we get*

$$x^{\frac{1}{4}}=y \qquad\qquad x^{\frac{1}{4}}=y$$

$$^{\frac{1}{4}}=\frac{2}{5} \qquad\qquad x^{\frac{1}{4}}=1$$

Taking power '4' on both siedes we get Taking power '4' on both siedes we get

$$\left(x^{\frac{1}{4}}\right)^4=\left(\frac{2}{5}\right)^4 \qquad\qquad \left(x^{\frac{1}{4}}\right)^4=(1)^4$$

$$x=\frac{2^4}{5^4} \qquad\qquad \left(x^{\frac{1}{4}}\right)^4=1$$

$$x=\frac{16}{625} \qquad\qquad x=1$$

Thus, solution set = $\left\{\frac{16}{625},1\right\}$

(4) $x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$

Solution:

$x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$

$x^{\frac{2}{3}}-15x^{\frac{1}{3}}+54=0 \qquad ......(i)$

Let $x^{\frac{1}{3}}=y$, *then* $x^{\frac{2}{3}}=y^2$

So eq.(i) becomes

$y^2-15y+54=0$

$y^2-9y-6y+54=0$

$y(y-9)-6(y-9)=0$

$(y-6)(y-9)=0$

Either $(y-9)=0$ *or* $(y-6)=0$

$y=9 \qquad y=6$

Put $y=9$ in $x^{\frac{1}{3}}=y$, we get

$x^{\frac{1}{3}}=y$

$x^{\frac{1}{3}}=9$

Taking cube on both siedes we get

$\left(x^{\frac{1}{3}}\right)^3=(9)^3$

$x=729$

Put $y=6$ *in* $x^{\frac{1}{3}}=y$, *we get*

$x^{\frac{1}{3}}=y$

$x^{\frac{1}{3}}=6$

Taking cube on both siedes we get

$\left(x^{\frac{1}{3}}\right)^3=(6)^3$

$x=216$

Thus, solution set = $\{729, 216\}$

(5) $3x^{-2}+5=8x^{-1}$

Solution:

$3x^{-2}+5=8x^{-1}$

$3x^{-2}-8x^{-1}+5=0 \qquad ......(i)$

Let $x^{-1}=y$, *then* $x^{-2}=y^2$

So eq.(i) becomes

$3y^2-8y+5=0$

$3y^2-5y-3y+5=0$

$y(3y-5)-1(3y-5)=0$

$(y-1)(3y-5)=0$

Either $y-1=0$ *or* $3y-5=0$

$y=1 \qquad 3y=5$

$y=1 \qquad y=\frac{5}{3}$

Put $v=1$ in $x^{-1}=y$, we get

$x^{-1}=y$

$x^{-1}=1$

$\frac{1}{x}=1$

$x=1$

Put $y=\frac{5}{3}$ *in* $x^{-1}=y$, *we get*

$x^{-1}=y$

$x^{-1}=\frac{5}{3}$

$\frac{1}{x}=\frac{5}{3}$

$x=\frac{3}{5}$

Thus, solution set = $\left\{1, \frac{3}{5}\right\}$

(6) $(2x^2+1)+\dfrac{3}{2x^2+1}=4$

Solution:

$(2x^2+1)+\dfrac{3}{2x^2+1}=4$ ......(i)

Let $2x^2+1=y,$

So eq.(i) becomes

$y+\dfrac{3}{y}=4$

Multiplying both sides by 'y', we get

$$\begin{aligned}
&y^2+3=4y\\
&y^2-4y+3=0\\
&y^2-3y-y+3=0\\
&y(y-3)-1(y-3)=0\\
&(y-1)(y-3)=0
\end{aligned}$$

Either $y-1=0$ *or* $y-3=0$

$y=1 \qquad y=3$

Put $y=1$ in $2x^2+1=y$, we get

$$\begin{aligned}
2x^2+1&=1\\
2x^2&=1-1\\
2x^2&=0\\
x^2&=0\\
x&=0
\end{aligned}$$

Put $y=3$ *in* $2x^2+1=y$, *we get*

$$\begin{aligned}
2x^2+1&=3\\
2x^2&=3-1\\
2x^2&=2\\
x^2&=1\\
x&=\pm1
\end{aligned}$$

Thus, solution set $=\{-1,0,1\}$

(7) $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$

Solution:

$\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$ ......(i)

Let $\dfrac{x}{x-3}=y,$

So eq.(i) becomes

$y+4\left(\dfrac{1}{y}\right)=4$

Multiplying both sides by 'y', we get

$$\begin{aligned}
&y^2+4=4y\\
&y^2-4y+4=0\\
&(y)^2-2(y)(2)+(2)^2=0\\
&(y-2)^2=0\\
&(y-2)=0
\end{aligned}$$

Put $y=2$ in $\frac{x}{x-3}=y$, we get

$$\begin{aligned}
\frac{x}{x-3}&=y\\
\frac{x}{x-3}&=2\\
2(x-3)&=x\\
2x-6&=x\\
2x-x&=6\\
x&=6
\end{aligned}$$

Thus, solution set $=\{6\}$

(8) $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$

Solution:

$$\begin{aligned}
&\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}\\
&\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=\frac{13}{6} \quad ......(i)
\end{aligned}$$

Let $\frac{4x+1}{4x-1}=y,$

So eq. (i) becomes

$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by '6y', we get

$$\begin{aligned}
&6y^2+6=13y\\
&6y^2-13y+6=0\\
&6y^2-9y-4y+6=0\\
&3y(2y-3)-2(2y-3)=0\\
&(3y-2)(2y-3)=0
\end{aligned}$$

