Exercise 5.1

1. If X = {1, 4, 7, 9} and Y = {2, 4, 5, 9)

Then find:

(i) $X \cup Y$

(ii) $X \cap Y$

(iii) $Y \cup X$

(iv) $Y \cap X$

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$

$= \{1, 2, 4, 7, 9\}$

(ii) $X \cap Y = \{l,4,7,9\} \cap \{2,4,5,9\}$

$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{l, 4, 7, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cup X = \{2, 4, 5, 9\} \cap \{l, 4, 7, 9\}$

$= \{4, 9\}$

2. If X Set of prime numbers less than or equal to 17 and Set of first 12 natural numbers, then find the following.

(i) $X \cup Y$ **(ii)** $Y \cup X$ **(iii)** $Z \cap Y$ **(iv)** $Y \cap X$

Solution:

$X = \{2, 3, 5, 7, 11, 13, 17\}$, $Y = \{l, 2, 3, 4, ,12\}$

(i) $X \cup Y = \{2, 3, 5, 7, 11, 13, 17\} \cup \{l, 2, 3, 4, \ldots.., 12\}$

$=\{1, 2, 3, 4, \ldots.., 12, 13, 17\}$

$=\{1, 2, 3, 4, \ldots..., 12\} \cup \{13, 17\}$

$= Y \cup \{13,17\}$

(ii) $Y \cup X = \{l, 2, 3, 4, \ldots, 12\} \cup \{2, 3, 5, 7, 11, 13, 17\}$

$= \{1, 2, 3, 4, \ldots., 12, 13, 17\}$

$= \{l, 2, 3, 4, \ldots, 12\} \cup \{l3, 17\}$

$=Y \cup \{13,17\}$

(iii) $X \cap Y = \{2, 3, 5, 7, 11, 13, 17\} \cap \{1, 2, 3, \ldots, 12\}$

$= \{2, 3, 5, 7, 11\}$

(iv) $Y \cap X = \{l, 2, 3, 4, \ldots., 12\} \cap \{2, 3, 5, 7, 11, 13, 17\}$

$= \{2, 3, 5, 7, 11\}$

3. If $X = \emptyset$, $Y = Z^+$, $F = O^+$, then

find: (i) $X \cup Y$ (ii) $X \cup T$ (iii) $Y \cup T$ (iv) $X \cap Y$

(v) $X \cap T$ (vi) $Y \cap T$

Solution:

$X = \emptyset, Y = Z^+, T = O^+$

(i) $X \cup Y = \emptyset \cup Z^+$

$= Z^+ = Y$

(ii) $X \cup T = \emptyset \cup O^+$

$= O^+ = T$

(iii) $Y \cup T = Z^+ \cup O^+$

$= Z^+ = Y$

(iv) $X \cap Y = \emptyset \cap Z^+$

$= \emptyset$

(v) $X \cap T = \emptyset \cap O^+$

$= \emptyset$

(vi) $Y \cap T = Z^+ \cap O^+$

$= O^+ = T$

4. If $U = \{x \mid x \in N \wedge 3 < x \leq 25\}$, $X = \{x \mid x \text{ is prime} \wedge 8 < x < 25\}$ and $Y = \{x \mid x \in W \wedge 4 \leq x \leq 17)$. Find the value of:

i) $(X \cup Y)'$ (ii) $X' \cap Y'$ (iii) $(X \cap Y)'$ (iv) $X' \cup Y'$

Solution:

$U = \{4, 5, 6, 7, \ldots, 24, 25\}$

$X = \{11, 13, 17, 19, 23\}$

$Y = \{4, 5, 6, 7, \ldots, 16, 17\}$

(i) $(X \cup Y)' = U - (X \cup Y)$

Now

$X \cup Y = \{11, 13, 17, 19, 23\} \; \{4, 5, 6, 7, \ldots, 16, 17\}$

$= \{4. 5, 6, 7, \ldots, 16, 17, 19, 23\}$

$(X \cup Y)' = \{4, 5. 6. 7,......,24, 25\} - \{4, 5, 6, 7,......,16, 17, 19, 23\}$

$= \{18, 20, 21, 22, 24, 25\}$

(ii) $X' \cap Y'$

Now $X' = U - X$

$= \{4, 5, 6, 7,....,24, 25\} - \{11, 13, 17, 19, 23\}$

$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$

$Y' = U - Y$

$= \{4, 5, 6, 7,....,24, 25\} - \{4, 5, 6, 7,.....,16, 17\}$

$= \{18, 19, 20,........, 24, 25\}$

$X' \cap Y' = \{4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 21, 22, 24, 25\}$

