Exercise 12.1

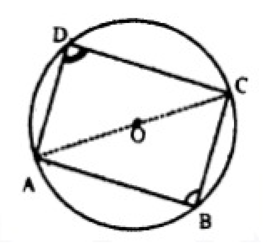
1. Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.

Solution:

Given:

A quadrilateral ABCD.

In scribed in a circle whose centre is O.



To Prove:

∠A + ∠C = 2∠rts.

And ∠B + ∠D = 2∠rts.

Construction:

Join A with O and O with C

Proof:

Arc ABC subtends ∠AOC at the centre O and.

∠ ADC at a point D on the remaining part of the circumference.

$m\angle ADC = \frac{1}{2}\angle AOC$ ....(i)

Similarly, arc ADC subtends reflex ∠AOC at the centre and ∠ABC on the circumferences.

$m\angle ABC = \frac{1}{2}\angle AOC$ ......(ii)

By Adding (i) & (ii)

$m\angle ADC + m\angle ABC = \frac{1}{2}[m\angle AOC + m\angle AOC]$

$m\angle D + m\angle B = \frac{1}{2}[4\angle rts]$

$m\angle D + m\angle B = 2rt\angle S.$ Proved.

Similarly, by Joining B with O and O with D it can be proved that

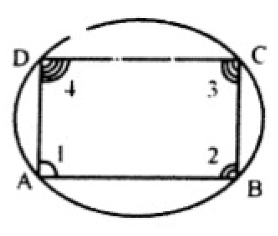
∠A + ∠C = 2∠rts.

2. Show that paralleloaram inscribed in a circle will be a rectangle.

Solution:

Given:

ABCD is a parallelogram inscribed in a circle with centre "O".



To Prove:

ABCD is a rectangle.

Proof

$m\angle 1 + m\angle 3 = 180°$ ________(i)

[cyclic quadrilateral]

But $m\angle 1 + m\angle 3 = 180°$ ________(ii)

[opp. ∠sof a parallelogram)

From (i) & (ii) we get

$2m\angle 1 = 180°$

⇒ $m\angle 1 = 90''$

∴ $m\angle 1 = m\angle 3 = 90°$.

By this

$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90°$.

Hence, the parallelogram ABCD is a rectangle.

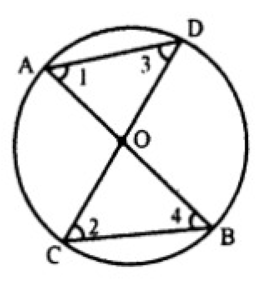
3. AOB and COD are two intersecting chords of a circle. Show that △s AOD and BOC are equiangular.

Solution:

Given:

Two chords AOB & COD

Intersecting each other at O.



To Prove:

△AOD and △COD are equiangular.

Proof:

△AOB and △COB

∠1 = ∠2.

∠3 = ∠4

[angles in the same segment of a circle].

∴∠AOB = ∠COB.

(Vertical angles).

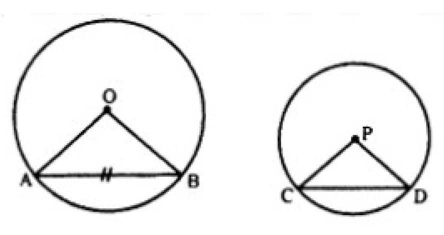
△AOD and △COD are equiangular triangles.

4. $\overline{AD}$, and $\overline{BC}$ are two parallel chords of a circle prove that arc $\overline{AB}$ ≅ arc $\overline{CD}$ and arc $\overline{AC}$ ≅ arc $\overline{BD}$.

Solution:

Given:

Two circles, C (O, r) and C (P, r) chord AB = chord CD.



To Prove:

$\overset{\frown}{AB} = \overset{\frown}{CD}$

Construction:

Join OA, OB and PC, PD.

Proof:

In △AOB & △CPD

OA ≅ PC

and OB ≅ PD [radii of equal circles]

AB = CD [given]

∴ △AOB ≅ △CPD S.S.S.