Either $3y-2=0$ *or* $2y-3=0$

$$\begin{aligned}
3y&=2 \qquad & 2y&=3\\
y&=\frac{2}{3} \qquad & y&=\frac{3}{2}
\end{aligned}$$

Put $y=\frac{2}{3}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\begin{aligned}
\frac{4x+1}{4x-1}&=y\\
\frac{4x+1}{4x-1}&=\frac{2}{3}\\
3(4x+1)&=2(4x-1)\\
12x+3&=8x-2\\
12x-8x&=-2-3\\
4x&=-5\\
x&=-\frac{5}{4}
\end{aligned}$$

Put $y=\frac{3}{2}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\begin{aligned}
\frac{4x+1}{4x-1}&=y\\
\frac{4x+1}{4x-1}&=\frac{3}{2}\\
3(4x-1)&=2(4x+1)\\
12x-3&=8x+2\\
12x-8x&=2+3\\
4x&=5\\
x&=\frac{5}{4}
\end{aligned}$$

Thus, solution set = $\left\{\pm\frac{5}{4}\right\}$

(9) $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

Solution:

$$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12} \qquad \text{......}(i)$$

Let $\frac{x-a}{x+a}=y,$

So eq.(i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$

Multiplying both sides by '12y', we get

$$12y^2-12=7y$$
$$12y^2-7y-12=0$$
$$12y^2-16y+9y-12=0$$
$$4y(3y-4)+3(3y-4)=0$$
$$(4y+3)(3y-4)=0$$

$$\begin{aligned} &\textit{Either } 4y+3=0 && \textit{or } 3y-4=0 \\ &4y=-3 && 3y=4 \\ &y=-\frac{3}{4} && y=\frac{4}{3} \end{aligned}$$

Put $y=-\frac{3}{4}$ in $\frac{x-a}{x+a}=y,$ we get

$$\frac{x-a}{x+a}=y$$
$$\frac{x-a}{x+a}=-\frac{3}{4}$$
$$4(x-a)=-3(x+a)$$
$$4x-4a=-3x-3a$$
$$4x+3x=4a-3a$$
$$7x=a$$
$$x=\frac{a}{7}$$

Put $y=\frac{4}{3}$ in $\frac{x-a}{x+a}=y,$ we get

$$\frac{x-a}{x+a}=y$$
$$\frac{x-a}{x+a}=\frac{4}{3}$$
$$4(x+a)=3(x-a)$$
$$4x+4a=3x-3a$$
$$4x-3x=-4a-3a$$
$$x=-7a$$

Thus, solution set = $\left\{-7a,\frac{a}{7}\right\}$

(10) $x^4-2x^3-2x^2+2x+1=0$

Solution:

$x^4-2x^3-2x^2+2x+1=0$

Dividing each term by x^2, *we get*

$$\frac{x^4}{x^2}-2\frac{x^3}{x^2}-2\frac{x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$
$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$
$$x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad .....(i)$$

Let $\quad x-\frac{1}{x}=y$

$$\left(x-\frac{1}{x}\right)^2=y^2$$
$$x^2+\frac{1}{x^2}-2=y^2$$
$$x^2+\frac{1}{x^2}=y^2+2$$

So eq. (i) becomes

$$y^2+2-2y-2=0$$
$$y^2-2y=0$$
$$y(y-2)=0$$

Either $y=0$ *or* $y-2=0, \Rightarrow y=2$

Put $y=0$ in $x-\frac{1}{x}=y$, we get

$$x-\frac{1}{x}=y$$
$$x-\frac{1}{x}=0$$
$$x^2-1=0$$
$$x^2=1$$
$$x=\pm1$$

Put $y=2$ in $x-\frac{1}{x}=y$, we get

$$x-\frac{1}{x}=y$$
$$x-\frac{1}{x}=2$$
$$x^2-1=2x$$
$$x^2-2x-1=0$$
$$x^2-2x-1=0$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{4+4}}{2}$$
$$x=\frac{2\pm\sqrt{8}}{2}$$
$$x=\frac{2\pm2\sqrt{2}}{2}$$
$$x=\frac{2\left(1\pm\sqrt{2}\right)}{2}$$
$$x=1\pm\sqrt{2}$$

Thus, solution set $=\left\{\pm1, 1\pm\sqrt{2}\right\}$

(11) $2x^4+x^3-6x^2+x+2=0$

Solution:

$2x^4+x^3-6x^2+x+2=0$

Dividing each term by x^2, *we get*

$$\frac{2x^4}{x^2}+\frac{x^3}{x^2}-\frac{6x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}=0$$
$$2x^2+x-6+\frac{1}{x}+\frac{2}{x^2}=0$$

$$2x^2+\frac{2}{x^2}+x+\frac{1}{x}-6=0$$

$$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-6=0 \;.....(i)$$

Let $x+\frac{1}{x}=y$

$$\left(x+\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}+2=y^2$$

$$x^2+\frac{1}{x^2}=y^2-2$$

So eq. (i) becomes

$$2\left(y^2-2\right)+y-6=0$$

$$2y^2-4+y-6=0$$

$$2y^2+y-10=0$$

$$2y^2+5y-4y-10=0$$

$$y(2y+5)-2(2y+5)=0$$

$$(y-2)(2y+5)=0$$

Either $y-2=0$ *or* $2y+5=0$

$y=2 \qquad 2y=-5$

$y=2 \qquad y=\frac{-5}{2}$

Put $y=2$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$

$$x+\frac{1}{x}=2$$

$$x^2+1=2x$$

$$x^2-2x+1=0$$

$$(x-1)^2=0$$

$$\Rightarrow \quad x-1=0$$

$$\Rightarrow \quad x=1$$

Put $y=-\frac{5}{2}$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$

$$x+\frac{1}{x}=-\frac{5}{2}$$

$$2x^2+2=-5x$$

$$2x^2+5x+2=0$$

$$2x^2+4x+x+2=0$$

$$2x(x+2)+1(x+2)=0$$

Either $2x+1=0$ or $x+2=0$

$2x=-1 \qquad x=-2$

$x=-\frac{1}{2}$

Thus, solution set $=\left\{1,-2,-\frac{1}{2}\right\}$

(12) $4.2^{2x+1}-9.2^{x}+1=0$

Solution:

$4.2^{2x+1}-9.2^x+1=0$

$4.2^{2x}.2-9.2^x+1=0 \qquad ....(i)$

Let $2^x=y$ Then $2^{2x}=y^2$

So eq. (i) becomes

$4.y^2.2-9.y+1=0$

$8y^2-9y+1=0$

$8y^2-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(8y-1)(y-1)=0$

Either $8y-1=0$ *or* $y-1=0$

$8y=1 \qquad y=1$

$y=\frac{1}{8} \qquad y=1$

Put $y=\frac{1}{8}$ in $2^x=y$, we get

$2^x=y$

$2^x=\frac{1}{8}$

$2^x=\frac{1}{2^3}$

$2^x=2^{-3}$

$x=-3$

Put $y=1$ in $2^x=y$, we get

$2^x=y$

$2^x=1$

$2^x=2^0$

$x=0$

Thus, solution set $=\{-3,0\}$

(13) $3^{2x+2}=12.3^x-3$

Solution:

$3^{2x+2}=12.3^x-3$

$3^{2x}.3^2-12.3^x+3=0 \qquad ....(i)$

Let $3^x=y$ Then $3^{2x}=y^2$

So eq. (i) becomes

$y^2.9-12y+3=0$

$9y^2-12y+3=0$

$9y^2-9y-3y+3=0$

$9y(y-1)-3(y-1)=0$

$(9y-3)(y-1)=0$

Either $9y-3=0$ *or* $y-1=0$

$9y=3 \qquad y=1$

$y=\frac{3}{9} \qquad y=1$

$y=\frac{1}{3} \qquad y=1$

Put $y=\frac{1}{3}$ in $3^x=y$, we get

$3^x=y$

$3^x=\frac{1}{3}$

$3^x=3^{-1}$

$x=-1$

Put $y=1$ in $3^x=y$, we get

$3^x=y$

$3^x=1$

$3^x=3^0$

$x=0$

Thus, solution set $=\{-1,0\}$

(14) $2^x + 64.2^{-x} - 20 = 0$

Solution:

$2^x + 64.2^{-x} - 20 = 0 \qquad \text{.........}(i)$

Let $2^x = y$, Then $2^{-x} = \frac{1}{y}$

So eq. (i) becomes

$y - 64.\frac{1}{y} - 20 = 0$

$y^2 - 64 - 20y = 0$

$y^2 - 20y - 64 = 0$
$y^2 - 16y - 4y - 64 = 0$
$y(y-16) - 4(y-16) = 0$
$(y-4)(y-16) = 0$

Either $y - 4 = 0$ *or* $y - 16 = 0$
$y = 4 \qquad y = 16$

Put $y = 4$ in $2^x = y$, we get
$2^x = y$
$2^x = 4$
$2^x = 2^2$

Put $y = 16$ in $2^x = y$, we get
$2^x = y$
$2^x = 16$
$2^x = 2^4$
$x = 4$

Thus, solution set $= \{2, 4\}$

(15) $(x+1)(x+3)(x-5)(x-7) = 192$

Solution:

$(x+1)(x+3)(x-5)(x-7) = 192$

As $1 - 5 = 3 - 7$
$4 = 4$

So $[(x+1)(x-5)][(x+3)(x-7)] = 192$
$[x^2 - 5x + x - 5][x^2 - 7x + 3x - 21] = 192$
$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \qquad \text{......}(i)$

Let $x^2 - 4x = y$

$$(y-5)(y-21)=192$$
$$y^2-21y-5y+105=192$$
$$y^2-26y+105-192=0$$
$$y^2-26y-87=0$$
$$y^2-29y+3y-87=0$$
$$y(y-29)+3(y-29)=0$$
$$(y+3)(y-29)=0$$

Either $y+3=0$ *or* $y-29=0$

$y=-3$ $\quad$ $y=29$

Put $y=-3$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=-3$$
$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ *or* $x-3=0$

$x=1$ $\quad$ $x=3$

Put $y=29$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=29$$
$$x^2-4x-29=0$$

Here $a=1, b=-4, c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+116}}{2}$$
$$x=\frac{4\pm\sqrt{132}}{2}$$
$$x=\frac{4\pm2\sqrt{33}}{2}$$
$$x=\frac{2\left(2\pm\sqrt{33}\right)}{2}$$
$$x=2\pm\sqrt{33}$$

Thus, solution set $=\left\{1,3,2\pm\sqrt{33}\right\}$

(16) $(x-1)(x-2)(x-8)(x+5)+360=0$

Solution:

$$(x-1)(x-2)(x-8)(x+5)+360=0$$

As $\quad -1-2=-8+5$

$$-3=-3$$

So $\quad \left[(x-1)(x-2)\right]\left[(x-8)(x+5)\right]+360=0$

$$\left[x^2-2x-x+2\right]\left[x^2+5x-8x-40\right]+360=0$$
$$\left(x^2-3x+2\right)\left(x^2-3x-40\right)+360=0 \quad ......(i)$$

