EXERCISE 3.1

1. **Express the following as a ratio a : b and as a fraction in its simplest (lowest) form.**

(i) Rs.750, Rs.1250

Solution:

$$
\begin{aligned}
& 750:1250 \\
&= \frac{750}{10}:\frac{1250}{10} \qquad (\text{divided by } 10) \\
&= 75:125 \\
&= \frac{75}{5}:\frac{125}{5} \qquad (\text{Divided by } 5) \\
&= 15:25 \\
&= \frac{15}{5}:\frac{25}{5} \qquad (\text{Divided by } 5) \\
&= 3:5 \\
&= \frac{3}{5}
\end{aligned}
$$

(ii) 450cm, 3m

Solution:

$$
\begin{aligned}
&= 450cm:300cm \qquad 1\text{m} = 100cm \\
&= 450:300 \\
&= \frac{450}{10}:\frac{300}{10} \qquad (\text{divided by } 10) \\
&= 45:30 \\
&= \frac{45}{5}:\frac{30}{5} \qquad (\text{Divided by } 5)
\end{aligned}
$$

$$
\begin{aligned}
&= 9:6 \\
&= \frac{9}{3}:\frac{6}{3} \qquad (\text{Divided by } 3) \\
&= 3:2 \\
&= \frac{3}{2}
\end{aligned}
$$

(iii) 4kg, 2kg 750gm

Solution:

$$
\begin{aligned}
& 4\text{kg}:2kg\ 750gm \\
&= 4000g:2750g \qquad 1kg = 1000g \\
&= 4000:2750 \\
&= \frac{4000}{10}:\frac{2750}{10} \qquad (\text{divided by } 10) \\
&= 400:275 \\
&= \frac{400}{5}:\frac{275}{5} \qquad (\text{Divided by } 5) \\
&= 80:55 \\
&= \frac{80}{5}:\frac{55}{5} \qquad (\text{Divided by } 5)
\end{aligned}
$$

$= 16 : 11$

$= \frac{16}{11}$

(iv) 27min. 30 sec, 1 hour

Solution:

$27\text{min. } 30\text{sec} : 1 \text{ hour}$

$= (27 \times 60 + 30)\text{sec} : (1 \times 60 \times 60)\text{sec}$

$= 1650 : 3600$

$= \frac{1650}{10} : \frac{3600}{10} \qquad (\text{divided by } 10)$

$= 165 : 360$

$= \frac{165}{5} : \frac{360}{5} \qquad (\text{Divided by } 5)$

$= 33 : 72$

$= \frac{33}{3} : \frac{72}{3} \qquad (\text{Divided by } 3)$

$= 11 : 24$

$= \frac{11}{24}$

(v) 75°, 225°

Solution:

$75^o : 225^o$

$= \frac{75}{5} : \frac{225}{5} \qquad (\text{divided by } 5)$

$= 15 : 45$

$= \frac{15}{5} : \frac{45}{5} \qquad (\text{divided by } 5)$

$= 3 : 9$

$= \frac{3}{3} : \frac{9}{3} \qquad (\text{Divided by } 5)$

$= 1 : 3$

$= \frac{1}{3}$

2. In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

(i) Boys to total students

Solution:

Total students = 60

Number of girls students = 25

Number of boys students = 60 — 25 = 35

$$\begin{aligned}35:60 &= \frac{35}{5}:\frac{60}{5} \qquad \text{(divided by 5)}\\ &= 7:12\end{aligned}$$

(ii) Boys to girls

Solution:

$$\begin{aligned}35:25 &= \frac{35}{5}:\frac{25}{5} \qquad \text{(divided by 5)}\\ &= 7:5\end{aligned}$$

3. If 3 (4x - 5y) = 2x - 7y, find the ratio x : y.

Solution:

$$3(4x-5y)=2x-7y$$
$$12x-15y=2x-7y$$
$$12x-2x=15y-7y$$
$$10x=8y$$
$$\frac{x}{y}=\frac{8}{10}$$
$$\frac{x}{y}=\frac{4}{5}$$
$$\Rightarrow x:y=4:5$$

4. Find the value of p, if the ratios 2p + 5 : 3p + 4 and 3 : 4 are equal.

Solution:

As the given ratios are equal, so

$$2p+5:3p+4=3:4$$
$$\frac{2p+5}{3p+4}=\frac{3}{4}$$
$$3(3p+4)=4(2p+5)$$
$$9p+12=8p+20$$

$$9p-8p=20-12$$
$$p=8$$

5. If the ratios 3x + 1 : 6 + 4x and 2 : 5 are equal. Find the value of x.

Solution:

As the given ratios are equal, so

$$3x+1:6+4x=2:5$$
$$\frac{3x+1}{6+4x}=\frac{2}{5}$$
$$5(3x+1)=2(6+4x)$$
$$15x+5=12+8x$$
$$15x-8x=12-5$$
$$7x=7$$

Dividing throughout by '7', we get

$$x=1$$

6. Two numbers are in the ratio 5 : 8. If 9 is added to each number, we get a new ratio 8:11. Find the numbers.

Solution:

Because the ratio of two numbers is 5 : 8.

Multiply each number of the ratio with x. then the numbers be 5x, 8x and the ratio becomes 5x : 8x.

Now according to the given condition, we have

$$\frac{5x+9}{8x+9}=\frac{8}{11}$$
$$8(8x+9)=11(5x+9)$$
$$64x+72=55x+99$$
$$64x-55x=99-72$$
$$9x=27$$

Dividing throughout by '9', we get
$$x=3$$

The required numbers are
$$5(x)=5(3)=15$$
$$8(x)=8(3)=24$$

7. If 10 is added in each number of the ratio 4 : 13, we get a new ratio 1 : 2. What are the numbers?

Solution:

Because the ratio of two numbers is 4 : 13.

Multiply each number of the ratio with x. then the numbers be 4x, 13x and the ratio becomes 4x : 13x.

Now according to the given condition, we have

$$\frac{4x+10}{13x+10}=\frac{1}{2}$$
$$1(13x+10)=2(4x+10)$$
$$13x+10=8x+20$$
$$13x-8x=20-10$$
$$5x=10$$

Dividing both sides by '5', we get
$$x=2$$

The required numbers are
$$4x=4(2)=8$$
$$13x=13(2)=26$$

8. Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250.

Solution:

Let the cost of 8 kg mangoes be x - rupees.

Then in proportion form, we have

8 kg : 5 kg :: Rs. x : Rs. 250

Product of means = Product of extremes

(5)(x) = (8)(250)

5x = 8 x 250

$$x = \frac{8\times 250}{5}$$

x = 8x50

x = Rs. 400

9. If a : b = 7 : 6, find the value of 3a + 5b : 7b - 5a.

Solution:

Given that $a{:}b = 7:6$

or $\frac{a}{b} = \frac{7}{6}$

$$\text{Now,}\quad 3a+5b:7b-5a = \frac{3a+5b}{7b-5a}$$

$$= \frac{\frac{3a+5b}{b}}{\frac{7b-5a}{b}} \quad \text{(Dividing numerator and denominator by 'b')}$$

$$= \frac{3\left(\frac{a}{b}\right)+5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right)-5\left(\frac{a}{b}\right)}$$

$$= \frac{3\left(\frac{7}{6}\right)+5}{7-5\left(\frac{7}{6}\right)}$$

$$= \frac{\frac{21}{6}+5}{7-\frac{35}{6}} = \frac{\frac{21+5\times 6}{6}}{\frac{7\times 6-35}{6}}$$

$$= \frac{\frac{21+30}{6}}{\frac{42-35}{6}} = \frac{\frac{51}{6}}{\frac{7}{6}}$$

$$= \frac{51}{6}\times\frac{6}{7} = \frac{51}{7}$$

Hence,

$3a+5b:7b-5a = 51:7$

10. Complete the following:

(i) If $\frac{24}{7} = \frac{6}{x}$, then $4x =$ ________

Solution:

$$\frac{24}{7} = \frac{6}{x}$$

$$(24)(x) = (6)(7)$$

$$(6\times 4)(x) = (6)(7)$$

$$4x = \frac{6\times 7}{6}$$

$$4x = 7$$

(ii) If $\frac{5a}{3x} = \frac{15b}{y}$, then $ay =$ ________

Solution:

$$\frac{5a}{3x} = \frac{15b}{y}$$
$$(5a)(y) = (3x)(15b)$$
$$(5)(ay) = (3x)(15b)$$
$$ay = \frac{(3x)(15b)}{5}$$
$$ay = 9bx$$

(iii) If $\frac{9pq}{2lm} = \frac{18p}{5m}$, then $5q =$ ________

Solution:

$$\frac{9pq}{2lm} = \frac{18p}{5m}$$
$$(5m)(9pq) = (2lm)(18p)$$
$$(9mp)(5q) = (2lm)(18p)$$
$$5q = \frac{(2lm)(18p)}{(9mp)}$$
$$5q = (2l)(2)$$
$$5q = 4l$$

11. Find x in the following proportions.

(i) 3x - 2 : 4 :: 2x + 3 : 7

Solution:

$$3x - 2 : 4 :: 2x + 3 : 7$$
$$3x - 2 : 4 = 2x + 3 : 7$$

$\therefore$ Product of extremes = Product of means

$$(3x - 2)(7) = (4)(2x + 3)$$
$$21x - 14 = 8x + 12$$
$$21x - 8x = 12 + 14$$
$$13x = 26$$

Dividing both sides by '13', we get

$$x = 2$$

(ii) $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

Solution:

$$\frac{3x-1}{7}:\frac{3}{5}::\frac{2x}{3}:\frac{7}{5}$$
$$\frac{3x-1}{7}:\frac{3}{5}=\frac{2x}{3}:\frac{7}{5}$$

Multiplying throughout by '105' (L.C.M. of 3,5,7), we get

$$105\times\frac{3x-1}{7}:105\times\frac{3}{5}=105\times\frac{2x}{3}:105\times\frac{7}{5}$$
$$15\times(3x-1):21\times3=35\times2x:21\times7$$
$$45x-15:63=70x:147$$

$\therefore$ Product of extremes = Product of means

$$(45x-15)(147)=(63)(70x)$$
$$6615x-2205=4410x$$
$$6615x-4410x=2205$$
$$2205x=2205$$

Dividing both sides by '2205', we get

$$x=1$$

(iii) $\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$

Solution:

$$\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$$
$$\frac{x-3}{2}:\frac{5}{x-1}=\frac{x-1}{3}:\frac{4}{x+4}$$

$\therefore$ Product of extremes = Product of means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right)=\left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$
$$\frac{2(x-3)}{x+4}=\frac{5}{3}$$
$$\frac{2x-6}{x+4}=\frac{5}{3}$$

$$3(2x-6)=5(x+4)$$
$$6x-18=5x+20$$
$$6x-5x=18+20$$
$$x=38$$

(iv) $p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$

Solution:

$$p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$$
$$p^2+pq+q^2:x=\frac{p^3-q^3}{p+q}:(p-q)^2$$

$\therefore$ Product of means = Product of extremes

$$x\times\frac{(p^3-q^3)}{p+q}=(p-q)^2(p^2+pq+q^2)$$
$$x=\frac{(p-q)^2(p^2+pq+q^2)(p+q)}{(p^3-q^3)}$$

$$x=\frac{(p-q)(p-q)(p^2+pq+q^2)(p+q)}{(p-q)(p^2+pq+q^2)}$$

$$x=(p-q)(p+q)$$

$$x=p^2-q^2$$

(v) $8-x:11-x::16-x:25-x$

Solution:

$$8-x:11-x::16-x:25-x$$

$$8-x:11-x=16-x:25-x$$

$\therefore$ Product of extremes = Product of means

$$(8-x)(25-x)=(11-x)(16-x)$$

$$200-8x-25x-x^2=176-11x-16x-x^2$$

$$200-33x-x^2=176-27x-x^2$$

$$-33x+27x=176-200$$

$$-6x=-24$$

$$x=4$$

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(c) Variation:

The word variation is frequently used in all sciences. There are two types of variations:

(i) Direct variation **(ii)** Inverse variation.