$\cap \{18, 19, 20,.....,24, 25\}$

$= \{18, 20, 21, 22, 24, 25\}$

(iii) $(X \cap Y)' = U - (X \cap Y)$

Now

$X \cap Y = \{11, 13, 17, 19, 23\} \cap \{4, 5, 6, 7,..., 16, 17\}$

$= \{11,13, 17\}$

$(X \cap Y)' = \{4, 5, 6, 7,...., 24, 25\} - \{11, 13, 17\}$

$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 18, 20, 21, 22, 24, 25\}$

$Y' = U - Y$

$= \{4, 5, 6, 7,....., 24, 25\} - \{4, 5, 6, 7,...., 16, 17\}$

$\{18, 19, 20,....,24, 25\}$

$Y' = U - Y$

$= \{4, 5, 6,...., 10, 12, 14, 15, 16, 18,....., 25\} \cup = \{ 18, 19, 20,...., 24, 25\}$

$= \{4, 5 .... ,10, 12, 14, 16, 18, ..., 25\}$

5. If $X = \{2, 4, 6, ..., 20\}$ and $Y = \{4, 8, 12,...., 24\}$, then find the following:

(i) $X - Y$ **(ii) $Y - X$**

Solution:

$X = \{2, 4, 6,...., 20\}$, $Y = \{4, 8, 12,...., 24\}$

(i) $X - Y$ $= \{2,4, 6,..., 20\} - \{4, 8, 12,..., 24\}$

$= \{2, 6,10, 14, 18\}$

(ii) Y - X $= \{4, 8, 12, \ldots, 24\} - \{2, 4, 6, \ldots, 20\}$

$= \{24\}$

6. If A = N and B = W, then find the value of

(i) A - B **(ii) B – A**

Solution:

$A = N$ and $B = W$

(i) A - B $= N - W$

$= \emptyset$

(ii) B - A $= W - N$

$= \{0, 1, 2, \ldots\} - \{1, 2, \ldots\}$

$= \{0\}$

Properties of Union and Intersection:

(a) Commutative property of union.

For any two sets A and B, prove that $A \cup B = B \cup A$.

Proof:

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$ (by definition of union of sets)

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$\Rightarrow A \cup B \subseteq B \cup A$ (i)

Now let $y \in B \cup A$

$\Rightarrow y \in B$ or $y \in A$ (by definition of union of sets)

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow B \cup A \subseteq A \cup B$ (ii)

From (i) and (ii), we have $A \cup B = B \cup A$. (by definition of equal sets)

(b) Commutative property of intersection

For any two sets A and B, prove that $A \cup B$

Proof: Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$ (by definition of intersection of sets)

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cap A$

$\Rightarrow A \cap B \subseteq B \cap A$ (i)

Now let $y \in B \cap A$

$\Rightarrow y \in B$ and $y \in A$ (by definition of intersection of sets)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A \cap B$

Therefore, $B \cap A \subseteq A \cap B$ (ii)

From (i) and (ii), we have $A \cap B = B \cap A$ (by definition of equal sets)**(c)**

Associative property of union

For any three sets A, B and C, prove that $(A \cup B) \cup C = A \cup (B \cup C)$

Proof: Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $A \in C$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A \cup (B \cap C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$ (i)

Similarly, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ (ii)

From (i) and (ii), we have

$(A \cup B) \cup C = A \cup (B \cup C)$

(d) Associative property of intersection

For any three sets A, B and C, prove that $(A \cap B) \cap C = A \cap (B\ C\}$

Proof: Let $\in (A \cap B) \cap C$.

$\Rightarrow X \in (A \cap B)$ and $X \in C$

$\Rightarrow (X \in A$ and $x \in 5)$ and $x \in C$

$\Rightarrow x \in A$ and $(x \in B)$ and $\in C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow X \in A \cap (B \cap C)$

$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C)$ (i)

Similarly, $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ (ii)

From (i) and (ii), we have

$(A \cap B) \cap C = A \cap (B \cap C)$

(e) Distributive property of union over intersection

For any three sets A, B and C, prove that $A \cup (B \cup C) = (A \cup B) \cap (A \cup C)$

Proof: Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $(x \in B$ and $X \in C)$

$\Rightarrow (X \in A$ or $x \in B)$ and $(x \in A$ or $X \in C)$

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Similarly, now let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \in A$ or $y \in C)$

$\Rightarrow y \in A$ or $(y \in B$ and $y \in C)$

$\Rightarrow y \in A$ or $y \in B \cap C$

$\Rightarrow y \in y \in \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ (ii)

From (i) and (ii), we have $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(f) Distributive property of intersection over union

For any three sets A, B and C, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof: Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A$ and $[x \in B$ or $x \in C]$