Let $\quad x^2-3x=y$

So eq.(i) *becomes*

$$(y+2)(y-40)+360=0$$
$$y^2-40y+2y-80+360=0$$
$$y^2-38y+280=0$$
$$y^2-28y-10y+280=0$$
$$y(y-28)-10(y-28)=0$$
$$(y-10)(y-28)=0$$

Either $y-10=0$ *or* $y-28=0$

$y=10$ $\quad$ $y=28$

Put $y=10$ in $x^2-3x=y$, we get

$$x^2-3x=y$$
$$x^2-3x=10$$
$$x^2-3x-10=0$$
$$x^2-5x+2x-10=0$$
$$x(x-5)+2(x-5)=0$$
$$(x+2)(x-5)=0$$

Either $x+2=0$ *or* $x-5=0$

$x=-2$ $\quad$ $x=5$

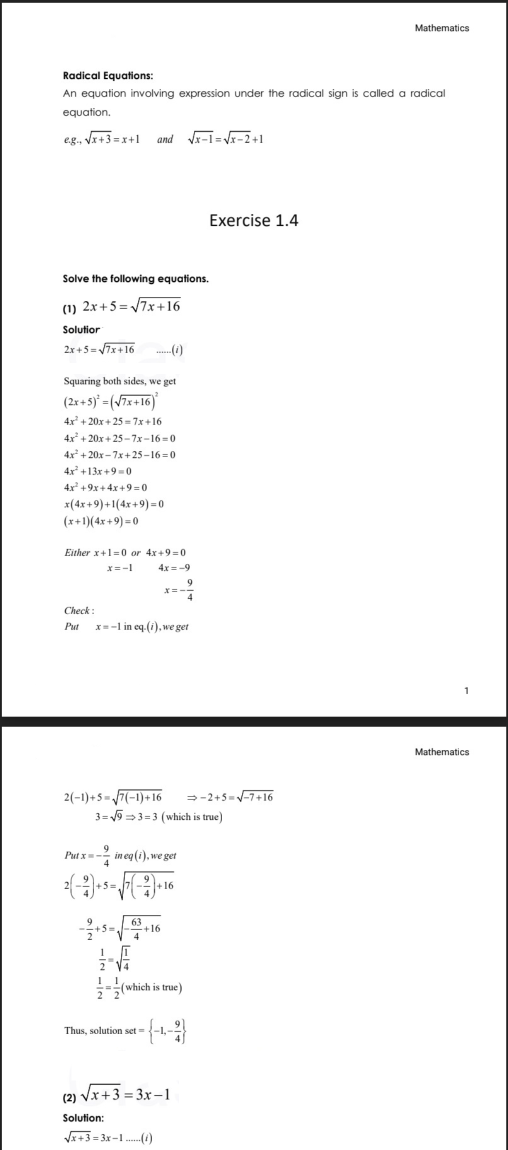
Put $y=28$ in $x^2-3x=y$, we get

$$x^2-3x=y$$
$$x^2-3x=28$$
$$x^2-3x-28=0$$
$$x^2-7x+4x-28=0$$
$$x(x-7)+4(x-7)=0$$
$$(x+4)(x-7)=0$$

Either $x+4=0$ *or* $x-7=0$

$x=-4$ $\quad$ $x=7$

Thus, solution set $=\{-4,-2,5,7\}$



Radical Equations:

An equation involving expression under the radical sign is called a radical equation.

e.g., $\sqrt{x+3}=x+1$ *and* $\sqrt{x-1}=\sqrt{x-2}+1$

Exercise 1.4

Solve the following equations.

(1) $2x+5=\sqrt{7x+16}$

Solution

$2x+5=\sqrt{7x+16}$ ......(i)

Squaring both sides, we get

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$
$$4x^2+20x+25=7x+16$$
$$4x^2+20x+25-7x-16=0$$
$$4x^2+20x-7x+25-16=0$$
$$4x^2+13x+9=0$$
$$4x^2+9x+4x+9=0$$
$$x(4x+9)+1(4x+9)=0$$
$$(x+1)(4x+9)=0$$

Either $x+1=0$ *or* $4x+9=0$

$x=-1$ $\quad 4x=-9$

$x=-\frac{9}{4}$

Check :

Put $x=-1$ in eq.(i), *we get*

$2(-1)+5=\sqrt{7(-1)+16} \quad \Rightarrow -2+5=\sqrt{-7+16}$

$3=\sqrt{9}\Rightarrow 3=3$ (which is true)

Put $x=-\frac{9}{4}$ *in eq*(i), *we get*

$$2\left(-\frac{9}{4}\right)+5=\sqrt{7\left(-\frac{9}{4}\right)+16}$$
$$-\frac{9}{2}+5=\sqrt{-\frac{63}{4}+16}$$
$$\frac{1}{2}=\sqrt{\frac{1}{4}}$$
$$\frac{1}{2}=\frac{1}{2}\text{(which is true)}$$

Thus, solution set $=\left\{-1,-\frac{9}{4}\right\}$

(2) $\sqrt{x+3}=3x-1$

Solution:

$\sqrt{x+3}=3x-1$ ......(i)

Squaring both sides, we get

$$\left(\sqrt{x+3}\right)^2 = (3x-1)^2$$
$$x+3 = 9x^2-6x+1$$
$$9x^2-6x+1-x-3=0$$
$$9x^2-7x-2=0$$
$$9x^2-9x+2x-2=0$$
$$9x(x-1)+2(x-1)=0$$
$$(9x+2)(x-1)=0$$

Either $9x+2=0$ *or* $x-1=0$

$$9x=-2 \qquad x=1$$
$$x=-\frac{2}{9} \qquad x=1$$

Check :