(i) Direct Variation

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

In other words, if a quantity y varies directly with regard to a quantity x. We say that y is directly proportional to x and is written as $y \propto x$ or y = kx.

i.e., $\frac{y}{x} = k, k \neq 0$.

The sign $\propto$ read as "varies as" is called the sign of proportionality or variation, while $k \neq 0$ is known as constant of variation.

e.g.,

(i) Faster the speed of a car, longer the distance it covers.

(ii) The smaller the radius of the circle, smaller the circumference is.

(ii) inverse Variation

If two quantities are related in such a way that when one quantity increases, the other decreases, then this variation is called inverse variation.

In other words, if a quantity y varies inversely with regard to quantity x. We say that y is inversely proportional to x or y varies inversely as x and is written as $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$

i.e., xy = k, where $k \neq 0$ is the constant of variation.

Exercise 3.2

1. If y varies directly as x, and y = 8 when x = 1, find

(i) y in terms of x

Solution:

Given that y varies directly as x.

Therefore $y \propto x$

$\Rightarrow$ y = kx ________(i)

Where k is constant of variation.

Put x = 2 and y = 8 in eq. (i), we have

8 = k (2)

or 2k = 8

$\Rightarrow$ k = 4

Put $k = 4$ in eq. (i), we get

$$y = 4x.$$

(ii) y when x = 5

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________(i)

Where k is constant of variation.

Put $x = 2$ and $y = 8$ in eq. (i), we have

$$8 = k\,(2)$$

or $\quad 2k = 8$

$\Rightarrow \quad k = 4$

Put $k = 4$ and $x = 5$ in eq. (i), we get

$$y = (4)(5) = 20$$

(iii) x when y = 28

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________(i)

Where k is constant of variation.

Put $x = 2$ and $y = 8$ in eq. (i), we have

$$8 = k\,(2)$$

or $\quad 2k = 8$

$\Rightarrow \quad k = 4$

Put $k = 4$ and $y = 28$ in eq. (i), we get

$$28 = 4x$$

or $\quad 4x = 28$

$\Rightarrow \quad x = 7$

2. If $y \propto x$, and $y = 7$ when $x = 3$ find

(i) y in terms of x

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________(i)

Where k is constant of variation.

Put $x = 3$ and $y = 7$ in eq. (i), we have

$7 = k(3)$

or $\quad 3k = 7$

$\Rightarrow \quad k = \frac{7}{3}$

Put $k = \frac{7}{3}$ in eq. (i), we get

$y = \frac{7}{3}x$

(ii) x when y = 35 and y when x = 18

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________(i)

Where k is constant of variation.

Put $x = 3$ and $y = 7$ in eq. (i), we have

$7 = k(3)$

or $\quad 3k = 7$

$\Rightarrow \quad k = \frac{7}{3}$

Put $k = \frac{7}{3}$ and $y = 35$ in eq. (i), we get

$35 = \frac{7}{3}x$

$x = 35 \times \frac{3}{7}$

$x = 5 \times 3 = 15$

Put $k = \frac{7}{3}$ and $x = 18$ in eq. (i), we get

$y = \left(\frac{7}{3}\right)(18)$

$y = 7 \times 6 = 42$

3. If R ∝ T and R = 5 when r = 8, find the equation connecting R and T. Also find R when T = 64 and F when R = 20.

Solution:

Given that $\quad R \propto T$

$\Rightarrow \quad R = kT$ ________(i)

Put $R = 5$ and $T = 8$ in eq. (i), we get

$5 = k(8)$

or $\quad 8k = 5$

$\Rightarrow \quad k = \frac{5}{8}$

Put $k = \frac{5}{8}$ in eq. (i), we get

$$R = \frac{5}{8}T \qquad \_\_\_\_\_\_\_\_\_\_\text{(ii)}$$

Put T = 64 and in eq. (ii), we get

$$R = \frac{5}{8}(64)$$

$$= 5 \times 8 = 40$$

Put R = 20 and in eq. (ii), we get

$$20 = \frac{5}{8}T$$

or $$\frac{5}{8}T = 20$$

$$T = 20 \times \frac{8}{5}$$

$$T = 4 \times 8 = 32$$

4. If $R \propto T^2$ and R = 8 when T = 3, find R when T = 6.

Solution:

Given that $R \propto T^2$

$$\Rightarrow \quad R = kT^2 \qquad \_\_\_\_\_\_\_\_\text{(i)}$$

Put R = 8 and T = 3 in eq. (i), we get

$$8 = k\,(3)^2$$

$$8 = 9\,k$$

or $$9k = 8$$

$$\Rightarrow \quad k = \frac{8}{9}$$

Put $k = \frac{8}{9}$, T = 6 in eq. (i), we get

$$R = \frac{8}{9}(6)^2$$

$$R = \frac{8}{9}(36)$$

$$R = 8 \times 4 = 32$$

5. If $V \propto R^3$ and V = 5 when R = 3, find R, when V = 625.

Solution:

Given that $V \propto R^3$

$\Rightarrow \quad V = kR^3$ ________(i)

Put V = 5 and R = 3 in eq. (i), we get

$$5 = k\,(3)^3$$

$$5 = 27\,k$$

or $\quad 27k = 5$

$\Rightarrow \quad k = \frac{5}{27}$

Put $k = \frac{5}{27}$, V = 625 in eq. (i), we get

$$625 = \frac{5}{27}\,R^3$$

or $\quad \frac{5}{27}\,R^3 = 625$

$$R^3 = 625 \times \frac{27}{5}$$

$$R^3 = 125 \times 27$$

$$R^3 = 5^3 \times 3^3$$

$$R^3 = (5 \times 3)^3$$

$\Rightarrow \quad R = 5 \times 3 = 15$

6. If w varies directly as u^3 and w = 81 when u = 3. Find w, when u = 5.

Solution:

Given that $w \propto u^3$

$\Rightarrow \quad w = ku^3$ ________(i)

Put w = 81 and u = 3 in eq. (i), we get

$$81 = k\,(3)^3$$

$$81 = 27k$$

or $\quad 27k = 81$

$\Rightarrow \quad k = 3$

Put k = 3 and u = 5 in eq. (i), we get

$$w = (3)\,(5)^3$$

$$w = 3 \times 125 = 375$$

7. If y varies inversely as x and y = 7 when x = 2, find y when x = 126.

Solution:

Given that $y \propto \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$ ________(i)

Put y = 7 and x = 2 in eq. (i), we get

$$7 = \frac{k}{2}$$

$$k = 14$$

Put k = 14 and x = 126 in eq. (i), we get

$$y = \frac{14}{126}$$

$$y = \frac{1}{9}$$

8. If $y \propto \frac{1}{x}$ and y = 4 when x = 3, find x when y = 24.

Solution:

Given that $y \propto \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$ ________(i)

Put y = 4 and x = 3 in eq. (i), we get

$$4 = \frac{k}{3}$$

$$k = 12$$

Put k = 12 and y = 24 in eq. (i), we get

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24} = \frac{1}{2}$$

9. If $w \propto \frac{1}{z}$ and w = 5 when z = 7, find w when z = $\frac{175}{4}$.

Solution:

Given that $w \propto \frac{1}{z}$

$\Rightarrow \quad w = \frac{k}{z}$ ________(i)

Put w = 5 and z = 7 in eq. (i), we get

$$5 = \frac{k}{7}$$

$$k = 35$$

Put k = 35 and z = $\frac{175}{4}$ in eq. (i), we get

$$w = \frac{35}{\frac{175}{4}}$$

$$w = 35 \times \frac{4}{175}$$

$$w = \frac{4}{5}$$

10. $A \propto \frac{1}{r^2}$ **and A = 2 when r = 3, find r when A = 72.**

Solution:

Given that $A \propto \frac{1}{r^2}$

$\Rightarrow$ $A = \frac{k}{r^2}$ ________(i)

Put A = 2 and r = 3 in eq. (i), we get

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$k = 18$$

Put k = 18 and A = 72 in eq. (i), we get

$$72 = \frac{18}{r^2}$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm\frac{1}{2}$$

11. $a \propto \frac{1}{b^2}$ **and a = 3 when b = 4, find a when b = 8.**

Solution:

Given that $a \propto \frac{1}{b^2}$

$\Rightarrow$ $a = \frac{k}{b^2}$ ________(i)

Put a = 3 and b = 4 in eq. (i), we get

$$3 = \frac{k}{(4)^2}$$

$$3 = \frac{k}{16}$$

$$k = 48$$

Put k = 48 and b = 8 in eq. (i), we get

$$a = \frac{48}{(8)^2} = \frac{48}{64}$$

$$a = \frac{3}{4}$$

12. $V \propto \frac{1}{r^3}$ **and V = 5 when r = 3, find V when r = 6 and r when V = 320.**

Solution:

Given that $V \propto \frac{1}{r^3}$

$\Rightarrow$ $V = \frac{k}{r^3}$ ________(i)

Put V = 5 and r = 3 in eq. (i), we get

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

k = 135

Put k = 135 and r = 6 in eq. (i), we get

$$V = \frac{135}{(6)^3}$$

$$V = \frac{135}{216}$$

$$V = \frac{5}{8}$$

Put k = 135 and V = 320 in eq. (i), we get

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320}$$

$$r^3 = \frac{27}{64}$$

$$r^3 = \left(\frac{3}{4}\right)^3$$

$$\Rightarrow \quad r = \frac{3}{4}$$

13. $m \propto \frac{1}{n^3}$ **and m = 2 when n = 4, find m when n = 6 and n when m = 432.**

Solution:

Given that $m \propto \frac{1}{n^3}$

$$\Rightarrow \quad m = \frac{k}{n^3} \qquad \text{\_\_\_\_\_\_\_\_(i)}$$

Put m = 2 and n = 4 in eq. (i), we get

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

k = 128

Put k = 128 and n = 6 in eq. (i), we get

$$m = \frac{128}{(6)^3}$$

$$m = \frac{128}{216}$$

$$m = \frac{16}{27}$$

Put k = 128 and m = 432 in eq. (i), we get

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432}$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow \quad n = \frac{2}{3}$$

Find 3rd, 4th, mean and continued proportion:

We are already familiar with proportions that if quantities a, b, c and d are in proportion, then a : b :: c : d

i.e., product of extremes = product of means

Third Proportional

If three quantities a, b and c are related as a : b :: b : c, then c is called the third proportion.