$\Rightarrow [x \in A$ and $X \in B]$ or $[x \in A$ and $X \in C]$

$\Rightarrow [x \in A \cap B]$ or $[x \in A \cap C]$

$\Rightarrow x \in (A \cap B) \cup (A \cup C)$

Hence by def. of subsets

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ (i)

Similarly $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ (ii)

From (i) and (ii). we have. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) De-Morgan's laws

For any two sets A and B, prove that

(i) $(A \cup B)' = A' \cap B'$

Proof: Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$ (by definition of complement of set)

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $X \in B'$

$\Rightarrow x \in A' \cap B'$ (by definition of intersection of sets)

$\Rightarrow (A \cup B) \subseteq (A \cup B)'$ (i)

Similarly, $A' \cap B' \subseteq (A \cup B)$ (ii)

Using (i) and (ii), we have $(A \cup B)' = A' \cap B'$

(ii) Let $x \in (A \cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$or $x \in B'$

$\Rightarrow x\ A' \cup B'$

$\Rightarrow (A \cap B)' \subseteq A' \cup B'$ (i)

Let $y \in A' \cap B'$

$\Rightarrow y \in A \cap B$

$\Rightarrow y \notin A$ or $x \notin B$

$\Rightarrow y \notin A \cap B$

$\Rightarrow y \in (A \cap B)'$

$\Rightarrow (A' \cap B)' \subseteq A' \cap B'$ (ii)

From (i) and (ii) we have proved that

$(A \cap B)' = A' \cup B'$

Exercise 5.2

1. If $X = \{1, 3, 5, 7, ...,19\}$, $Y = (0, 2, 4, 6, 8,...,20)$

$Z = (2, 3, 5, 7, 11, 13, 17, 19, 23)$, then find the following.

(i) $X \cup (Y \cup Z)$

Solution:

$Y \cup Z = \{0, 2, 4, 6, 8,...,20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{0, 2, 3, 4, ..., 17, 19, 20, 23)$

$X \cup (Y \cup Z) = \{1, 3, 5, 7, ..., 19) \cup \{ 0, 2, 3, 4, ..., 17, 19, 20, 23)$

$= \{0, 1, 2, 3, ..., 30, 33\}$

(ii) $(X \cup Y) \cup Z$

Solution:

$X \cup Y = \{1, 3, 5, 7, .., 19) \cup \{0, 2, 4, 6, 8, ..., 20\}$

$= \{0, 1, 2, 3, ..., 19, 20)$

$(X \cup Y) \cup Z = \{0, 1, 2, 3, ..., 19, 20) \cup \{ 2, 3, 5, 7, 11, 13, 17, 19, 23)$

$= \{0, 1, 2, 3,....., 20, 23)$

(iii) $X \cap (Y \cap Z)$

Solution:

$Y \cap Z = \{0, 2, 4, 6, 8, ..., 20) \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \emptyset$

$X \cap (Y \cap Z)$

$X \cap Y = \{1, 3, 5, 7,...., 19\} \cap \emptyset$

$= \emptyset$

(iv) $(X \cap Y) \cap Z$

Solution:

$X \cap Y = \{1, 3, 5, 7, ..., 19) \cap \{0, 2, 4, 6, 8, ...,20\}$

$= \emptyset$

$(X \cap Y) \cap Z = \emptyset \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23)$

$= \emptyset$

(v) $X \cup (Y \cap Z)$

Solution:

$Y \cap Z = \{0, 2, 4, 6, 8, ..., 20) \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{2)$

$X \cup (Y \cap Z) = \{1, 3, 5, 7,...,19\} \cup \{2\}$

$= \{1, 2, 3, 5, 7,...,19\}$

(vi) $(X \cup Y) \cap (X \cup Z)$

Solution:

$X \cup Y = \{1, 3, 5, 7,....,19\} \cup \{0, 2, 4, 6, 8,....,20\}$

$= \{0, 1, 2, 3,...., 19, 20)$

$X \cup Z = \{1, 3, 5, 7, ..., 19) \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23)$

$= \{ 1, 2, 3, 5, 7, ..., 17, 19, 23)$

$(X \cup Y) \cap (X \cup Z) = \{0, 1, 2, 3, ..., 19, 20) \cap \{l, 2, 3, 5, 7, ..., 17, 19, 23)$

$= \{1, 2, 3, 5, 7, ..., 19\}$

(vii) $X \cap (Y \cup Z)$

Solution:

$Y \cup Z = \{0, 2, 4, 6, 8, ..., 20\} \cup (2, 3, 5, 7, 11, 13, 17, 19, 23)$

$= \{0, 2, 3, 4, 5, 6, ..., 19, 20, 23)$

$X \cap (Y \cup Z) = \{1, 3, 5, 7,...., 19\} \cap \{0, 2, 3, 4, 5, 6, ..., 19,20\}$

$= \{3, 5, 7, ..., 19\} \cap \{0, 2, 4, 6, 8,...., 20\}$

$= \emptyset$

$X \cap Z = \{1, 3, 5, 7, ..., 19) \cap \{2, 3, 5, 7, 11, 13, 17, 19,23)$

$= \{3, 5, 7, 11, 13, 17, 19)$

(viii) $(X \cap Y) \cup (X \cap Z)$

Solution:

$X \cap Y = \{1, 3, 5, 7,....,19) \cap \{0, 2, 4, 6, 8, ..., 20\}$

$= \emptyset$

$X \cap Z = \{1, 3, 5, 7,...., 19\} \cap (2, 3, 5, 7, 11, 13, 17, 19, 23)$

$= \{3, 5, 7, 11, 13, 17, 19\}$

2. If $4 = (1, 2, 3, 4, 5, 6)$, $B = (2, 4, 6, 8)$, $C= \{1, 4, 8)$.

Prove the following identities:

(ii) $A \cap B = B \cap A$

Solution:

L.H.S. $= A \cap B$

$= \{1, 2, 3, 4, 5, 6) \cap (2, 4, 6, 8)$

$= (2, 4, 6)$ .....(i)

R.HS. $= B \cap A$

$= (2, 4, 6, 8) \cap (1, 2, 3, 4, 5, 6)$

$= (2, 4, 6\}$ .....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence proved.

(ii) $A \cup B = B \cup A$

Solution:

L.H.S. $= A \cup B$

$= (1, 2, 3, 4, 5, 6) \cup \{2, 4, 6, 8)$

$=\{1, 2, 3, 4, 5, 6, 8)$ ....(i)

R.HS. $= B \cup A$

$= \{2, 4, 6, 8) \cup (1, 2, 3, 4, 5, 6)$

$= \{1, 2, 3, 4, 5, 6, 8)$ ......(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence proved.

(iii) $A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution:

L.H.S. $= A \cap (B \cup C)$

$= \{1, 2, 3, 4, 5, 6\} \cap (\{2, 4, 6, 8) \cap \{1, 4. 8\})$

$=\{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$

$=\{1, 2, 3, 4, 5, 6\}$ ....(i)

R.H.S. $= (A \cap B) \cup (A \cup C)$

$= (\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}) \cup (\{1, 2, 3, 4, 5\ 6\} \cap \{1, 4, 8\})$

$= \{2, 4, 6\} \cup \{1, 4\}$

$= \{1, 2, 3, 4, 5, 6)$ ....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

L.H.S.$= A \cup (B \cap C)$

$= \{1, 2, 3, 4, 5, 6\} \cup (\{2, 4, 6, 8) \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6,\} \cup \{4, 8)$

$= \{1, 2, 3, 4, 5, 6, 8\}$ ....(i)

R.HS. $= (A \cup B) \cap (A \cup C)$

R.HS. $= (A \cup B) \cap (A \cup C)$

$= (\{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}) \cap (\{1,2, 3, 4, 5, 6\} \cap \{1, 4, 8\})$

$= \{1,2, 3, 4, 6, 8\} \cap \{1.2, 3. 4. 5, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6,8\}$ ....(ii)

From (i) and (ii), we have L.H.S = R.H.S.

Hence Proved.

3. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, then verify the De-Morgan's Laws.

i.e., $(A \cap B) = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Solution:

L.H.S. $= A' \cup B'$

$= U - (A \cap B)$

$= \{1,2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1.3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8. 9, 10\} - \{3, 5, 7\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$ ....(i)

R.H.S. $= A' \cup B'$

$= [U - A] \cup [U - B]$

$= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\})$

$\cup (\{1,2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\})$

$= \{2, 4, 6, 8, 10\} \cup \{1,4, 6, 8, 9, 10\}$

$= \{1, 2, 4, 6, 8, 9,10\}$ ....(ii)

From (j) and (ii), we have L.H.S = R.H.S.