∴ ∠AOB ≅ ∠CPD

There are the angles subtended by the AB and CD at the centre of equal circle

$\overset{\frown}{AB} = \overset{\frown}{CD}$

THEOREM 1

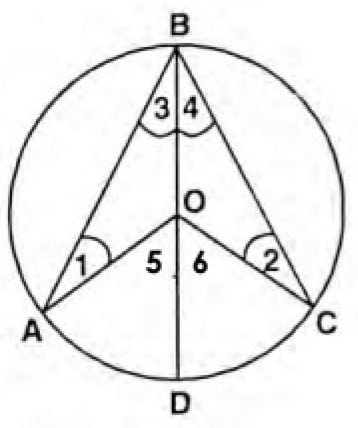
12.1 (i) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Given:

AC is an arc of a circle with centre O.

Whereas ∠AOC is the central angle

and ∠ABC is circum angle.



To prove:

m∠AOC = 2m∠ABC

Construction:

Join B with O and produce it to meet the circle at D.

Write angles ∠1, ∠2, ∠3, ∠4, ∠5 and ∠6 as shown in the figure.

Proof

Statements	Reasons
As m∠1 = m∠3 (i)	Angles opposite to equal sides in △OAB
And m∠2 = m∠4 (ii)	Angles opposite to equal sides in △OBC
Now m∠5 = m∠1 + m∠3 (iii)	External angle is the sum of internal opposite angles.
Similarly m∠6 = m∠2 + m∠4 (iv)	
Again m∠5 = m∠3 + m∠3 = 2m∠3 (v)	Using (i) and (iii)
and m∠6 = m∠4 + m∠4 = 2m∠4 (vi)	Using (ii) and (iv)
Then from figure	
⇒m∠5 + m∠6 = 2m∠3 + 2m∠4	Adding (v) and (vi)

⇒ m∠AOC = 2(m∠3 + m∠4) = 2m∠ABC	

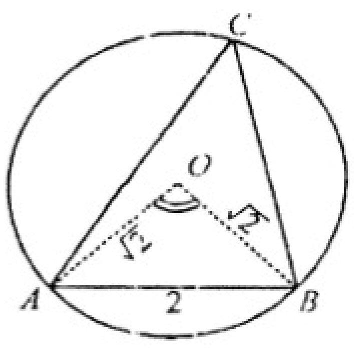
Example:

The radius of a circle is $\sqrt{2}$ cm. A chord 2 cm in length divides the circle into two segments. Prove that the angle of large segment is 45°.

Given:

In a circle with centre O and radius mOA = mOB = $\sqrt{2}$ cm,

The length of chord AB = 2 cm divides the circle into two segments with ACB as larger one.



To prove:

m∠ACB = 45°

Construction:

Join O with A and O with B.

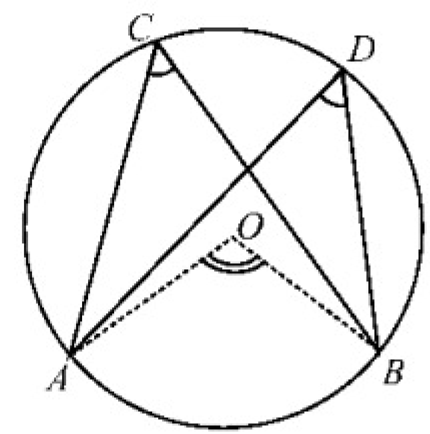
Proof:

Statements	Reasons
In △OAB	
$(\overline{OA})^2 + (\overline{OB})^2 = (\sqrt{2})^2 + (\sqrt{2})^2$	$m\overline{OA} = m\overline{OB} = \sqrt{2}$ cm
$= 2 + 2 = 4$	
$= (2)^2 = (\overline{AB})^2$	Given mAB = 2 cm
△AOB is right angled triangle	
With m∠AOB = 90°	Which being a central angle standing on an arc AB

Then m∠ACB = $\frac{1}{2}$ m∠AOB	By theorem 1
$= \frac{1}{2}$ (90°) = 45°	Circum angle is half of the central angle.

THEOREM 2

12.1 (ii) Any two angles in the same segment of a circle are equal.



Given:

$\angle ACB$ and $\angle ADB$ are the circum angles in the same segment of a circle with centre O.

To prove:

$m\angle ACB = m\angle ADB$

Construction:

Join O with A and O with B.

So that $\angle AOB$ is the central angle.