Fourth Proportional

If four quantities a, b, c and d are related as a: b :: c : d

Then d is called the fourth proportional.

Mean Proportional

If three quantities a, b and c are related as a : b :: b : c, then b is called the mean proportional.

Continued Proportion

If three quantities a, b and c are related as

a : b :: b : c

where a is first, b is the mean and c is the third proportional, then a, b and c are in continued proportion.

Exercise 3.3

1. Find a third proportional to

(i) 6, 12

Solution:

Let C be the third proportional, then

$6:12 :: 12:C$

$\therefore$ Product of extremes = Product of means

$6C = 12 \times 12$

$6C = 144$

$C = \frac{144}{6}$

$C = 24$

(ii) $a^3, 3a^2$

Solution:

Let C be the third proportional, then

$a^3 : 3a^2 :: 3a^2 : C$

$\therefore$ Product of extremes = Product of means

$$(C)(a^3)=(3a^2)(3a^2)$$
$$C=\frac{(3a^2)(3a^2)}{a^3}$$
$$C=\frac{9a^4}{a^3}$$
$$C=9a$$

(iii) $a^2-b^2,\ a-b$

Solution:

Let C be the third proportional, then

$$a^2-b^2 : a-b :: a-b : C$$

$\therefore$ Product of extremes = Product of means

$$(a^2-b^2)(C)=(a-b)(a-b)$$
$$C=\frac{(a-b)(a-b)}{(a^2-b^2)}$$
$$C=\frac{(a-b)(a-b)}{(a-b)(a+b)}$$
$$C=\frac{(a-b)}{(a+b)}$$

(iv) $(x-y)^2, x^3-y^3$

Solution:

Let C be the third proportional, then

$$(x-y)^2 : x^3-y^3 :: x^3-y^3 : C$$

$\therefore$ Product of extremes = Product of means

$$(C)(x-y)^2=(x^3-y^3)(x^3-y^3)$$
$$C=\frac{(x-y)(x^2+xy+y^2)(x-y)(x^2+xy+y^2)}{(x-y)^2}$$
$$C=\frac{(x-y)^2(x^2+xy+y^2)^2}{(x-y)^2}$$
$$C=(x^2+xy+y^2)^2$$

(v) $(x+y)^2, x^2-xy-2y^2$

Solution:

Let C be the third proportional, then

$$(x+y)^2 : x^2-xy-2y^2 :: x^2-xy-2y^2 : C$$

$\therefore$ Product of extremes = Product of means

$$(C)(x+y)^2=(x^2-xy-2y^2)(x^2-xy-2y^2)$$

$$C=\frac{\left(x^2-xy-2y^2\right)^2}{(x+y)^2}$$

$$C=\frac{\left(x^2-2xy+xy-2y^2\right)^2}{(x+y)^2}$$

$$C=\frac{\left[x(x-2y)+y(x-2y)\right]^2}{(x+y)^2}$$

$$C=\frac{\left[(x-2y)(x+y)\right]^2}{(x+y)^2}$$

$$C=\frac{(x-2y)^2(x+y)^2}{(x+y)^2}$$

$$C=(x-2y)^2$$

(vi) $\dfrac{p^2-q^2}{p^3+q^3},\dfrac{p-q}{p^2-pq+q^2}$

Solution:

Let C be the third proportional, then

$$\frac{p^2-q^2}{p^3+q^3}:\frac{p-q}{p^2-pq+q^2}::\frac{p-q}{p^2-pq+q^2}:C$$

$\therefore$ Product of extremes = Product of means

$$(C)\left(\frac{p^2-q^2}{p^3+q^3}\right)=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)$$

$$C=\frac{(p-q)^2}{\left(p^2-pq+q^2\right)^2}\times\frac{p^3+q^3}{p^2-q^2}$$

$$C=\frac{(p-q)^2}{\left(p^2-pq+q^2\right)^2}\times\frac{(p+q)\left(p^2-pq+q^2\right)}{(p+q)(p-q)}$$

$$C=\frac{p-q}{p^2-pq+q^2}$$

2. Find a fourth proportional to

(i) 5, 8, 15

Solution:

Let x be the fourth proportional, then

$$5:8::15:x$$

$\therefore$ Product of extremes = Product of means

$$(5)(x)=(8)(15)$$

$$x=\frac{(8)(15)}{(5)}$$

$$x=8\times3=24$$

(ii) $4x^4, 2x^3, 18x^5$

Solution:

[redacted]

Let C be the fourth proportional, then

$$4x^4 : 2x^3 :: 18x^5 : C$$

$\therefore$ Product of extremes = Product of means

$$\left(4x^4\right)(C)=\left(2x^3\right)\left(18x^5\right)$$
$$C=\frac{36x^8}{4x^4}$$
$$C=9x^4$$

(iii) $15a^5b^6, 10a^2b^5, 21a^3b^3$

Solution:

Let x be the fourth proportional, then

$$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : x$$

$\therefore$ Product of extremes = Product of means

$$x\left(15a^5b^6\right)=\left(10a^2b^5\right)\left(21a^3b^3\right)$$
$$x=\frac{10\times 21a^5b^8}{15a^5b^6}$$

$$x=2\times 7b^2$$
$$x=14b^2$$

(iv) $x^2-11x+24, (x-3), 5x^4-40x^3$

Solution:

Let x be the fourth proportional, then

$$x^2-11x+24 : (x-3) :: 5x^4-40x^3 : C$$

$\therefore$ Product of extremes = Product of means

$$C\left(x^2-11x+24\right)=(x-3)\left(5x^4-40x^3\right)$$
$$C=\frac{(x-3)\left[5x^3(x-8)\right]}{x^2-11x+24}$$
$$C=\frac{5x^3(x-3)(x-8)}{x^2-8x-3x+24}$$
$$C=\frac{5x^3(x-3)(x-8)}{x(x-8)-3(x-8)}$$
$$C=\frac{5x^3(x-3)(x-8)}{(x-3)(x-8)}$$
$$C=5x^3$$

(v) $p^3+q^3, p^2-q^2, p^2-pq+q^2$

Solution:

Let C be the fourth proportional, then

$$p^3+q^3 : p^2-q^2 :: p^2-pq+q^2 : C$$

$\therefore$ Product of extremes = Product of means

$$C\left(p^3+q^3\right)=\left(p^2-q^2\right)\left(p^2-pq+q^2\right)$$
$$C=\frac{(p-q)(p+q)\left(p^2-pq+q^2\right)}{\left(p^3+q^3\right)}$$

$$C=\frac{(p-q)\left(p^3+q^3\right)}{\left(p^3+q^3\right)}$$

$$C=p-q$$

(vi) $\left(p^2-q^2\right)\left(p^2+pq+q^2\right), p^3+q^3, p^3-q^3$

Solution:

Let x be the fourth proportional, then

$$\left(p^2-q^2\right)\left(p^2+pq+q^2\right): p^3+q^3 :: p^3-q^3 : x$$

∴ Product of extremes = Product of means

$$x\left(p^2-q^2\right)\left(p^2+pq+q^2\right)=\left(p^3+q^3\right)\left(p^3-q^3\right)$$

$$x=\frac{\left(p^3+q^3\right)\left(p^3-q^3\right)}{\left(p^2-q^2\right)\left(p^2+pq+q^2\right)}$$

$$x=\frac{\left(p^3+q^3\right)\left(p^3-q^3\right)}{(p+q)(p-q)\left(p^2+pq+q^2\right)}$$

$$C=\frac{(p+q)\left(p^2-pq+q^2\right)\left(p^3-q^3\right)}{(p+q)\left(p^3-q^3\right)}$$

$$C=p^2-pq+q^2$$

3. Find a mean proportional between

(i) 20,45

Solution:

Let m be the mean proportional, then

$$20 : m :: m : 45$$

∴ Product of means = Product of extremes

$$m\times m=20\times 45$$

$$m^2=900$$

$$m=\pm\sqrt{900}$$

$$m=\pm 30$$

(ii) $20x^3y^5, 5x^7y$

Solution:

Let m be the mean proportional, then

$$20x^3y^5 : m :: m : 5x^7y$$

∴ Product of means = Product of extremes

$$m\times m=20x^3y^5\times 5x^7y$$

$$m^2=100x^{10}y^6$$

$$m=\pm\sqrt{100x^{10}y^6}$$

$$m=\pm\left(100x^{10}y^6\right)^{\frac{1}{2}}$$

$$m=\pm\left(10^2\right)^{\frac{1}{2}}\left(x^{10}\right)^{\frac{1}{2}}\left(y^6\right)^{\frac{1}{2}}$$

$$m=\pm 10x^5y^3$$

(iii) $15p^4qr^3, 135q^5r^7$

Solution:

Let m be the mean proportional, then

$$15p^4qr^3 : m :: m : 135q^5r^7$$

∴ Product of means = Product of extremes

$$m\times m=15p^4qr^3\times 135q^5r^7$$

$$m \times m = 15 \times 135 p^4 q^6 r^{10}$$
$$m^2 = 2025 p^4 q^6 r^{10}$$
$$m = \pm\sqrt{2025 p^4 q^6 r^{10}}$$
$$m = \pm\left(2025 p^4 q^6 r^{10}\right)^{\frac{1}{2}}$$
$$m = \pm\left(45^2\right)^{\frac{1}{2}}\left(p^4\right)^{\frac{1}{2}}\left(q^6\right)^{\frac{1}{2}}\left(r^{10}\right)^{\frac{1}{2}}$$
$$m = \pm 45 p^2 q^3 r^5$$

(iv) $x^2 - y^2, \dfrac{x-y}{x+y}$

Solution:

Let m be the mean proportional, then

$$x^2 - y^2 : m :: m : \frac{x-y}{x+y}$$

∴ Product of means = Product of extremes

$$m \times m = x^2 - y^2 \times \frac{x-y}{x+y}$$
$$m^2 = (x-y)(x+y) \times \frac{x-y}{x+y}$$
$$m^2 = (x-y)(x-y)$$
$$m^2 = (x-y)^2$$
$$m = \pm\sqrt{(x-y)^2}$$
$$m = \pm(x-y)$$

4. Find the values of the letter involved in the following continued proportions.

(i) 5, p, 45

Solution:

Since 5, p and 45 are in continued proportions.

$$5 : p :: p : 45$$

∴ Product of means = Product of extremes

$$p \times p = 5 \times 45$$
$$p^2 = 225$$
$$p = \pm\sqrt{225}$$
$$p = \pm 15$$

(ii) 8, x, 18

Solution:

Since 8, x and 18 are in continued proportions.

$$8 : x :: x : 18$$

$\therefore$ Product of means = Product of extremes

$$\begin{aligned} x\times x &= 8\times 18 \\ x^2 &= 144 \\ x &= \pm\sqrt{144} \\ x &= \pm 12 \end{aligned}$$

(iii) 12, 3p-6, 27

Solution:

Since $12, 3p-6$ and 27 are in continued proportions.

$$12 : 3p-6 :: 3p-6 : 27$$

$\therefore$ Product of means = Product of extremes

$$\begin{aligned} (3p-6)(3p-6) &= 12\times 27 \\ (3p-6)^2 &= 324 \\ \sqrt{(3p-6)^2} &= \pm\sqrt{324} \\ 3p-6 &= \pm 18 \end{aligned}$$

$$\begin{array}{lcl} 3p-6=-18 & or & 3p-6=18 \\ 3p=6-18 & or & 3p=18+6 \\ 3p=-12 & or & 3p=24 \\ p=-4 & or & p=8 \end{array}$$

(iv) 7, m-3, 28

Solution:

Since $7, \mathrm{m}-3$ and 28 are in continued proportions.

$$7 : \mathrm{m}-3 :: \mathrm{m}-3 : 28$$

$\therefore$ Product of means = Product of extremes

$$\begin{aligned} (\mathrm{m}-3)(\mathrm{m}-3) &= 7\times 28 \\ (\mathrm{m}-3)^2 &= 196 \\ \sqrt{(\mathrm{m}-3)^2} &= \pm\sqrt{196} \\ \mathrm{m}-3 &= \pm 14 \end{aligned}$$

$$\mathrm{m}-3=-14 \quad or \quad \mathrm{m}-3=14$$

$$\begin{array}{lcl} m=3-14 & or & m=3+14 \\ m=-11 & or & m=17 \end{array}$$

Theorems on Proportions:

If four quantities a, b, c and d form a proportion, then many other useful properties may be deduced by the properties of fractions.