Put $x=-\frac{2}{9}$ in eq.(i), *we get*

$$\sqrt{-\frac{2}{9}+3} = 3\left(-\frac{2}{9}\right)-1$$
$$\sqrt{\frac{25}{9}} = -\frac{2}{3}-1$$
$$\sqrt{\frac{25}{9}} = -\frac{5}{3}$$
$$\frac{5}{3} \neq -\frac{5}{3} \text{ (which is not true)}$$

Put $x=1$ *in* $eq(i)$, *we get*

$$\sqrt{1+3} = 3(1)-1$$
$$\sqrt{4} = 3-1$$
$$2=2 \text{ (which is true)}$$

Thus, solution set = $\{1\}$

(3) $4x = \sqrt{13x+14}-3$

Solution:

$$4x = \sqrt{13x+14}-3 \qquad \text{......}(i)$$
$$4x+3 = \sqrt{13x+14}$$

Squaring both sides, we get

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$
$$16x^2+24x+9 = 13x+14$$
$$16x^2+24x-13x+9-14=0$$
$$16x^2+11x-5=0$$
$$16x^2+16x-5x-5=0$$
$$16x(x+1)-5(x+1)=0$$
$$(16x-5)(x+1)=0$$

$$\begin{aligned} &\textit{Either } 16x-5=0 \quad \textit{or} \quad x+1=0 \\ &\qquad 16x=5 \qquad\qquad x=-1 \\ &\qquad x=\frac{5}{16} \qquad\qquad x=-1 \end{aligned}$$

Check :

Put $x=\frac{5}{16}$ in eq.(i), *we get*

$$\begin{aligned} 4\left(\frac{5}{16}\right) &= \sqrt{13\left(\frac{5}{16}\right)+14}-3 \\ \frac{5}{4} &= \sqrt{\frac{65}{16}+14}-3 \\ \frac{5}{4} &= \sqrt{\frac{289}{16}}-3 \\ \frac{5}{4} &= \frac{17}{4}\text{-}3 \\ \frac{5}{4} &= \frac{5}{4} \quad (\text{which is true}) \end{aligned}$$

Put $x=-1$ *in eq*(i), *we get*

$$\begin{aligned} 4(-1) &= \sqrt{13(-1)+14}-3 \\ -4 &= \sqrt{-13+14}-3 \\ -4 &= \sqrt{1}-3 \\ -4 &= 1-3 \\ -4 &\neq -2 \quad (\text{which is not true}) \end{aligned}$$

Thus, solution set = $\left\{\frac{5}{16}\right\}$

(4) $\sqrt{3x+100}-x=4$

Solution:

$$\sqrt{3x+100}-x=4 \qquad \ldots\ldots(i)$$

$$\sqrt{3x+100}=4+x$$

Squaring both sides, we get

$$\begin{aligned} &\left(\sqrt{3x+100}\right)^2=(x+4)^2 \\ &3x+100=x^2+8x+16 \\ &x^2+8x+16-3x-100=0 \\ &x^2+5x-84=0 \\ &x^2+12x-7x-84=0 \\ &x(x+12)-7(x+12)=0 \\ &(x-7)(x+12)=0 \end{aligned}$$

$$\begin{aligned} &\textit{Either } x-7=0 \quad \textit{or} \quad x+12=0 \\ &\qquad x=7 \qquad\qquad x=-12 \end{aligned}$$

Check :

Put $x=7$ in eq.(i), *we get*

$$\sqrt{13(7)+100}-7=4 \qquad \Rightarrow \sqrt{21+100}-7=4$$

$$\sqrt{121}-7=4 \qquad \Rightarrow 11-7=4$$

$$4=4 \quad (\text{which is true})$$

Put $x=-12$ *in eq*(i), *we get*

$$\sqrt{3(-12)+100}-(-12)=4 \qquad \Rightarrow \sqrt{-36+100}+12=4$$

$$\sqrt{64}+12=4 \qquad \Rightarrow 8+12=4$$

$$20\neq 4 \quad (\text{which is not true})$$

Thus, solution set = $\{7\}$

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$
$$(x+5)+(x+21)+2\sqrt{(x+5)(x+21)}=x+60$$
$$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$$
$$2x+26+2\sqrt{x^2+26x+105}=x+60$$

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \quad ......(i)$$

Squaring both sides, we get

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$
$$2\sqrt{x^2+26x+105}=-x+34$$
$$2\sqrt{x^2+26x+105}=-(x-34)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2+26x+105}\right)^2=\left[-(x-34)\right]^2$$
$$4\left(x^2+26x+105\right)=x^2-68x+1156$$
$$4x^2+104x+420=x^2-68x+1156$$
$$4x^2-x^2+104x+68x+420-1156=0$$
$$3x^2+172x-736=0$$

Here a=3, b=172, c=-736

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$
$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$
$$x=\frac{-172\pm\sqrt{38416}}{6}$$
$$x=\frac{-172-196}{6} \quad or \quad x=\frac{-172+196}{6}$$
$$x=-\frac{368}{6} \qquad x=\frac{24}{6}$$
$$x=-\frac{184}{3} \qquad x=4$$

Check :

Put $x=-\frac{184}{3}$ in eq.(i), *we get*

$$\sqrt{-\frac{184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}}$$
$$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$
$$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}\ \text{(which is not true)}$$

$Put\ x=4\ \ in\, eq(i), we\ get$

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$
$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$
$$3+5=8$$
$$8=8\qquad\text{(which is true)}$$

Thus, solution set = $\{4\}$

(6) $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Solution:

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}\qquad ......(i)$$

Squaring both sides, we get

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$
$$(x+1)+(x-2)+2\sqrt{(x+1)(x-2)}=x+6$$
$$x+1+x-2+2\sqrt{x^2-x-2}=x+6$$
$$2x-1+2\sqrt{x^2-x-2}=x+6$$
$$2\sqrt{x^2-x-2}=-x+7$$
$$2\sqrt{x^2-x-2}=-(x-7)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2-x-2}\right)^2=\left[-(x-7)\right]^2$$
$$4\left(x^2-x-2\right)=x^2-14x+49$$
$$4x^2-4x-8=x^2-14x+49$$
$$4x^2-x^2-4x+14x-8-49=0$$
$$3x^2+10x-57=0$$

Here a=3, b=10, c= -57

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-10\pm\sqrt{(10)^2-4(3)(-57)}}{2(3)}$$
$$x=\frac{-10\pm\sqrt{100+684}}{6}$$
$$x=\frac{-10\pm\sqrt{784}}{6}$$
$$x=\frac{-10\pm 28}{6}$$
$$x=\frac{-10-28}{6}\ or\qquad x=\frac{-10+28}{6}$$
$$x=\frac{-38}{6}\qquad x=\frac{18}{6}$$
$$x=-\frac{19}{3}\qquad x=3$$

Check :

$Put\qquad x=-\frac{19}{3}$ in eq.(i), *we get*

$$\sqrt{-\frac{19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{-\frac{19}{3}+6}$$
$$\sqrt{-\frac{16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{-\frac{1}{3}}$$

$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$$\frac{9i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \text{(which is not true)}$$

Put $x=3$ *in eq* (i), *we get*

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$

$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

$$2+1=3$$

$$3=3 \quad \text{(which is true)}$$

Thus, solution set $=\{3\}$

(7) $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

Solution:

$$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x} \quad ......(i)$$

Squaring both sides, we get

$$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$

$$(11-x)+(6-x)-2\sqrt{(11-x)(6-x)}=27-x$$

$$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$$

$$17-2x-2\sqrt{x^2-17x+66}=27-x$$

$$-2\sqrt{x^2-17x+66}=27-x-17+2x$$

$$-2\sqrt{x^2-17x+66}=10+x$$

Squaring both sides, we get

$$\left(-2\sqrt{x^2-17x+66}\right)^2=(10+x)^2$$

$$4\left(x^2-17x+66\right)=100+20x+x^2$$

$$4x^2-68x+264=x^2+20x+100$$

$$4x^2-x^2-68x-20x+264-100=0$$

$$3x^2-88x+164=0$$

Here a = 3, b = -88, c = 164

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$

$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$

$$x=\frac{88\pm\sqrt{5776}}{6}$$

$$x=\frac{88\pm 76}{6}$$

$$x=\frac{88-76}{6} \quad or \quad x=\frac{88+76}{6}$$

$$x=\frac{12}{6} \qquad x=\frac{164}{6}$$
$$x=2 \qquad x=\frac{82}{3}$$

Check :

$$Put \quad x=2 \text{ in eq.}(i), we\,get$$
$$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$$
$$\sqrt{9}-\sqrt{4}=\sqrt{25}$$
$$3-2=5$$
$$1=5 \qquad \text{(which is not true)}$$

$$Put\; x=\frac{82}{3}\; in\,eq(i), we\,get$$
$$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$
$$\sqrt{-\frac{49}{3}}-\sqrt{-\frac{64}{3}}=\sqrt{-\frac{1}{3}}$$
$$\frac{7i}{\sqrt{3}}-\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$
$$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \text{(which is not true)}$$

Thus, solution set = $\{\ \}$

(8) $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution:

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a} \qquad \ldots\ldots(i)$$

Squaring both sides, we get

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$
$$(4a+x)+(a-x)-2\sqrt{(4a+x)(a-x)}=a$$
$$4a+x+a-x-2\sqrt{4a^2-3ax-x^2}=a$$
$$5a-2\sqrt{4a^2-3ax-x^2}=a$$
$$-2\sqrt{4a^2-3ax-x^2}=a-5a$$

$$-2\sqrt{4a^2-3ax-x^2}=-4a$$
$$\Rightarrow \sqrt{4a^2-3ax-x^2}=2a$$

Squaring both sides, we get

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$
$$4a^2-3ax-x^2=4a^2$$
$$4a^2-4a^2-3ax-x^2=0$$
$$-3ax-x^2=0$$
$$-x(x+3a)=0$$
$$x(x+3a)=0$$

$$Either\; x=0 \;\; or \;\; x+3a=0$$
$$x=-3a$$

Check :

$$Put \quad x=0 \text{ in eq.}(i), we\,get$$
$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$
$$\sqrt{4a}-\sqrt{a}=\sqrt{a}$$
$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$
$$\sqrt{a}=\sqrt{a} \qquad \text{(which is true)}$$

$$Put \quad x=-3a \text{ in eq.}(i), we\,get$$

$$\sqrt{4a+(-3a)}-\sqrt{a-(-3a)}=\sqrt{a}$$
$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$
$$\sqrt{a}-\sqrt{4a}=\sqrt{a}$$
$$\sqrt{a}-2\sqrt{a}=\sqrt{a}$$
$$-\sqrt{a}=\sqrt{a} \qquad \text{(which is not true)}$$

Thus, solution set $=\{0\}$

(9) $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \quad ......(i)$$

Let $x^2+x=y$

So eq.(i) becomes
$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides, we get
$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=1$$
$$(y+1)+(y-1)-2\sqrt{(y+1)(y-1)}=1$$
$$y+1+y-1-2\sqrt{y^2-1}=1$$
$$2y-2\sqrt{y^2-1}=1$$
$$-2\sqrt{y^2-1}=1-2y$$