Proof:

Statements	**Reasons**
Standing on the same arc AB of a circle.	
$\angle AOB$ is the central angle whereas	Construction
$\angle ACB$ and $\angle ADB$ are circum angles	Given
$m\angle AOB = 2m\angle ACB$ (i)	By theorem 1
and $m\angle AOB = 2m\angle ADB$ (ii)	By theorem 1
$\Rightarrow$ $2\angle ACB = 2m\angle ADB$	Using (i) and (ii)
Hence, $m\angle ACB = m\angle ADB$	

THEOREM 3

12.1 (iii) The angle

- **in a semi-circle is a right angle,**
- **in a segment greater than a semi-circle is less than a right angle,**
- **in a segment less than a semi-circle is greater than a right angle;**

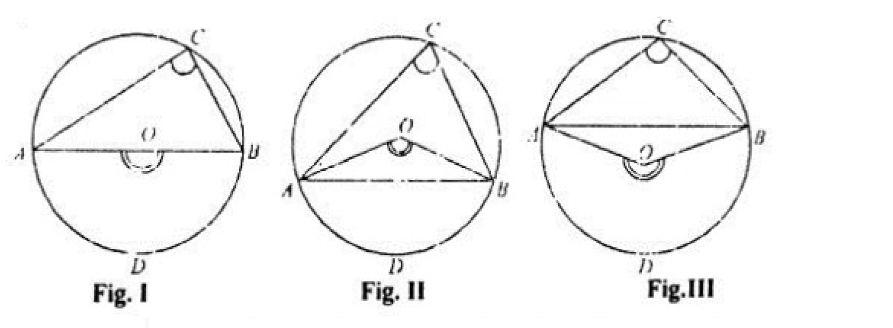


Fig. I Fig. II Fig.III

Given:

AB is the chord corresponding to an arc ADB

Whereas ∠AOB is a central angle and ∠ACB is

a circum angle of a circle with centre O.

To prove:

In fig. (I) If sector ACB is a semi-circle then m∠ACB = 1∠rt

In fig (II) If sector ACB is greater than a semi-circle then m∠ACB < 1∠rt

In fig (III) If sector ACB is less than a semi-circle then m∠ACB > 1∠rt

Proof:

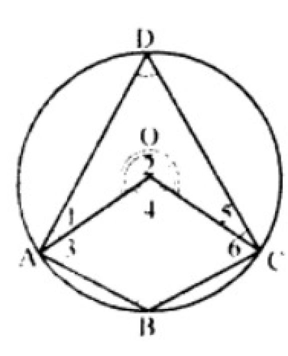
Statements	Reasons
In each figure, $\overline{AB}$ is the chord of a circle with centre O.	Given
∠AOB is the central angle standing on an arc ADB.	
Whereas ∠ACB is the circum angle	Given
Such that m∠AOB = 2m∠ACB (i)	By theorem 1
Now in fig (I) m∠AOB = 180°	A straight angle
∴ m∠AOB = 2∠rt (ii)	
⇒ m∠ACB = 1∠rt	Using (i) and (ii)
In fig (II) m∠AOB < 180°	
∴ m∠AOB < 2∠rt (iii)	
⇒ m∠ACB <1∠rt	Using (i) and (iii)
In fig (III) m∠AOB > 180°	
∴ m∠AOB > 2∠rt (iv)	
⇒ m∠ACB > 1∠rt	Using (i) and (iv)

Corollary 1. The angles subtended by an arc at the circumference of a circle are equal.

Corollary 2. The angles in the same segment of a circle are congruent.

Theorem 4

12.1 (iv) The opposite angles of any quadrilateral inscribed in a circle are supplementary.



Given.

ABCD is a quadrilateral inscribed in a circle with centre O.

To prove:

$$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$.

Write ∠l, ∠2, ∠3, ∠4, ∠5 and ∠6 as shown in the figure.

Proof:

Statements	Reasons
Standing on the same arc ADC, ∠2 is a central angle	Arc ADC of the circle with centre O.