(1) Theorem of Invertendo

If $a : b = c : d$, then $b : a = d : c$

(2) Theorem of Alternando

If $a : b = c : d$, then $a : c = b : d$

(3) Theorem of Componendo

If $a : b = c : d$, then

(i) $a + b : b = c + d : d$

and (ii) $a : a + b = c : c + d$

(4) Theorem of Dividendo

If $a : b = c : d$, then

(i) $a - b : b = c - d : d$

and (ii) $a : a - b = c : c - d$

(5) Theorem of Componendo dividendo

If $a : b = c : d$, then

(i) $a + b : a - b = c + d : c - d$

and (ii) $a - b : a + b = c - d : c + d$

Exercise 3.4

1. Prove that a : b = c : d, if

(i) $\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$

Solution:

$Given\ \frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$

By componendo-dividendo theorem, we have

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)}=\frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b}=\frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

$$\frac{8a}{10b}=\frac{8c}{10d}$$

Multiplying both sides by $\frac{10}{8}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence proved.

(ii) $\dfrac{2a+9b}{2a-9b}=\dfrac{2c+9d}{2c-9d}$

Solution:

Given $\dfrac{2a+9b}{2a-9b}=\dfrac{2c+9d}{2c-9d}$

By componendo-dividendo theorem, we have

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)}=\frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{2a+9b+2a-9b}{2a+9b-2a+9b}=\frac{2c+9d+2c-9d}{2c+9d-2c+9d}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

Multiplying both sides by $\dfrac{18}{4}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence proved.

(iii) $\dfrac{ac^2+bd^2}{ac^2-bd^2}=\dfrac{c^3+d^3}{c^3-d^3}$

Solution:

Given $\dfrac{ac^2+bd^2}{ac^2-bd^2}=\dfrac{c^3+d^3}{c^3-d^3}$

By componendo-dividendo theorem, we have

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}=\frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}=\frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2}=\frac{2c^3}{2d^3}$$

$$\Rightarrow \frac{ac^2}{bd^2}=\frac{c^3}{d^3}$$

Multiplying both sides by $\dfrac{d^2}{c^2}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence proved.

(iv) $\dfrac{a^2c+b^2d}{a^2c-b^2d}=\dfrac{ac^2+bd^2}{ac^2-bd^2}$

Solution:

Given $\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$

By componendo-dividendo theorem, we have

$$\frac{(a^2c+b^2d)+(a^2c-b^2d)}{(a^2c+b^2d)-(a^2c-b^2d)}=\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}$$
$$\frac{a^2c+b^2d+a^2c-b^2d}{a^2c+b^2d-a^2c+b^2d}=\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}$$
$$\frac{2a^2c}{2b^2d}=\frac{2ac^2}{2bd^2}$$
$$\Rightarrow\frac{a^2c}{b^2d}=\frac{ac^2}{bd^2}$$

Multiplying both sides by $\frac{bd}{ac}$, we get

$$\frac{a}{b}=\frac{c}{d}$$
$$\Rightarrow a:b=c:d$$

Hence proved.

(v) $pa+qb:pa-qb=pc+qd:pc-qd$

Solution:

Given $pa+qb:pa-qb=pc+qd:pc-qd$

By componendo-dividendo theorem, we have

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)}=\frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$
$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb}=\frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$
$$\frac{2pa}{2qb}=\frac{2pc}{2qd}$$

Multiplying both sides by $\frac{2q}{2p}$, we get

$$\frac{a}{b}=\frac{c}{d}$$
$$\Rightarrow a:b=c:d$$

Hence proved.

(vi) $\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$

Solution:

Given

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$
$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$
$$\frac{a+b+c+d+a+b-c-d}{a+b+c+d-a-b+c+d}=\frac{a-b+c-d+a-b-c+d}{a-b+c-d-a+b+c-d}$$
$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$
$$\frac{2(a+b)}{2(c+d)}=\frac{2(a-b)}{2(c-d)}$$
$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$
$$or\ \frac{a+b}{a-b}=\frac{c+d}{c-d}$$

By componendo-dividendo theorem, we have

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)}=\frac{(c+d)+(c-d)}{(c+d)-(c-d)}$$

$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b}=\frac{2c}{2d}$$

$$\Rightarrow \frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence proved.

(vii) $$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

Solution:

Given

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a-3b+2c+3d}=\frac{2a-3b+2c-3d+2a-3b-2c+3d}{2a-3b+2c-3d-2a+3b+2c-3d}$$

$$\frac{4a+6b}{4c+6d}=\frac{4a-6b}{4c-6d}$$

$$or\ \frac{4a+6b}{4a-6b}=\frac{4c+6d}{4c-6d}$$

By componendo-dividendo theorem, we have

$$\frac{(4a+6b)+(4a-6b)}{(4a+6b)-(4a-6b)}=\frac{(4c+6d)+(4c-6d)}{(4c+6d)-(4c-6d)}$$

$$\frac{4a+6b+4a-6b}{4a+6b-4a+6b}=\frac{4c+6d+4c-6d}{4c+6d-4c+6d}$$

$$\frac{8a}{12b}=\frac{8c}{12d}$$

Multiplying both sides by $\frac{12}{8}$, we get

$$\Rightarrow \frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence proved.

(viii) $$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

Solution:

Given

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By componendo-dividendo theorem, we have

$$\frac{\left(a^2+b^2\right)+\left(a^2-b^2\right)}{\left(a^2+b^2\right)-\left(a^2-b^2\right)}=\frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}=\frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$

$$\frac{2a^2}{2b^2}=\frac{2ac}{2bd}$$

Multiplying both sides by $\frac{2b}{2a}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence proved.

2. Using theorem of componendo-dividendo

(i) Find the value of $\dfrac{x+2y}{x-2y}+\dfrac{x+2z}{x-2z}, if\ x=\dfrac{4yz}{y+z}$

Solution:

Given

$$x=\frac{4yz}{y+z}\quad or\quad x=\frac{(2y)(2z)}{y+z}$$

$$or\ \frac{x}{2y}=\frac{2z}{y+z}$$

Applying componendo-dividendo theorem, we get

$$\frac{x+2y}{x-2y}=\frac{2z+(y+z)}{2z-(y+z)}$$

$$\frac{x+2y}{x-2y}=\frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y}=\frac{y+3z}{z-y}\qquad ....(i)$$

Now,

$$x=\frac{4yz}{y+z}\quad or\quad x=\frac{(2y)(2z)}{y+z}$$

$$or\ \frac{x}{2z}=\frac{2y}{y+z}$$

Applying componendo-dividendo theorem, we get

$$\frac{x+2z}{x-2z}=\frac{2y+(y+z)}{2y-(y+z)}$$

$$\frac{x+2z}{x-2z}=\frac{2y+y+z}{2y-y-z}$$

$$\frac{x+2z}{x-2z}=\frac{3y+z}{y-z}\qquad ....(ii)$$

Adding eq.(i) and eq.(ii), we get

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{y+3z}{z-y}+\frac{3y+z}{y-z}$$

$$=\frac{y+3z}{z-y}+\frac{3y+z}{-(z-y)}$$

$$=\frac{y+3z}{z-y}-\frac{3y+z}{z-y}$$

$$=\frac{(y+3z)-(3y+z)}{z-y}$$

$$=\frac{y+3z-3y-z}{z-y}$$

$$=\frac{2z-2y}{z-y}$$

$$=\frac{2(z-y)}{z-y}=2$$

(ii) Find the value of $\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}, if\ m=\frac{10np}{n+p}$

Solution:

Given

$$m=\frac{10np}{n+p} \quad or \quad m=\frac{(2p)(5n)}{n+p}$$

$$or\ \frac{m}{5n}=\frac{2p}{n+p}$$

Applying componendo-dividendo theorem, we get

$$\frac{m+5n}{m-5n}=\frac{2p+(n+p)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n} \quad ....(i)$$

Now,

$$m=\frac{10np}{n+p} \quad or \quad m=\frac{(5p)(2n)}{n+p}$$

$$or\ \frac{m}{5p}=\frac{2n}{n+p}$$

Applying componendo-dividendo theorem, we get

$$\frac{m+5p}{m-5p}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p} \quad ....(ii)$$

Adding eq.(i) and eq.(ii), we get

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$

$$=\frac{3p+n}{p-n}+\frac{3n+p}{-(p-n)}$$

$$=\frac{3p+n}{p-n}-\frac{3n+p}{p-n}$$

$$=\frac{(3p+n)-(3n+p)}{p-n}$$

$$=\frac{3p+n-3n-p}{p-n}$$

$$=\frac{2p-2n}{p-n}$$

$$=\frac{2(p-n)}{p-n}=2$$

(iii) Find the value of $\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}, if\ x=\frac{12ab}{a-b}$

Solution:

Given

$$x=\frac{12ab}{a-b} \quad or \quad x=\frac{(6a)(2b)}{a-b}$$

$$or\ \frac{x}{6a}=\frac{2b}{a-b}$$

Applying componendo-dividendo theorem, we get

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$

$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$

$$\frac{x+6a}{x-6a}=\frac{a+b}{3b-a}$$

$$or\ \frac{x-6a}{x+6a}=\frac{3b-a}{a+b} \quad \text{..........}(i)$$

Now,

$$x=\frac{12ab}{a-b} \quad or \quad x=\frac{(2a)(6b)}{a-b}$$

$$or\ \frac{x}{6b}=\frac{2a}{a-b}$$

Applying componendo-dividendo theorem, we get

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b} \quad \text{.......}(ii)$$

Subtracting eq.(ii) from eq.(i), we have

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$

$$=\frac{(3b-a)-(3a-b)}{a+b}$$

$$=\frac{3b-a-3a+b}{a+b}$$

$$=\frac{4b-4a}{a+b}$$

$$=\frac{4(b-a)}{a+b}$$

(iv) Find the value of $\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}, if\ x=\frac{3yz}{y-z}$

Solution:

Given

$$x=\frac{3yz}{y-z} \quad or \quad x=\frac{(3y)(z)}{y-z}$$

$$or\ \frac{x}{3y}=\frac{z}{y-z}$$

Applying componendo-dividendo theorem, we get

$$\frac{x+3y}{x-3y}=\frac{z+(y-z)}{z-(y-z)}$$

$$\frac{x+3y}{x-3y}=\frac{z+y-z}{z-y+z}$$

$$\frac{x+3y}{x-3y}=\frac{y}{2z-y}$$

$$or\ \frac{x-3y}{x+3y}=\frac{2z-y}{y} \qquad ....(i)$$

Now,

$$x=\frac{3yz}{y-z} \qquad or \qquad x=\frac{(3z)(y)}{y-z}$$

$$or \qquad \frac{x}{3z}=\frac{y}{y-z}$$

Applying componendo-dividendo theorem, we get

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$\frac{x+3z}{x-3z}=\frac{y+y-z}{y-y+z}$$

$$\frac{x+3z}{x-3z}=\frac{2y-z}{z} \qquad ....(ii)$$

Subtracting eq.(ii) from eq.(i), we get

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

$$=\frac{2z^2-yz-2y^2+yz}{yz}$$

$$=\frac{2z^2-2y^2}{yz}=\frac{2(z^2-y^2)}{yz}$$

(v) Find the value of $\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}, if\ s=\frac{6pq}{p-q}$

Solution:

Given

$$s=\frac{6pq}{p-q} \qquad or \qquad s=\frac{(3p)(2q)}{p-q}$$

$$or\ \frac{s}{3p}=\frac{2q}{p-q}$$

Applying componendo-dividendo theorem, we get

$$\frac{s+3p}{s-3p}=\frac{2q+(p-q)}{2q-(p-q)}$$

$$\frac{s+3p}{s-3p}=\frac{2q+p-q}{2q-p+q}$$

$$\frac{s+3p}{s-3p}=\frac{p+q}{3q-p}$$

$$or\ \frac{s-3p}{s+3p}=\frac{3q-p}{p+q}\qquad\text{..........}(i)$$

Now,

$$s=\frac{6pq}{p-q}\qquad or\qquad s=\frac{(3q)(2p)}{p-q}$$

$$or\ \frac{s}{3q}=\frac{2p}{p-q}$$

Applying componendo-dividendo theorem, we get

$$\frac{s+3q}{s-3q}=\frac{2p+(p-q)}{2p-(p-q)}$$

$$\frac{s+3q}{s-3q}=\frac{2p+p-q}{2p-p+q}$$

$$\frac{s+3q}{s-3q}=\frac{3p-q}{p+q}\qquad\text{.......}(ii)$$

Adding eq.(i) and eq.(ii), we get

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$

$$=\frac{3q-p+3p-q}{p+q}$$

$$=\frac{2p+2q}{p+q}$$

$$=\frac{2(p+q)}{p+q}=2$$

(vi) Solve $\dfrac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\dfrac{12}{13}$

Solution:

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

Applying componendo-dividendo theorem, we get

$$\frac{\left[(x-2)^2-(x-4)^2\right]+\left[(x-2)^2+(x-4)^2\right]}{\left[(x-2)^2-(x-4)^2\right]-\left[(x-2)^2+(x-4)^2\right]}=\frac{12+13}{12-13}$$

$$\frac{(x-2)^2-(x-4)^2+(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2-(x-2)^2-(x-4)^2}=\frac{25}{-1}$$

$$\frac{2(x-2)^2}{-2(x-4)^2}=\frac{25}{-1}$$

$$\Rightarrow\frac{(x-2)^2}{(x-4)^2}=25$$

Taking square root on both sides, we get

$$\sqrt{\frac{(x-2)^2}{(x-4)^2}}=\pm\sqrt{25}$$

$$\frac{(x-2)}{(x-4)}=\pm5$$

$$\Rightarrow\frac{(x-2)}{(x-4)}=-5\qquad or\qquad\frac{(x-2)}{(x-4)}=5$$

$$x-2=-5(x-4)\qquad\qquad 5(x-4)=x-2$$

$$x-2=-5x+20\qquad\qquad 5x-20=x-2$$

$$x+5x=2+20\qquad\qquad 5x-x=20-2$$

$$6x=22\qquad\qquad 4x=18$$

$$x=\frac{22}{6}\qquad\qquad x=\frac{18}{4}$$

$x=\frac{11}{3}$ $\qquad\qquad$ $x=\frac{9}{2}$

Thus, solution set $=\left\{\frac{9}{2},\frac{11}{3}\right\}$

(vii) Solve $\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=\frac{2}{1}$$

Applying componendo-dividendo theorem, we get

$$\frac{\left[\sqrt{x^2+2}+\sqrt{x^2-2}\right]+\left[\sqrt{x^2+2}-\sqrt{x^2-2}\right]}{\left[\sqrt{x^2+2}+\sqrt{x^2-2}\right]-\left[\sqrt{x^2+2}-\sqrt{x^2-2}\right]}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}}=3$$

$$\frac{\sqrt{x^2+2}}{\sqrt{x^2-2}}=3$$

Squaring both sides, we get

$$\frac{x^2+2}{x^2-2}=9$$

$$9\left(x^2-2\right)=x^2+2$$

$$9x^2-18=x^2+2$$

$$9x^2-x^2=18+2$$

$$8x^2=20$$

$$x^2=\frac{20}{8}$$

$$x^2=\frac{5}{2}$$

$$x=\pm\sqrt{\frac{5}{2}}$$

Thus, solution set $=\left\{\pm\sqrt{\frac{5}{2}}\right\}$

(viii) Solve $\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}=\frac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}=\frac{1}{3}$$

Applying componendo-dividendo theorem, we get

$$\frac{\left[\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right]+\left[\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right]}{\left[\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right]-\left[\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right]}=\frac{1+3}{1-3}$$

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}+\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}-\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}=\frac{4}{-2}$$

$$\frac{2\sqrt{x^2+8p^2}}{-2\sqrt{x^2-p^2}}=-2$$

$$\Rightarrow\frac{\sqrt{x^2+8p^2}}{\sqrt{x^2-p^2}}=2$$

Squaring both sides, we get

$$\frac{x^2+8p^2}{x^2-p^2}=4$$

$$4\left(x^2-p^2\right)=x^2+8p^2$$

$$4x^2-4p^2=x^2+8p^2$$

$$4x^2-x^2=8p^2+4p^2$$

$$3x^2=12p^2$$

$$x^2=\frac{12p^2}{3}$$

$$x^2=4p^2$$

$$\Rightarrow x=\pm 2p$$

Thus, solution set $=\{+2p,-2p\}$

(ix) Solve $\dfrac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\dfrac{13}{14}$

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$$

Applying componendo-dividendo theorem, we get

$$\frac{\left[(x+5)^3-(x-3)^3\right]+\left[(x+5)^3+(x-3)^3\right]}{\left[(x+5)^3-(x-3)^3\right]-\left[(x+5)^3+(x-3)^3\right]}=\frac{13+14}{13-14}$$

$$\frac{(x+5)^3-(x-3)^3+(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3-(x+5)^3-(x-3)^3}=\frac{27}{-1}$$

$$\frac{2(x+5)^3}{-2(x-3)^3}=-27$$

$$\Rightarrow\frac{(x+5)^3}{(x-3)^3}=27$$

Taking power '$\frac{1}{3}$' on both sides, we get

$$\left[\frac{(x+5)^3}{(x-3)^3}\right]^{\frac{1}{3}}=\left[(3)^3\right]^{\frac{1}{3}}$$

$$\frac{(x+5)}{(x-3)}=3$$

$$3(x-3)=(x+5)$$

$$3x-9=x+5$$

$$3x-x=9+5$$

$$2x=14$$

$$\Rightarrow x=7$$

Thus, solution set $=\{7\}$

(i) Joint variation

A combination of direct and inverse variations of one or more than one variable forms joint variation.

If a variable y varies directly as x and varies inversely as z.

Then $y \propto x$ and $y \propto \frac{1}{z}$

In joint variation, we write it as

$$y \propto \frac{x}{z}$$

i.e, $$y = k\frac{x}{z}$$

Where $k \neq 0$ is the constant of variation.

Exercise 3.5

1. If s varies directly as u^2 and inversely as v and s = 7 when u = 3, v = 2. Find the value of s when u = 6 and v = 10.

Solution:

Given that s varies directly as u^2, so

$$s \propto u^2$$

Also given that S varies inversely as v, so

$$s \propto \frac{1}{v}$$

In joint variation, we can write

$$s \propto \frac{u^2}{v}$$

$$\Rightarrow \quad s = k\frac{u^2}{v} \quad \text{............}(i)$$

Put $s = 7, u = 3$ and $v = 2$ in eq.(i), we get

$$7 = k\frac{(3)^2}{2}$$

$$7 = \frac{9k}{2}$$

$$or\ \frac{9k}{2} = 7$$

Multiplying both sides by $\frac{2}{9}$, we get

$$k = 7 \times \frac{2}{9}$$

$$k = \frac{14}{9}$$

Put $k = \frac{14}{9}$ in eq.(i), we get

$$s = \frac{14u^2}{9v} \quad \text{........}(ii)$$

Put $u = 6$ and $v = 10$ in eq.(ii), we get

$$S = \frac{14 \times (6)^2}{9(10)}$$
$$= \frac{14 \times 36}{9 \times 10} = \frac{504}{90} = \frac{28}{5}$$

2. If w varies jointly as x, y² and 2 and w = 5 when x == 2, y = 3, z = 10. Find w When x = 4, y = 7 and z = 3.

Solution:

Given that s varies directly as x, y^2 and z.

Therefore $w \propto xy^2z$

$$\Rightarrow \quad w = kxy^2z \quad ....(i)$$

Put $w = 5, x = 2$ and $y = 3$ and $z = 10$ in eq.(i), we get

$$5 = k(2)(3)^2(10)$$
$$5 = k(2)(9)(10)$$
$$5 = 180k$$
$$180k = 5$$
$$k = \frac{5}{180} = \frac{1}{36}$$

Put $k = \frac{1}{36}$ in eq.(i), we get

$$w = \frac{1}{36}xy^2z \quad ....(ii)$$

Put $x = 4$ and $y = 7$ and $z = 3$ in eq.(ii), we get

$$w = \frac{1}{36}(4)(7)^2(3)$$
$$= \frac{1}{36}(4)(49)(3)$$
$$= \frac{588}{36} = \frac{49}{3}$$

3. If y varies directly as x³ and inversely as z² and t, and y = 16 when x = 4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

Solution:

Given that s varies directly as x^3.

Therefore $y \propto x^3$

Also given that y varies inversely as z^2 and t.

Therefore $y \propto \frac{1}{z^2t}$

In joint variation, we can write

$$y \propto \frac{x^3}{z^2 t}$$

$$\Rightarrow \quad y = k\frac{x^3}{z^2 t} \quad \text{.........}(i)$$

Put $y = 16, x = 4$ and $z = 2$ and $t = 3$ in eq.(i), we get

$$16 = k\frac{(4)^3}{(2)^2(3)}$$

$$16 = k\frac{64}{4\times 3}$$

$$16 = k\frac{64}{12}$$

$$or \ \frac{64}{12}k = 16$$

$$k = 16\times\frac{12}{64}$$

$$k = 3$$

Put $k = 3$ in eq.(i), we get

$$y = \frac{3x^3}{z^2 t} \quad \text{........}(ii)$$

Put $x = 2$ and $z = 3$ and $t = 4$ in eq.(ii), we get

$$y = \frac{3(2)^3}{(3)^2(4)}$$

$$= \frac{3\times 8}{9\times 4} = \frac{2}{3}$$

4. If u varies directly as x² and inversely as the product yz³, and u = 2 when x = 8, y = 7, z = 2. Find the value of u when x = 6, y = 3, z = 2.