Squaring both sides, we get
$$\left(-2\sqrt{y^2-1}\right)^2=(1-2y)^2$$
$$4\left(y^2-1\right)=1-4y+4y^2$$
$$4y^2-4=1-4y+4y^2$$
$$4y^2-4-1+4y-4y^2=0$$
$$4y-5=0$$
$$y=\frac{5}{4}$$

Put $y=\frac{5}{4}$ *in* $x^2+x=y$, *we get*
$$x^2+x=\frac{5}{4}$$
$$4x^2+4x=5$$
$$4x^2+4x-5=0$$

Here a=4, b=4, c=-5
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$
$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$
$$x=\frac{-4\pm4\sqrt{6}}{8}=\frac{4\left(-1\pm\sqrt{6}\right)}{8}=\frac{-1\pm\sqrt{6}}{2}$$

Thus, solution set $=\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

(10) $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3 \qquad ......(i)$$

Let $x^2+3x=y$

So eq.(i) becomes

$$\sqrt{y+8}+\sqrt{y+2}=3$$

Squaring both sides, we get

$$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$$
$$(y+8)+(y+2)+2\sqrt{(y+8)(y+2)}=9$$
$$y+8+y+2+2\sqrt{y^2+10y+16}=9$$
$$2y+10+2\sqrt{y^2+10y+16}=9$$
$$2\sqrt{y^2+10y+16}=9-2y-10$$
$$2\sqrt{y^2+10y+16}=-2y-1$$
$$2\sqrt{y^2+10y+16}=-(2y+1)$$

Squaring both sides, we get

$$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$$
$$4\left(y^2+10y+16\right)=4y^2+4y+1$$
$$4y^2+40y+64=4y^2+4y+1$$
$$4y^2-4y^2+40y-4y+64-1=0$$
$$36y=-63$$
$$y=-\frac{63}{36}$$

Put $y=-\frac{63}{36}$ *in* $x^2+3x=y$*, we get*

$$x^2+3x=-\frac{63}{36}$$
$$\Rightarrow 36x^2+108x=-63$$
$$36x^2+108x+63=0$$

Here a=36, b=108, c=63

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$
$$x=\frac{-108\pm\sqrt{11664+9072}}{72}$$

$$x=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{-108\pm36\sqrt{2}}{72}$$

$$x=\frac{36\left(-3\pm\sqrt{2}\right)}{72}$$

$$x=\frac{-3\pm\sqrt{2}}{2}$$

Thus, Solution set $=\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

(11) $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5 \qquad ......(i)$$

Let $x^2+3x=y$

So eq.(i) becomes

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring both sides, we get

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=(5)^2$$

$$(y+9)+(y+4)+2\sqrt{(y+9)(y+4)}=25$$

$$y+9+y+4+2\sqrt{y^2+13y+36}=25$$

$$2y+13+2\sqrt{y^2+13y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=-2y+12$$

$$2\sqrt{y^2+13y+36}=-2(y-6)$$

$$\Rightarrow\sqrt{y^2+13y+36}=-(y-6)$$

Squaring both sides, we get

$$\left(\sqrt{y^2+13y+36}\right)^2=\left[-(y-6)\right]^2$$

$$y^2+13y+36=y^2-12y+36$$

$$y^2-y^2+13y+12y+36-36=0$$

$$25y=0$$

$$\Rightarrow y=0$$

Put $y=0$ *in* $x^2+3x=y$, *we get*

$$x^2+3x=y$$

$$x^2+3x=0$$

$$x(x+3)=0$$

Either $x=0$ *or* $x+3=0$

$$x = -3$$

Thus, Solution set = $\{-3, 0\}$

Solved Miscellaneous Exercise. 1

Q1. Multiple Choice Questions:

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is:

(a) $bx+c=0, b\neq 0$ **(b)** $ax^2+bx+c=0, a\neq 0$

(c) $ax^2=bx, a\neq 0$ **(d)** $ax^2=0, a\neq 0$

(ii) The number of terms in a standard quadratic equation $ax^2+bx+c=0$ is

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

(iii) The number of methods to solve a quadratic equation is:

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

(iv) The quadratic formula is:

(a) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **(b)** $\frac{b\pm\sqrt{b^2-4ac}}{2a}$

(c) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ **(d)** $\frac{b\pm\sqrt{b^2-4ac}}{2a}$

(v) Two linear factors of $x^2-15x+56$ are:

(a) (x-7) and (x+8) **(b)** (x+7) and (x-8)

(c) (x-7) and (x-8) **(d)** (x+7) and (x+8)

(vi) An equation which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an

(a) Exponential equation **(b)** Reciprocal equation

(c) Radical Equation **(d)** None of these

(vii) An equation of the type $3^{x}+3^{2-x}+6=0$ is a/an:

(a) Exponential equation **(b)** Radical Equation

(c) Reciprocal equation **(d)** None of these

(viii) The solution set of equation $4x^2-16=0$ is

(a) $\{\pm 4\}$ **(b)** $\{4\}$

(c) $\{\pm 2\}$ **(d)** ± 2

(ix) An equation of the form $2x^4-3x^3+7x^2-3x+2=0$ is called a/an

(a) Reciprocal equation **(b)** Radical Equation

(c) Exponential equation **(d)** None of these

Answers:

(i)	b	**(ii)**	c	**(iii)**	c	**(iv)**	a	**(v)**	c

(vi)	b	**(vii)**	a	**(viii)**	c	**(ix)**	a		

Q2. Write short answers of the following question.