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. $= A' \cup B'$

$= U - (A \cup B)$

$= \{1, 2, 3,4, 5, 6, 7, 8, 9, 10\} - (\{1,3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 5, 7,9\}$

- $=\{4, 6, 8, 9,10\}$ ....(i)

R.H.S. $= A' \cap B'$

$= [U - A] \cap [U - B]$

$= (\{1, 2, 3,......,10\} - \{1,3, 5, 7, 9\}) \cap (\{1, 2, 3,......,10\} - \{2, 3, 5, 7\})$

$=\{2, 4, 6. 8. 10\} \cap \{1,4 ,6, 8, 9. 10\}$

$= \{4, 6, 8, 9,10\}$ .....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

4. If $U = \{1, 2, 3, ...,20)$, $X = \{1, 3, 7, 9,15, 18, 20\}$ and

$Y = (1, 3, 5,..., 17\}$, then show that

(i) $X - Y = X \cap Y'$

Solution:

L.H.S. = X ∩ Y'

= {1, 3, 5, 7, 9, 15, 18, 20} ∩ (U - Y)

= {1, 2, 5, 7, 9, 15, 18, 20} ∩ ({ 1, 2, 3,....,20} - {1, 3, 5, 17})

= {1, 3, 5, 7, 9, 15, 18, 20} ∩ {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

= {18, 20} ....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

(ii) Y - X = Y ∩ X'

Solution:

L.H.S = Y - X

= {1, 3, 5,.....,17} - {1, 2, 5, 7, 9, 15, 18, 20}

= {5, 11, 13, 17} ....(i)

R.H.S. = Y ∩ X'

= Y ∩ (U - X)

= {1, 3, 5,...., 17} ∩ ({1, 3,....., 20} - {1, 3, 5, 7, 9, 15, 18. 20}

= {1,3, 5,....., 17} ∩ {2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 19}

= {5, 11,13, 17} .....(ii)

From (i) and (ii). we have

L.H.S = R.H.S.

Hence Proved.

Verify the fundamental properties for given sets:

(a) A and B are any two subsets of U, then A ∪ B = B ∪ A (commutative law).

For example

A = {1, 3, 5, 7} and B = {2, 3, 5, 7}

then A ∪ B = {1, 3, 5, 7} ∪ {2, 3, 5, 7} = {1, 2, 3, 5, 7}

and B ∪ A = {2, 3, 5, 7} ∪ {1,3, 5, 7} = {1, 2, 3, 5, 7}

Hence, verified that A ∪ B = B ∪ A.

(b) Commutative property of intersection

For example A = {1, 3, 5, 7} and B = [2, 3, 5, 7}

Then A ∩ B = {1, 3, 5, 7} ∩ {2, 3, 5, 7} = {3, 5, 7}

and B ∩ A = {2, 3, 5, 7} ∩ {1,3, 5, 7} = {3, 5, 7}

Hence, verified that A ∩ B = B ∩ A.

(c) If A, B and C are the subsets of U, then $(A \cup B) \cup C = A \cup (B \cup C)$.

(Associative law)

Suppose $A = \{1, 2, 4, 8\}$; $B = \{2, 4, 6\}$

And $C = \{3, 4, 5, 6\}$

Then L.H.S. $= (A \cup B) \cup C$

$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cup \{3, 4, 5, 6\}$

$= \{1, 2, 4, 6, 8\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

and R.H.S. $= A \cup (B \cup C)$

$= \{42, 4, 8\} \cup (\{2, 4, 6\} \cup (3, 4, 5, 6\})$

$= \{1, 2, 4, 8\} \cup \{2, 3, 4, 5, 6\}$

$= \{42, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S

Hence, union of Sets is associative.

(d) If A, B and C are the subsets of U, then $(A \cap B) \cap C = A \cap (B \cap C)$

(Associative Law).

Suppose., $A = \{1, 2, 4, 8\}$; $5 = \{2, 4, 6\}$ and $C = \{3, 4, 5, 6\}$

then L.H. S, $= (A \cap B) \cap C$

$= (\{1, 2, 4, 8\} \cap \{2, 4, 6)) \cap \{3, 4, 5, 6\}$

$= \{2, 4\} \cap \{3, 4, 5, 6\} = \{4\}$

and R.H.S. $= A \cap (B \cap C)$

$= \{1, 2, 4, 8\} \cap (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$

$= (1, 2, 4, 8\}; = \{4, 6\} - \{4\}$

L.H.S. = R.H.S.
Hence, intersection of sets is associative.