Solution:

Given that u varies directly as x^2.

Therefore $\quad u \propto x^2$

Also given that u varies inversely as yz^3.

Therefore $\quad u \propto \frac{1}{yz^3}$

In joint variation, we can write

$$u \propto \frac{x^2}{yz^3}$$

$$\Rightarrow \quad u = k\frac{x^2}{yz^3} \quad \text{.........}(i)$$

Put $u = 2, x = 8$ and $y = 7$ and $z = 2$, we get

$$2 = k\frac{(8)^2}{(7)(2)^3}$$

$$2 = \frac{64}{56}k$$

$$\frac{64}{56}k = 2$$

$$k = 2\times\frac{56}{64} = \frac{7}{4}$$

Put $k = \frac{7}{4}$ in eq.(i), we get

$$u = \frac{7x^2}{4yz^3} \quad \text{........}(ii)$$

Put $x = 6$ and $y = 3$ and $z = 2$ in eq.(ii), we get

$$u = \frac{7(6)^2}{4(3)(2)^3}$$

$$= \frac{7\times 36}{4\times 3\times 8} = \frac{252}{96} = \frac{21}{8}$$

5. If v varies directly as the product xy³ and inversely as z² and v = 27 when x = 7, y = 6, z = 7. Find the value of v when x = 6, y = 2, z = 3.

Solution:

Given that v varies directly as xy^3.

Therefore $v \propto xy^3$

Also given that u varies inversely as z^2

Therefore $v \propto \frac{1}{z^2}$

In joint variation, we can write

$$v \propto \frac{xy^3}{z^2}$$

$$\Rightarrow \quad v = k\frac{xy^3}{z^2} \quad ....(i)$$

Put $v = 27, x = 7$ and $y = 6$ and $z = 7$, we get

$$27 = \frac{k(7)(6)^3}{(7)^2}$$

$$27 = \frac{k(7)(216)}{49}$$

$$27 = \frac{1512k}{49}$$

$$or\ k = 27\times\frac{49}{1512}$$

$$k = \frac{1323}{1512} = \frac{7}{8}$$

Put $k = \frac{7}{8}$ in eq.(i), we get

$$v = \frac{7xy^3}{8z^2} \quad ....(ii)$$

Put $x = 6$ and $y = 2$ and $z = 3$ in eq.(ii), we get

$$v = \frac{7(6)(2)^3}{8(3)^2}$$

$$= \frac{336}{72} = \frac{14}{3}$$

6. If w varies inversely as the cube of u, and w = 5 when u = 3. Find w, when u = 6.

Solution:

Given that w varies inversely as u^3.

Therefore $w \propto \frac{1}{u^3}$

$\Rightarrow \quad w = \frac{k}{u^3}$ ....(i)

Put $w = 5, u = 3$ in eq.(i), we get

$5 = \frac{k}{(3)^3}$

$k = 27 \times 5 = 135$

Put $k = 135$ in eq.(i), we get

$w = \frac{135}{u^3}$ ....(ii)

Put $u = 6$ in eq.(ii), we get

$w = \frac{135}{(6)^3}$

$= \frac{135}{216} = \frac{5}{8}$

K-Method:

(i) Use k - method to prove conditional equalities involving proportions.

If a : b :: c : d is a proportion, then putting each ratio equal to k

$$i.e., \frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k \text{ and } \frac{c}{d}=k$$

$$a=bk \text{ and } c=dk$$

Using the above equations, we can solve certain problems relating to proportions more easily. This method is known as k-method.

Exercise 3.6

1. If a: b = c : d, (a, b, c. d ≠ 0), then show that

(i) $\dfrac{4a-9b}{4a+9b}=\dfrac{4c-9d}{4c+9d}$

Solution:

$$\begin{aligned}
&Given \quad a:b=c:d\\
&Let \quad \frac{a}{b}=\frac{c}{d}=k\\
&\Rightarrow \quad \frac{a}{b}=k \text{ and } \quad \frac{c}{d}=k\\
&\qquad a=bk \text{ and } \quad c=dk\\
&L.H.S=\frac{4a-9b}{4a+9b} \qquad R.H.S=\frac{4c-9d}{4c+9d}\\
&\qquad =\frac{4bk-9b}{4bk+9b} \qquad =\frac{4dk-9d}{4dk+9d}\\
&\qquad =\frac{b(4k-9)}{b(4k+9)} \qquad =\frac{d(4k-9)}{d(4k+9)}\\
&\qquad =\frac{4k-9}{4k+9} \;.....(i) \qquad =\frac{4k-9}{4k+9} \;.......(ii)
\end{aligned}$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\dfrac{4a-9b}{4a+9b}=\dfrac{4c-9d}{4c+9d}$

(ii) $\dfrac{6a-5b}{6a+5b}=\dfrac{6c-5d}{6c+5d}$

Solution:

$$\begin{aligned}
&Given \quad a:b=c:d\\
&Let \quad \frac{a}{b}=\frac{c}{d}=k\\
&\Rightarrow \quad \frac{a}{b}=k \text{ and } \quad \frac{c}{d}=k\\
&\qquad a=bk \text{ and } \quad c=dk\\
&L.H.S=\frac{6a-5b}{6a+5b} \qquad R.H.S=\frac{6c-5d}{6c+5d}\\
&\qquad =\frac{6bk-5b}{6bk+5b} \qquad =\frac{6dk-5d}{6dk+5d}\\
&\qquad =\frac{b(6k-5)}{b(6k+5)} \qquad =\frac{d(6k-5)}{d(6k+5)}
\end{aligned}$$

$=\dfrac{6k-5}{6k+5}$ .....(i) $\quad =\dfrac{6k-5}{6k+5}$ ....(ii)

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\dfrac{6a-5b}{6a+5b}=\dfrac{4c-5d}{4c+5d}$

(iii) $\dfrac{a}{b}=\sqrt{\dfrac{a^2+c^2}{b^2+d^2}}$

Solution:

Given $a:b=c:d$

Let $\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow \dfrac{a}{b}=k$ and $\dfrac{c}{d}=k$

$a=bk$ and $c=dk$

$$L.H.S=\frac{a}{b}=\frac{bk}{b}=k \quad .....(i)$$

$$R.H.S=\sqrt{\frac{a^2+c^2}{b^2+d^2}}=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}=\sqrt{\frac{k^2\left(b^2+d^2\right)}{b^2+d^2}}=\sqrt{k^2}=k \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\dfrac{a}{b}=\sqrt{\dfrac{a^2+c^2}{b^2+d^2}}$

(iv) $a^6+c^6:b^6+d^6=a^3c^3:b^3d^3$

Solution:

Given $a:b=c:d$

Let $\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow \dfrac{a}{b}=k$ and $\dfrac{c}{d}=k$

$a=bk$ and $c=dk$

$$L.H.S=a^6+c^6:b^6+d^6=\frac{a^6+c^6}{b^6+d^6}$$

$$R.H.S=a^3c^3:b^3d^3=\frac{a^3c^3}{b^3d^3}$$

$$= \frac{b^6k^6 + d^6k^6}{b^6 + d^6}$$
$$= \frac{k^6\left(b^6 + d^6\right)}{b^6 + d^6}$$
$$= k^6 \quad .....(i)$$

$$= \frac{\left(b^3k^3\right)\left(d^3k^3\right)}{b^3d^3}$$
$$= \frac{\left(b^3d^3\right)\left(k^6\right)}{b^3d^3}$$
$$= k^6 \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

(v) $p(a+b)+qb : p(c+d)+qd = a:c$

Solution:

Given $a:b = c:d$

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk$ and $c = dk$

$$L.H.S = p(a+b)+qb : p(c+d)+qd$$
$$= \frac{p(a+b)+qb}{p(c+d)+qd}$$
$$= \frac{p(bk+b)+qb}{p(dk+d)+qd}$$
$$= \frac{pbk+pb+qb}{pdk+pd+qd}$$
$$= \frac{b(pk+p+q)}{d(pk+p+q)}$$
$$= \frac{b}{d} \quad ....(i)$$

$$R.H.S = a:c$$
$$= \frac{a}{c}$$
$$= \frac{bk}{dk}$$
$$= \frac{b}{d} \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $p(a+b)+qb : p(c+d)+qd = a:c$

(vi) $a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$

Solution:

Given $a:b = c:d$

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk$ and $c = dk$

$$L.H.S = a^2 + b^2 : \frac{a^3}{a+b}$$
$$= a^2 + b^2 \div \frac{a^3}{a+b}$$
$$= a^2 + b^2 \times \frac{a+b}{a^3}$$
$$= \left(b^2k^2 + b^2\right) \times \frac{bk+b}{b^3k^3}$$
$$= \left(b^2\right)\left(k^2+1\right) \times \frac{b(k+1)}{b^3k^3}$$
$$= \frac{b^3}{b^3k^3}\left(k^2+1\right)(k+1)$$

$$R.H.S = c^2 + d^2 : \frac{c^3}{c+d}$$
$$= c^2 + d^2 \div \frac{c^3}{c+d}$$
$$= c^2 + d^2 \times \frac{c+d}{c^3}$$
$$= \left(d^2k^2 + d^2\right) \times \frac{dk+d}{d^3k^3}$$
$$= \left(d^2\right)\left(k^2+1\right) \times \frac{d(k+1)}{d^3k^3}$$
$$= \frac{d^3}{d^3k^3}\left(k^2+1\right)(k+1)$$

$=\frac{1}{k^3}\left(k^2+1\right)(k+1)$ ....(i) $\quad$ $=\frac{1}{k^3}\left(k^2+1\right)(k+1)$ ....(ii)

From (i) and (ii) we have

L.H.S = R.H.S

Hence $a^2+b^2:\frac{a^2}{a+b}=c^2+d^2:\frac{c^2}{c+d}$

(vii) $\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$

Solution:

Given $a:b=c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k$ and $\frac{c}{d}=k$

$a=bk$ and $c=dk$

$$
\begin{aligned}
L.H.S &=\frac{a}{a-b}:\frac{a+b}{b}\\
&=\frac{a}{a-b}\div\frac{a+b}{b}\\
&=\frac{a}{a-b}\times\frac{b}{a+b}\\
&=\frac{bk}{bk-b}\times\frac{b}{bk+b}\\
&=\frac{bk}{b(k-1)}\times\frac{b}{b(k+1)}\\
&=\frac{k}{(k-1)(k+1)}\\
&=\frac{k}{k^2-1} \quad \text{.........}(i)
\end{aligned}
$$

$$
\begin{aligned}
R.H.S &=\frac{c}{c-d}:\frac{c+d}{d}\\
&=\frac{c}{c-d}\div\frac{c+d}{d}\\
&=\frac{c}{c-d}\times\frac{d}{c+d}\\
&=\frac{dk}{dk-d}\times\frac{d}{dk+d}\\
&=\frac{dk}{d(k-1)}\times\frac{d}{d(k+1)}\\
&=\frac{k}{(k-1)(k+1)}\\
&=\frac{k}{k^2-1} \quad \text{.........}(ii)
\end{aligned}
$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$

2. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}\left(a,b,c,d,e,f\neq 0\right)$ **then show that**

(i) $\frac{a}{b}=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

Solution:

$$\text{Let} \quad \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$$

$$\Rightarrow \quad \frac{a}{b}=k \quad \text{and} \quad \frac{c}{d}=k \quad \text{and} \quad \frac{e}{f}=k$$

$$a=bk \qquad c=dk \qquad e=fk$$

$$\begin{aligned} L.H.S &= \frac{a}{b} \\ &= \frac{bk}{b} \\ &= k \quad \text{........}(i) \end{aligned}$$

$$\begin{aligned} R.H.S &= \sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}} \\ &= \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}} \\ &= \sqrt{\frac{k^2\left(b^2+d^2+f^2\right)}{b^2+d^2+f^2}} \\ &= \sqrt{k^2} \\ &= k \quad \text{........}(ii) \end{aligned}$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{a}{b}=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$

Solution:

$$\text{Let} \quad \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$$

$$\Rightarrow \quad \frac{a}{b}=k \quad \text{and} \quad \frac{c}{d}=k \quad \text{and} \quad \frac{e}{f}=k$$

$$a=bk \qquad c=dk \qquad e=fk$$

$$\begin{aligned} L.H.S &= \frac{ac+ce+ea}{bd+df+fb} \\ &= \frac{(bk)(dk)+(dk)(fk)+(fk)(bk)}{bd+df+fb} \\ &= \frac{bdk^2+dfk^2+fbk^2}{bd+df+fb} \\ &= \frac{k^2\left(bd+df+fb\right)}{bd+df+fb} \\ &= k^2 \quad \text{........}(i) \end{aligned}$$

$$\begin{aligned} R.H.S &= \left[\frac{ace}{bdf}\right]^{\frac{2}{3}} \\ &= \left[\frac{(bk)(dk)(fk)}{bdf}\right]^{\frac{2}{3}} \\ &= \left[\frac{bdf\left(k^3\right)}{bdf}\right]^{\frac{2}{3}} \\ &= \left[k^3\right]^{\frac{2}{3}} \\ &= k^2 \quad \text{........}(ii) \end{aligned}$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$

(iii) $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

Solution:

$$Let \quad \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$$

$$\Rightarrow \quad \frac{a}{b}=k \text{ and } \quad \frac{c}{d}=k \text{ and } \quad \frac{e}{f}=k$$

$$a=bk \qquad \mathrm{c}=dk \qquad e=fk$$

$$
\begin{aligned}
L.H.S &=\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}\\
&=\frac{(bk)(dk)}{bd}+\frac{(dk)(fk)}{df}+\frac{(fk)(bk)}{fb}\\
&=\frac{bd\left(k^2\right)}{bd}+\frac{df\left(k^2\right)}{df}+\frac{fb\left(k^2\right)}{fb}\\
&=k^2+k^2+k^2\\
&=3k^2 \quad ....(i)
\end{aligned}
$$

$$
\begin{aligned}
R.H.S &=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}\\
&=\frac{b^2k^2}{b^2}+\frac{d^2k^2}{d^2}+\frac{f^2k^2}{f^2}\\
&=k^2+k^2+k^2\\
&=3k^2 \quad ....(ii)
\end{aligned}
$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}=\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

Exercise 3.7

1. The surface area A of a cube varies directly as the square of the length l of an edge and A = 27 square units when l = 3 units.

Find

- **(i) A when l = 4 units**
- **(ii) l when A = 12 sq. units.**

Solution:

Given that $A \propto l^2$

$\Rightarrow$ $A = kl^2$ ....(1)

Put $A = 27$ and $l = 3$ in eq.(1), we get

$$27 = k(3)^2$$
$$27 = 9k$$
$$9k = 27$$
$$k = \frac{27}{9} = 3$$

Put $k = 3$ in eq.(1), we get

$$A = 3l^2 \text{ ....}(2)$$

(i) Put $l = 4$ in eq(2), we get

$$A = 3(4)^2$$
$$= 3(16) = 48 \text{ sq.units}$$

(ii) Put $A = 12$ in eq.(2), we get

$$12 = 3l^2$$

or $3l^2 = 12$

$\Rightarrow$ $l^2 = 4$

$$l = 2$$

2. The surface area S of the sphere varies directly as the square of radius r, and S = 16π when r = 2. Find r when S = 36π.

Solution:

Given that $S \propto r^2$

$\Rightarrow$ $S = kr^2$ ....(i)

Put $S = 16\pi$ and $r = 2$ in eq.(i), we get

$$16\pi = k(2)^2$$
$$16\pi = 4k$$
$$4k = 16\pi$$
$$k = 4\pi$$

Put $k = 4\pi$ in eq.(i), we get

$$S = 4\pi r^2 \text{ ....}(ii)$$

Put $S = 36\pi$ in eq(ii), we get

$$36\pi = 4\pi r^2$$

or $4\pi r^2 = 36\pi$

$$r^2 = \frac{36\pi}{4\pi}$$
$$r^2 = 9$$
$$r = 3$$

3. In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and F = 32 lb when s = 1.6 in.

Find

(i) S when F = 50 lb

(ii) F when S = 0.8 in.

Solution:

Given that $F \propto S$

$$\Rightarrow \quad F = kS \quad ....(1)$$

Put $F = 32$ and $S = 1.6$ in eq.(1), we get

$$32 = k(1.6)$$

or $$1.6k = 32$$

$$k = \frac{32}{1.6} = 20$$

Put $k = 20$ in eq.(1), we get

$$F = 20S \quad ....(2)$$

(i) Put $F = 50$ in eq(2), we get

$$50 = 20S$$

$$\Rightarrow \quad S = 2.5in$$

(ii) Put $S = 0.8$ in eq(2), we get

$$F = 20(0.8) = 16lb$$

4. The intensity I of light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12ft. from the source, find the intensity at a point 8ft. from the source.

Solution:

Given that $I \propto \frac{1}{d^2}$

$$\Rightarrow \quad I = \frac{k}{d^2} \quad ....(i)$$

Put $I = 20$ and $d = 12$ in eq.(i), we get

$$20 = \frac{k}{(12)^2}$$

or $$20 = \frac{k}{144}$$

$$k = 20 \times 144 = 2880$$

Put $k = 2880$ in eq.(i), we get

$$I=\frac{2880}{d^2} \qquad \text{.........}(ii)$$

Put $d=8$ in eq(ii), we get

$$I=\frac{2880}{(8)^2}$$
$$=\frac{2880}{64}=45$$

5. The pressure P in a body of fluid varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must the fluid be to exert a pressure of 9 lb/sq. in?

Solution:

Given that $\quad P\propto d$

$\Rightarrow \quad P=kd \qquad \text{........}(i)$

Put $P=2.25$ and $\mathrm{d}=5$ in eq.(i), we get

$$2.25=k(5)$$

or $\quad 5k=2.25$

$\Rightarrow \quad \mathrm{k}=\frac{2.25}{5}$

$$=\frac{225}{500}=\frac{9}{20}$$

Put $\mathrm{k}=\frac{9}{20}$ in eq.(i), we get

$$P=\frac{9}{20}d \qquad \text{.........}(ii)$$

Put $\mathrm{P}=9$ in eq(ii), we get

$$9=\frac{9}{20}d$$
$$d=9\times\frac{20}{9}$$
$$d=20\ ft.$$

6. Labor costs c varies jointly as the number of workers n and the average number of days d. if the cost of 800 workers for 13 days is Rs. 286000, then find the labor cost of 600 workers for 18 days.

Solution:

Given that $\quad c\propto nd$

$\Rightarrow \quad c=knd \qquad \text{.......}(i)$

Put $n=800$ and $\mathrm{d}=13$ and $\mathrm{c}=286000$ in eq.(i), we get

$$286000=k(800)(13)$$
$$10400k=286000$$
$$\mathrm{k}=\frac{286000}{10400}=\frac{55}{2}$$

Put $\mathrm{k}=\frac{55}{2}$ in eq.(i), we get

$$c=\frac{55}{2}nd \qquad \text{.........}(ii)$$

Put $n=600$ and $\mathrm{d}=18$ in eq(ii), we get

$$c=\frac{55}{2}\times600\times18$$
$$c=297000$$

7. The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4-inch pillar must be to support a load of 28 tons?

Solution:

Given that $c \propto \frac{d^4}{l^2}$

$\Rightarrow$ $c = k\frac{d^4}{l^2}$ ....(i)

Put $d = 6, l = 30$ and $c = 63$ in eq.(i), we get

$$63 = k\frac{(6)^4}{(30)^2}$$

$$63 = \frac{1296}{900}k$$

or $$k = 63 \times \frac{900}{1296}$$

$$k = \frac{175}{4}$$

Put $k = \frac{175}{4}$ in eq.(i), we get

$c = \frac{175d^4}{4l^2}$ ....(ii)

Put $d = 4$ and $c = 28$ in eq(ii), we get

$$28 = \frac{175(4)^4}{4l^2}$$

$$l^2 = \frac{175 \times 256}{4 \times 28} = 400$$

$$l = 20\,ft$$

8. The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies and inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec.?

Solution:

Given that $T \propto wd$

Also given that $T \propto \frac{1}{p}$

In joint variation, we can write

$$T \propto \frac{wd}{p}$$

$\Rightarrow$ $T = k\frac{wd}{p}$ ....(i)

Put $T = 25, p = 4, w = 500$ and $d = 40$ in eq.(i), we get

$$25=\frac{k\times 500\times 40}{4}$$

$$k=\frac{25\times 4}{500\times 40}$$

or $$\mathrm{k}=\frac{1}{200}$$

Put $k=\frac{1}{200}$ in eq.(i), we get

$$T=\frac{wd}{200p} \qquad \text{........}(ii)$$

Put $\mathrm{w}=800, d=120$ and $\mathrm{T}=40$ in eq(ii), we get

$$40=\frac{800\times 120}{200p}$$

$$p=\frac{800\times 120}{200\times 40}$$

$$p=12hp$$

9. The kinetic energy (K.E.) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec.

Solution:

Given that $K.E \propto mv^2$

$\Rightarrow$ $K.E=kmv^2 \quad \text{........}(i)$

Put $K.E=4320, m=45, v=24$ in eq.(i), we get

$$4320=k(45)(24)^2$$

$$k=\frac{4320}{45\times 576}$$

or $$\mathrm{k}=\frac{1}{6}$$

Put $k=\frac{1}{6}$ in eq.(i), we get

$$K.E=\frac{1}{6}mv^2 \qquad \text{........}(ii)$$

Put $m=3000$ and $\mathrm{v}=44$ in eq(ii), we get

$$K.E=\frac{1}{6}(3000)(44)^2$$

$$P=968000$$

Miscellaneous Exercise 3

1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) In a ratio a : b, a is called

(a) relation **(b)** antecedent

(c) consequent **(d)** None of these

(ii) In a ratio x : y, y is called

(a) relation **(b)** antecedent

(c) consequent **(d)** None of these

(iii) In a proportion a : b :: c : d, a and d are called,

(a) means **(b)** extremes

(c) third proportional **(d)** None of these

(iv) In a proportion a : b :: c : d, b and c are called

(a) means **(b)** extremes

(c) fourth proportional **(d)** None of these

(v) In continued proportion a : b = b : c, ac = b^2, b is said to be ________ proportional.