i. $x^2+2x-2=0$

Answer:

$x^2+2x-2=0$

Compare it with

$ax^2+bx+c=0$

Here $a=1, b=2, c=-2$

By using quadratic equation, we get

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+8}}{2}$$

$$x=\frac{-2\pm\sqrt{12}}{2}$$

$$x=\frac{-2\pm2\sqrt{3}}{2}$$

$$x=\frac{2\left(-1\pm\sqrt{3}\right)}{2}=-1\pm\sqrt{3}$$

Thus, the solution set $=\left\{-1\pm\sqrt{3}\right\}$

ii. **Solve by factorization** $5x^2=15x$

Answer:

$5x^2=15x$

$5x^2-15x=0$

$5x(x-3)=0$

Either $5x=0$ *or* $x-3=0$

$x=0$ $x=3$

Thus, the solution set $=\{0,3\}$

iii. **Write in standard form** $\frac{1}{x+4}+\frac{1}{x-4}=3$

Solution:

$\frac{1}{x+4}+\frac{1}{x-4}-3$ $\frac{x-4+x+4}{(x+4)(x-4)}=3$

$\frac{2x}{x^2-16}=3$ $3\left(x^2-16\right)=2x$

$3x^2-48-2x=0$

$3x^2-2x-48=0$

iv. **Write the names of the method for solving a quadratic equation.**

Answer: Solution of quadratic equations:

To find solution set of a quadratic equation, following methods are used:

(i) Factorization **(ii)** Completing square

(iii) Quadratic Formula

v. $Solve\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Answer:

$$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$$

Taking square root on both sides, we get

$$2x-\frac{1}{2}=\pm\frac{3}{2}$$
$$2x=\frac{1}{2}\pm\frac{3}{2}$$

$$\begin{array}{llll} Either & 2x=\frac{1}{2}+\frac{3}{2} & or & 2x=\frac{1}{2}-\frac{3}{2} \\ & 2x=2 & & 2x=-1 \\ & x=1 & & x=-\frac{1}{2} \end{array}$$

Thus, the solution set= $\left\{-\frac{1}{2},1\right\}$

vi. $Solve\ \sqrt{3x+18}=x$

Answer:

$$\sqrt{3x+18}=x$$

Taking square root on both sides, we get

$$3x+18=x^2$$
$$x^2-3x-18=0$$
$$x^2-6x+3x-18=0$$
$$x(x-6)+3(x-6)=0$$
$$(x-6)(x+3)=0$$

$$\begin{array}{llll} Either & x-6=0 & or & x+3=0 \\ & x=6 & & x=-3 \end{array}$$

Thus, the solution set= $\{-3,6\}$

vii. Define quadratic equation.

Ans: Quadratic Equation:

An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a **quadratic equation** or an equation of the second degree.

viii. Define reciprocal equation.

Ans: Reciprocal equations of the type:

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)=c=0 \text{ or } ax^4+bx^3+cx^2+bx+a=0$$

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Replacing x by $\frac{1}{x}$ in $ax^4-bx^3+cx^2-bx+a=0$

$a\left(\frac{1}{x}\right)^4-b\left(\frac{1}{x}\right)^3+c\left(\frac{1}{x}\right)^2-b\left(\frac{1}{x}\right)+a=0$ which is simplified as

$a-bx+cx^2-by^3+ax^4=0$ we get the same equation.

Thus $ax^4-bx^3+cx^2-bx+a=0$ is a reciprocal equation.

ix. Define exponential equation.

Ans: Exponential equation:

In exponential equations, variable occurs in exponent.

For example, $5^{1+x}+5x^{1-x}=26$.

x. Define radical equation.

Ans: Radical equations:

An equation involving expression under the radical sign is called a **radical equation.**

e.g., $\sqrt{x+3}=x+1$ and $\sqrt{x-1}=\sqrt{x-2}+1$

Q3. Fill in the blanks:

(i) The standard form of quadratic equation is ____________.

(ii) The number of methods to solve a quadratic equation are ___________.

(iii) The name of the method to derive a quadratic formula is ___________.

(iv) The solution of the equation $ax^2+bx+c=0, a\neq 0$ is ____________.

(v) The solution set of $25x^2-1=0$ is __________.

(vi) An equation of the form $2^{2x}-3.2^x+5=0$ is called a/an __________ equation.

(vii) The solution set of the equation $x^2-9=0$ is __________.

(viii) An equation of the type $x^4+x^3+x^2+x+1=0$ called a/an ____________.

(ix) A root of an equation, which do not satisfy the equation is called __________root.

(x) An equation involving impression of the variable under ______________is called radical.

Answer

(i)	$ax^2+bx+c=0$	**(ii)**	3	**(iii)**	Completing Square
(iv)	$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	**(v)**	$\left\{\pm\frac{1}{5}\right\}$	**(vi)**	Exponential
(vii)	$\{\pm 3\}$	**(viii)**	Reciprocal	**(ix)**	Extraneous
(x)	Radical Sign				

Summary

- ✓ An equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation or an equation of the second degree.
- ✓ A second-degree equation in one variable x, $ax^2+bx+c=0$
- ✓ Where $a\neq 0$ and a, b, care real numbers, is called the general or standard form of a quadratic equation.
- ✓ An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.
- ✓ In exponential equations, variable occur in exponents.
- ✓ An equation involving expression under the radical sign is called a radical equation.
- ✓ Quadratic formula for $ax^2+bx+c=0$, $a\neq 0$ is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
- ✓ Any quadratic equation is solved by the following three methods.
- ✓ **(i)** Factorization **(ii)** Completing square **(iii)**Quadratic formula