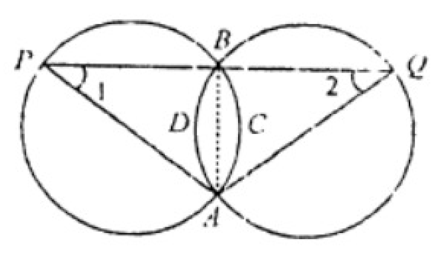
Whereas ∠B is the circum angle	
∴ $m\angle B = \frac{1}{2}(m\angle 2)$ (i)	By theorem 1
Standing on the same arc ABC, ∠4 is a central angle whereas ∠D) is the circum angle	Arc ABC of the circle with centre O.
$m\angle D = \frac{1}{2}(m\angle 4)$ (ii)	By theorem 1
$m\angle B + m\angle D = \frac{1}{2}m\angle 2 + \frac{1}{2}m\angle 4$	Adding (i) and (ii)
$= \frac{1}{2}(m\angle 2 + m\angle 4)$	
$= \frac{1}{2}$ (Total central angle)	
i.e., $m\angle B + m\angle D = \frac{1}{2}(4\angle rt) = 2\angle rt$	
Similarly, m∠A + m∠C = 2∠rt	

Corollary 1. In equal circles or in the same circle if two minor arcs are equal then angles inscribed by their corresponding major arcs arc also equal.

Corollary 2. In equal circles or in the same circle, two equal arcs subtend equal angles at the circumference and vice versa.

Example 1:

Two equal circles intersect in A and B. Through B, a straight line is drawn to meet the circumferences at P and Q respectively. Prove that mAP = mAQ.



Given:

Two equal circles cut each other at points A and B. A straight line PBQ drawn through B meets the circles at P and Q respectively.

To prove:

$m\overline{AP} = m\overline{AQ}$

Construction:

Join the points A and B. Also $\overline{AP}$ and $\overline{AQ}$

Write ∠l and ∠2 as shown in the figure.

Proof:

Statements	Reasons
∴ $m\overset{\frown}{ACB} = m\overset{\frown}{ADB}$	Arcs about the common chord AB.
∴ m∠1 = m∠2	Corresponding angles made by opposite arcs
So, $m\overline{AQ} = m\overline{AP}$	Sides opposite to equal angles in △APQ.
Or $m\overline{AP} = m\overline{AQ}$	

Example 2:

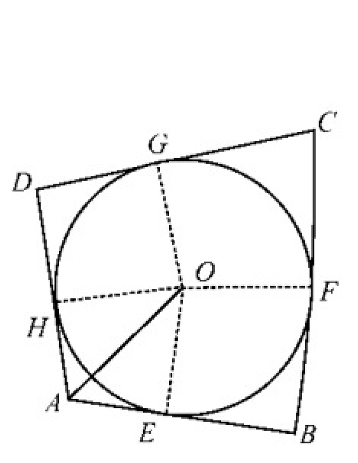
ABCD is a quadrilateral circumscribed about a circle.

Show that $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Given:

ABCD is a quadrilateral circumscribed about a circle with centre O.

So that each side becomes tangent to the circle.



To prove:

$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

Construction:

Drawn $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{BC}$, $\overline{OG} \perp \overline{CD}$ and $\overline{OH} \perp \overline{DA}$

Proof:

Statements	Reasons
∴ $m\overline{AE} = m\overline{HA}$; $m\overline{EB} = m\overline{BF}$ ...(i)	Since tangents drawn from a point to the circle are equal in length
$m\overline{CG} = m\overline{FC}$ and $m\overline{GD} = m\overline{DH}$ ...(ii)	
$(m\overline{AE} + m\overline{EB}) + (m\overline{CG} + m\overline{GD})$	Adding (i) & (ii)
$= (m\overline{BF} + m\overline{FC}) + (m\overline{DH} + m\overline{HA})$	
or $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$	

MISCELLANEOUS EXERCISE 12

Q1. Multiple Choice Questions:

Four possible answers are given for the following question.

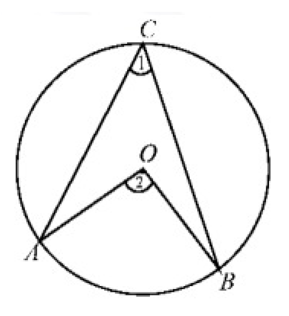
Tick (✓) the correct answer.