Distributive laws
(e) Union is distributive over intersection of sets
If A, B and C are the subsets of universal set U, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **Solution:** Suppose $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6\}$ and $C = \{3, 4, 5, 6\}$
then L.H.S $= A \cup (B \cap C)$

$=\{1, 2, 4, 8\} \cup (\{2, 4, 6) \cap \{3, 4, 5, 6\})$

$= \{1, 2, 4, 8) \cup \{4, 6\} - \{1, 2, 4, 6, 8\}$

and R.H.S $= (A \cup B) \cap (A \cup C)$

$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cap (\{1, 2, 4, 8\} \cup \{3, 4, 5, 6\})$

$= (1, 2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$

$= (1, 2, 4, 6, 8\}$

L.H.S = R.H.S

(f) Intersection is distributive over union of sets

To prove $\cap (B \cup C) = (A \cap B) \cup (A \cap B)$

Suppose $A = \{1, 2, 3, 4, 5, ..., 20\}$

$B = \{5, 10, 15, 20, 25, 30\}$

$C = \{3, 9, 15, 21, 27, 33\}$

L.H.S. $= A \cap (B \cup C)$

$= \{1, 2, 3, 4, 5, ..., 20\} \cap (\{5, 10, 15, 20, 25, 30\} \cup \{3, 9, 15, 21, 27, 33\})$

$= \{1, 2, 3, 4, 5, ..., 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$

$= \{3, 5, 9, 10, 15, 20\}$

R.H.S. $= (A \cap B) \cup (A \cap C)$

$= (\{1, 2, 3, 4, ..., 20\} \cap \{5, 10, 15, 20, 25, 30\})$

$\cup (\{1, 2, 3, 4, 5, ..., 20\} \cap \{3, 9, 15, 21, 27, 33\})$

$= \{5, 10, 15, 20\} \cup \{3, 9, 15\} = \{3, 5, 9, 10, 15, 20\}$,

L.H.S. = R.H.S.

(g) De Morgan's Laws $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Suppose $U = \{1, 2, 3, 4, ..., 10\}$

$A = \{2, 4, 6, 8, 10\}$ $\Rightarrow \{1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 5, 6\}$ $\Rightarrow B' = \{7, 8, 9, 10\}$

Now consider $A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6\}$

. $=\{2, 4, 6\}$

Then L.H.S. $= (A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, ..., 10) - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

and R.H.S. $= A \cup B'$

$= \{1, 3, 5, 7, 9\} \cup \{7, 8, 9, 10\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

L.H.S. = R.H.S.

$(A \cup B)' = A' \cap B'$.

Suppose $U = \{1, 2, 3, 4, ..., 10\}$

$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{-1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$

Now consider $A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8, 10\}$

L.H.S. $= (A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, ..., 10\} - \{1, 2, 3, 4, 5, 6, 8, 10\}$

$= \{7, 9\}$

and R.H.S $A' = B' = \{1, 3, 5, 7, 9\} \cap \{7, 8, 9, 10\}$

$= \{7, 9\}$

L.H.S. = R.H.S.

Venn Diagram:

British mathematician John Venn (1834 - 1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

Use Venn diagrams to represent:

(a) Union and intersection of sets

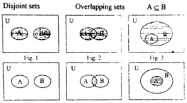
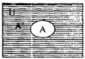


Fig. 1 Fig. 2 Fig. 3

Fig. 4 Fig. 5 Fig. 6

(Regions shown by horizontal line segments in figures 1 to 6.)

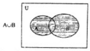
(b) Complement of a set

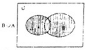


U – A = A' is shown by horizontal line segments.

Use Venn diagram to verily:

(a) Commutative law for union and intersection of sets.

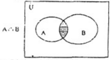


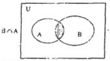


A ∪ B is shown by horizontal line segments,

B ∪ A is shown by vertical line segments.

The regions shown in both cases are equal.
Thus A ∪ B = B ∪ A.,

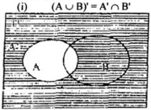




A ∩ B is shown by horizontal line segments. B ∩ A is shown by vertical line segments.
The regions shown in both cases are equal.
Thus A ∩ B = B ∩ A.

(b) De Morgan's laws

(i) (A ∪ B)' = A' ∩ B' (ii) (A ∩ B)' = A' ∪ B'



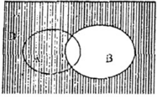
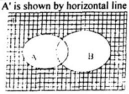


Fig. 1: A' is shown by horizontal line segments Fig. 2: B' is shown by vertical line segments



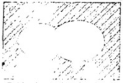
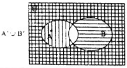


Fig. 3: A' ∩ B' is shown by squares Fig. 4: (A ∪ B)' is shown by slanting line segments

Regions shown in Fig. 3 and Fig. 4 are equal.
Thus (A ∪ B)' = A' ∩ B'

(A ∩ B)' = A' ∪ B'



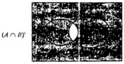


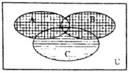
Fig. 5 A′ ∪ B′ is shown by squares, horizontal and vertical line segments.

Fig.6 U – (A ∩ B) = (A ∩ B) is shown by squares, horizontal

Regions shown in Fig. 5 and Fig. 6 are equal.