(a) third **(b)** fourth

(c) means **(d)** None of these

(vi) In continued proportion a : b = b : c, c is said to be ________ proportional to a and b.

(a) third **(b)** fourth

(c) means **(d)** None of these

(vii) Find x in proportion 4 : x :: 5 : 15

(a) $\frac{75}{4}$ **(b)** $\frac{4}{3}$

(c) $\frac{3}{4}$ **(d)** 12

(viii) If $u \propto v^2$,then

(a) $u = v^2$ **(b)** $u = kv^2$

(c) $uv^2 = k$ **(d)** $uv^2 = 1$

(ix) If $y^2 \propto \frac{1}{x^3}$ **,then**

(a) $y^2 = \frac{k}{x^3}$

(b) $y^2 = \frac{1}{x^3}$

(c) $y^2 = x^2$

(d) $y^2 = kx^3$

(x) If $\frac{u}{v} = \frac{v}{w} = k$ **,then**

(a) $u = wk^2$

(b) $u = vk^2$

(c) $u = w^2k$

(d) $u = v^2k$

(xi) The third proportional of x^2 *and* y^2 **is:**

(a) $\frac{y^2}{x^2}$

(b) x^2y^2

(c) $\frac{y^4}{x^2}$

(d) $\frac{y^2}{x^4}$

(xii) The fourth proportional w of x : y :: v : w is:

(a) $\frac{xy}{v}$

(b) $\frac{vy}{x}$

(c) xyv

(d) $\frac{x}{vy}$

(xiii) If a : b = x : y, then alternando property is:

(a) $\frac{a}{x} = \frac{b}{y}$

(b) $\frac{a}{b} = \frac{x}{y}$

(c) $\frac{a+b}{b} = \frac{x+y}{y}$

(d) $\frac{a-b}{x} = \frac{x-y}{y}$

(xiv) If a : b = x : y, then invertendo property is:

(a) $\frac{a}{x} = \frac{b}{y}$

(b) $\frac{a}{a-b} = \frac{x}{x-y}$

(c) $\frac{a+b}{b} = \frac{x+y}{y}$

(d) $\frac{b}{a} = \frac{y}{x}$

(xv) If $\frac{a}{b} = \frac{c}{d}$ **, then componendo property is:**

(a) $\frac{a}{a+b} = \frac{c}{a+d}$

(b) $\frac{a}{a-b} = \frac{c}{a-d}$

(c) $\frac{ad}{bc}$

(d) $\frac{a-b}{b} = \frac{c-d}{d}$

Answers:

(i)	b	**(ii)**	c	**(iii)**	b	**(iv)**	a	**(v)**	c
(vi)	a	**(vii)**	d	**(viii)**	b	**(ix)**	a	**(x)**	a
(xi)	c	**(xii)**	b	**(xiii)**	a	**(xiv)**	d	**(xv)**	a

2. Write short answers of the following question.

(i) Define ratio and give one example.

Ans: Ratio:

A relation between two quantities of the same kind (measured in same unit) is called ratio. If a and b are two quantities of the same kind and b is not zero, then the ratio of a and b is written as a : b or in fraction $\frac{a}{b}$.

e.g., If a hockey team wins 4 games and losses 5, then the ratio of the games won to games lost is 4 : 5 or in fraction $\frac{4}{5}$.

(ii) Define proportion.

Ans: Proportion:

A proportion is a statement, which is expressed as equivalence of two ratios.

(iii) Define direct variation.

Ans: Direct variation:

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

(iv) Define inverse variation.

Ans: Inverse Variation:

If two quantities are related in such a way that when one quantity increases, the other decreases, then this variation is called inverse variation.

(v) State theorem of componendo-dividendo.

Ans: The theorem of componendo-dividendo is

If $a : b = c : d$, then

$a + b : a - b = c + d : c - d$

(vi) Find x, if 6 : x : : 3 : 5,

Ans: $6 : x :: 3 : 5$

Product of means = Product of extremes

$(x)\ (3) = (6)\ (5)$

$3x = 30$

$\Rightarrow\ x = 10$

(vii) If x and y² varies directly, and x = 27 when y = 4. Find the value of y when x = 3.

Solution:

Given that $x \propto y^2$

$\Rightarrow$ $x = ky^2$ $\quad$ ....(i)

Put $x = 27$ and $y = 4$ in eq.(i), we get

$$27 = k(4)^2$$
$$27 = 16k$$
$$16k = 27$$
$$k = \frac{27}{16}$$

Put $k = \frac{27}{16}$ in eq.(i), we get

$$x = \frac{27}{16}y^2 \quad \text{........}(ii)$$

Put $x = 3$ in eq(ii), we get

$$3 = \frac{27}{16}y^2$$
$$y^2 = 3 \times \frac{16}{27}$$
$$y^2 = \frac{16}{9} \quad \Rightarrow \quad y = \pm\frac{4}{3}$$

(viii) If u and v varies inversely, and u = 8, when v = 3. Find v when u = 12.

Solution:

Given that $u \propto \frac{1}{v}$

$$\Rightarrow \quad u = \frac{k}{v} \;\ldots\ldots(i)$$

Put $u = 8$ and $v = 3$ in eq.(i), we get

$$8 = \frac{k}{3}$$
$$k = 24$$

Put $k = 24$ in eq.(i), we get

$$u = \frac{24}{v} \;\ldots\ldots(ii)$$

Put $u = 12$ in eq(ii), we get

$$12 = \frac{24}{v} \Rightarrow v = \frac{24}{12}$$
$$v = 2$$

(ix) Find the fourth proportional to 8, 7, 6.

Solution:

Let x be the fourth propotional, then

$$8:7::6:x$$

Product of extremes = Product of means

$$(8)(x) = (7)(6)$$
$$8x = 42$$
$$x = \frac{42}{8} \quad \Rightarrow x = \frac{21}{4}$$

(x) Find a mean proportional to 16 and 49.

Solution:

Let m be the mean propotional, then

$$16:m::m:49$$

Product of means = Product of extremes

$$m.m = 16 \times 49$$
$$m^2 = 784$$
$$m = \sqrt{784} = \pm 28$$

(xi) Find a third proportional to 28 and 4.

Solution:

Let C be the third propotional, then

$$28:4::4:C$$

Product of extremes = Product of means

$$(28)(C) = (4)(4)$$
$$28C = 16$$

$$C=\frac{16}{28}$$

$$C=\frac{4}{7}$$

(xii) If $y \propto \frac{x^2}{z}$ and y = 28 when x = 7, z = 2, then find y.

Solution:

Given that $y \propto \frac{x^2}{z}$

$$y = k\frac{x^2}{z} \qquad ....(i)$$

Put $x = 7,\ y = 28$ and $z = 2$ in eq.(i), we get

$$28 = k\frac{(7)^2}{2}$$

$$28 = k\frac{49}{2}$$

$$\text{k} = \frac{28\times 2}{49}$$

$$\text{k} = \frac{56}{49}$$

$$\text{k} = \frac{8}{7}$$

Put $\text{k} = \frac{8}{7}$ in eq.(i), we get

$$y = \frac{8x^2}{7z} \qquad ....(ii)$$

(xiii) If z $\propto$ xy and z = 36 when x = 2, y = 3, then find z.

Solution:

Given that $z \propto xy$

$$z = kxy \qquad ....(i)$$

Put $z = 36, x = 2$ and $y = 3$ in eq.(i), we get

$$36 = k(2)(3)$$

or $36 = 6k$

$\Rightarrow$ $\text{k} = 6$

Put $\text{k} = 6$ in eq.(i), we get

$$z = 6xy$$

(xiv) If $w \propto \frac{1}{v^2}$ and w = 2 when v = 3, then find w.

Solution:

Given $w \propto \frac{1}{v^2}$

$$w = \frac{k}{v^2} \ldots\ldots(i)$$

Put $w = 2$ and $v = 3$ in eq.(i), we get

$$2 = \frac{k}{(3)^2}$$

or $2 = \frac{k}{9}$

$$k = 18$$

Put $k = 18$ in eq.(i), we get

$$w = \frac{18}{v^2}$$

Q3. Fill in the blanks.

(i) The simplest form of the ratio $\frac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$ is ____________.

(ii) In a ratio x : y ; x is called ______

(iii) In a ratio a : b; b is called ______

(iv) In a proportion a : b : : x : y; a and y are called _________.

(v) In a proportion p : q : : m : n; q and m are called ___________.

(vi) In proportion 7 : 4 :: p : 8, p= __________.

(vii) If 6: m : : 9 : 12, then m = ___________.

(viii) If x and y varies directly, then x = __________.

(ix) If v varies directly as u^3, then u^3 = _____________.

(x) If w varies inversely as p^2, then k = ____________.

(xi) A third proportional of 12 and 4, is ____________.

(xii) The fourth proportional of 15, 6, 5 is ____________.

(xiii) The mean proportional of $4m^2 n^4$ and p^6 is _________.

(xiv) The continued proportion of 4, m and 9 is __________.

Answers:

(i)	$\frac{x+y}{x-y}$	**(ii)**	Antecedent	**(iii)**	Consequent
(iv)	Extremes	**(v)**	Means	**(vi)**	p = 14
(vii)	m = 8	**(viii)**	ky	**(ix)**	$\frac{v}{k}$
(X)	p^2w	**(xi)**	$\frac{4}{3}$	**(xii)**	2
xiii)	$\pm 2mn^2p^2$	**(xiv)**	m = ±6		

Summary

✓A relation between two quantities of the same kind is called ratio.

✓ A proportion is a statement, which is expressed as equivalence of two ratios.

If two ratios a: b and c: d are equal, then we can write a : b = c : d

✓ If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then the variation is called direct variation.

✓ If two quantities are related in such a way that when one quantity increases, the other decreases, then the variation is called inverse variation.

✓ Theorem on proportions:

(1) Theorem of Invertendo

If a : b = c : d, then b : a = d : c

(2) Theorem of Alternando

If a : b = c : d, then a : c = b : d

(3) Theorem of Componendo

If a : b = c : d, then

(i) a + b : b = c + d : d

and (ii) a : a + b = c : c + d

(4) Theorem of Dividendo

If a : b = c : d, then

(i) a – b : b = c – a :d

(ii) a : a - b=c : c – d

(5) Theorem of Componendo-dividendo

If a : b = c : d, then

a + b : a – b = c + d : c – d

✓A combination of direct and inverse variations of one or more than one variable forms joint variation.

✓K-Method

(a) $If\ \frac{a}{b}=\frac{c}{d}$

$Then\ \frac{a}{b}=\frac{c}{d}=k \qquad or \qquad a=bk \text{ and } c=dk$

(b) $If\ \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k, \qquad then \qquad a=bk, c=dk\ and\ e=fk$

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