(i) A circle passes through the vertices of a right angled △ABC with mAC = 3 cm and mBC = 4 cm, m∠C = 90°. Radius of the circle is:

(a) 1.5 cm **(b)** 2.0 cm

(c) 2.5 cm **(d)** 3.5 cm

(ii) In the adjacent circular figure, central and inscribed angles stand on the same arc $\overline{AB}$. Then



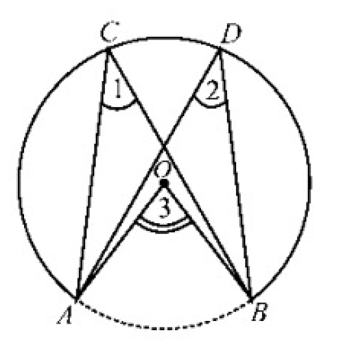
(a) m∠1 = m∠2 **(b)** m∠1 = 2m∠2

(c) m∠2 = 3m∠1 **(d)** m∠2 = 2m∠1

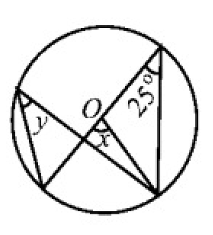
(iii) In the adjacent figure if m∠3 = 75°, then find m∠1 and m∠2.

(a) $37\frac{1^\circ}{2}, 37\frac{1^\circ}{2}$ **(b)** $37\frac{1^\circ}{2}, 75^\circ$

(c) $75^\circ, 37\frac{1^\circ}{2}$ **(d)** $75^\circ, 75^\circ$



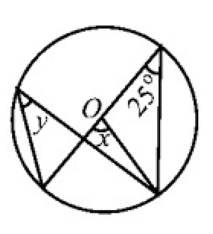
(iv) Given that O is the centre of the circle. The angle marked x will be:



(a) $12\frac{1^\circ}{2}$ **(b)** 25°

(c) 50° **(d)** 75°

(v) Given that O is the centre of the circle. The angle marked y will be:

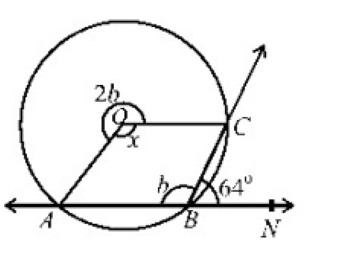


(a) $12\frac{1^\circ}{2}$ **(b)** 25°

(c) 50° **(d)** 75°

(vi) In the figure, O is the centre of the circle and $\overleftrightarrow{ABN}$ is a straight line.

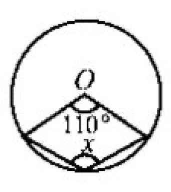
The obtuse angle AOC = x is:



(a) 32° **(b)** 64°

(c) 96° **(d)** 128°

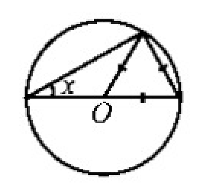
(vii) In the Figure, O is the centre of the circle, then the angle x is:



(a) 55° **(b)** 110°

(c) 220° **(d)** 125°

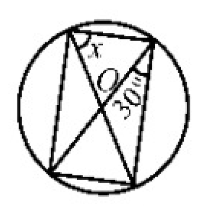
(viii) In the figure, O is the centre of the circle then angle x is:



(a) 15° **(b)** 30°

(c) 45° **(d)** 60°

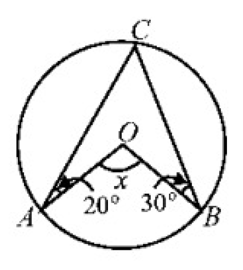
(ix) In the figure, O is the centre of the circle then the angle x is:



(a) 15° **(b)** 30°

(c) 45° **(d)** 60°

(x) In the figure, O is the centre of the circle then the angle x is:



(a) 50° **(b)** 75°

(c) 100° **(d)** 125°

Answers:

(i)	c	**(ii)**	d	**(iii)**	a	**(iv)**	c	**(v)**	b
(vi)	d	**(vii)**	d	**(viii)**	b	**(ix)**	d	**(x)**	c

Summary

✓ The angle subtended by an arc at the centre of a circle is called is central angle.

✓ A central angle is subtended by two radii with the vertex at the centre of the circle.

✓ The angle subtended by an arc of a circle at its circumference is called a circumangle.

✓ A circumangle is subtended between any two chords of a circle, having common point on its circumference.

✓ A quadrilateral is called cyclic when a circle can be drawn through its four vertices.

✓ The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

✓ Any two angles in the same segment of a circle are equal.

✓ The angle

- in a semi-circle is a right angle,
- in a segment greater than a semi-circle is less than a right angle.
- in a segment less than a semi-circle is greater than a right angle.

✓ The opposite angles of any quadrilateral inscribed in a circle are supplementary.