Thus $(A \cap B)' = A' \cup B'$

(c) Associative law:



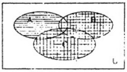
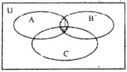


Fig. 1
(A ∪ B) ∪ C is shown in the above figure,

Fig. 2
A ∪ (5 ∪ C) is shown in the above figure.

Regions shown in fig. 1 and fig. 2 by different ways are equal.

Thus $(A \cup B) \cup C = A \cup (B \cup C)$



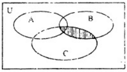


Fig. 3

Fig. 4

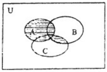
(A ∩ B) ∩ C is shown in figure 3 by double

A ∩ (B ∩ C) is shown in figure 4 by double crossing line segments

Regions shown in Fig. 3 and fig. 4 are equal.

Thus $(A \cap B)' \cap C' = A \cap (B \cap C)$

(d) Distributive law:



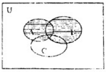
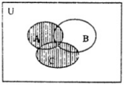


Fig. 1: $A \cup (B \cap C)$ is shown by horizontal line segments in the above figure.

Fig. 2: A u B is shown by horizontal line segments in the above figure.



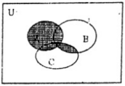
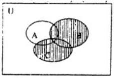


Fig. 3: $A \cup C$ is shown by vertical line segments in Fig. 3,

Fig. 4: $(A \cup B) \cap (A \cup C)$ is shown by double crossing line segments in Fig. 4.

Regions shown in Fig. 1 and Fig. 4 are equal.

Thus $A \cup (B \cap C) = (A \cup B') \cap (A \cup C)$



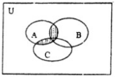


Fig. 5: $B \cup C$ is shown by vertical line segments in Fig. 5.

Fig. 6: $A \cap (B \cup C)$ is shown in Fig. 6 by vertical line segments.

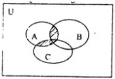


Fig. 7: $(A \cap B) \cup (A \cap C)$ is shown in Fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig, 7 are equal.

Thus $A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$

Exercise 5.3

1. If U = {1, 2, 3, 4, ..., 10)

A = {1, 3, 5, 7, 9)

B = (1, 4, 7, 10) then verify the following questions,

(i) A – B = A ∩ B'

L.H.S. = A - B

= {1, 3, 5, 7, 9) — {1, 4, 7, 10}

= {3, 5, 9} .....(i)

R.H.S. = A ∩ B'

=A ∩ (U - B)

= {1, 3, 5, 7, 9} ∩ ({1, 2, 3, 4,...., 10} – {1, 4, 7, 10})

= (1, 3, 5, 7, 9) ∩ (2, 3, 5, 6, 8, 9)

= (3, 5, 9) .....(ii)

From (1) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(ii) B – A = B ∩ A'

L.H.S. = B - A

= (1, 4, 7, 10) - (1, 3, 5, 7, 9)

= (4, 10) ....(i)

R.H.S. = B ∩ A'

=B ∩ (U - A)

= (1, 4, 7, 10) ∩ {1, 2, 3, 4,...., 10} - (1, 3, 4, 5, 7, 9))

= (1, 4, 7, 10) ∩ (2, 4, 6, 8, 10)

= (4, 10) ....(ii)

From (i) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(iii) (A ∪ B)' = A' ∩ B'

L.H.S. = (A ∪ B)'

= U - (A ∪ B)

= {1, 2, 3, 4, ..., 10} – ({1, 3, 5, 7, 9} ∪ {1, 4, 7, 10})

= (1, 2, 3, 4, ...10) - (1, 3, 4, 5, 7, 9, 10)

= (2, 6, 8) .....(i)

R.H.S. = A' ∩ B'

= (U - A) ∩ (U - B)

= ({1, 2, 3, 4, ..., 10} – {1, 3, 5, 7, 9}) ∩ (1, 2, 3, 4,...., 10) – {1, 4, 7, 10})

= (2, 4, 6, 8, 10} ∩ (2, 3, 5, 6, 8, 9)

= (2, 6, 8) ......ii)

From (i) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(iv) $(A \cap B)' = A' \cup B'$

L.H.S. = $(A \cup B)'$

$= U - (A \cap B)$

$= (1, 2, 3, 4, \ldots, 10) - (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\})$

$= (1, 2, 3, 4, \ldots 10) - \{1, 7\}$

$= (2, 3, 4, 5, 6, 8, 9, 10)$ ....(i)

R.H.S. = $A' \cup B'$

$= (U - A) \cup (U - B)$

$= (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\}) \cup (\{1, 2, 3, 4, \ldots, 10) - \{1, 4, 7, 10))$

$= \{2, 4, 6, 8, 10) \cup (2, 3, 5, 6, 8, 9)$

$= \{2, 3, 4, 5, 6, 8, 9, 10)$ .....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S. Hence Proved

(v) $(A - B)' = A' \cup B$

L.H.S. = $(A - B)'$

$= U - (A - B)$

$= \{1, 2, 3, 4, \ldots, 10) - (\{1, 3, 5, 7, 9) - \{1, 4, 7, 10\})$

$= \{1, 2, 3, 4, \ldots 10\} - \{3, 5, 9)$

$= \{1, 2, 3, 6, 7, 8, 10)$ .....(i)

R.H.S. = $A' \cup B'$

$= (U - A) \cup B$

$= (\{1, 2, 3, 4, \ldots, 10) - \{1, 3, 5, 7, 9\}) \cup \{1, 4, 7, 10)$

$= \{2, 4, 6, 8, 10) \cup \{1, 4, 7, 10)$

$= \{1, 2, 4, 6, 7, 8, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S. Hence Proved

(VI) $(B - A)' = B' \cup A$

L.H.S. = $(B \cup A)'$

$= U - (B - A)$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\})$

$= \{1, 2, 3, 4, \ldots, 10) - \{4, 10)$

$= \{1, 2, 3, 5, 6, 7, 8, 9\}$ .....(i)

R.H.S. = $B' \cup A$

$= (U - B) \cup A$

$=(\{1, 2, 3, 4, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\}) \cup \{1, 3, 5, 7, 9)$

$= \{2, 3, 5, 6, 8, 9\} \cup \{1,3, 5, 7, 9)$

$= \{1, 2, 3, 5, 6, 7, 8, 9\}$ .....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

2. If $U = (1,2, 3,4,-,10)$; $A = (1, 3, 5, 7, 9)$; $B = (1, 4, 7,10)$; $C = \{1, 5, 8, 10)$ then verify the following:

Solution:

(i) $(A \cup B)\cup C = A \cup (B \cup C)$

L.H.S.= $(A \cup B) \cup C$

$= (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10)) \cup \{l, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 9, 10\} \cup \{1, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ ....(i)

R.H.S. = $A \cup (B \cup C)$

$= \{1, 3, 5, 7, 9) \cup (\{1, 4, 7, 10) \cup \{1, 5, 8, 10))$

$=\{l, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8,10)$

$= \{1, 3, 4, 5, 7, 8, 9, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S. = $(A \cap B) \cap C$

$=(\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$

$= \{1, 7\} \cap \{1, 5, 8, 10)$

$= \{1\}$ ......(i)

R.H.S. = $A \cap (B \cap C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$

$= (1, 3, 5, 7, 9) \cap \{1, 10\}$

$= \{1)$ ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(iii) $A \cup (B \cup C) = (A \cup B) \cap (A \cup C)$

L.H.S. = $A \cup (B \cap C)$

$= \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$

$= \{1, 3, 5, 7, 9\} \cup \{1, 10)$

$= \{1, 3, 5, 7, 9, 10)$ ......(i)

R.H.S. = $(A \cup B \cap (A \cup C)$

$=(\{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\}) \cap (\{1, 3, 5, 7\} \cup \{1, 5, 8, 10\})$

$= \{1, 3, 4, 5, 7, 9, 10\} \cap \{1, 3, 5, 7, 8, 9, 10\}$

$= \{1, 3, 5, 7, 9, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. = $A \cap (B \cup C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cup \{1, 5, 8, 10\})$

$=\{1, 3, 5, 7, 9\} \cap \{1, 4, 5, 7, 8, 10)$

$= \{1, 5, 7)$ ....(i)

R.H.S. = $(A \cap B \cup (A \cap C)$

$=(\{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\}) \cup (\{1, 3, 5, 7, 9\} \cap \{1, 5, 8, 10\})$

$= \{1, 7) \cup \{1, 5)$

$= \{1, 5, 7)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

3. If $U = N$; then verify De-Morgan's laws by using $A = \emptyset$ and $B = P$. Solution:

Solution:

$U = N$, $A = \emptyset$, $B = P$

(i) $(A \cap B)' = A' \cup B'$

L.H.S. = $(A \cap B)'$

$=U - (A \cap B)$

$=N - (\emptyset \cap P)$

$=N - \emptyset$

$= N$ ....(i)

R.H.S. = $A' = B'$

$= (U - A) \cup (U - B)$

$=(N - \emptyset) \cup (N - P)$

$= N \cup (N - P)$

$= N$ ....(ii)

From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. = $(A \cup B)'$

$= U - (A \cup B)$

$= N - (\emptyset \cup P)$

$=N - \emptyset$

$= N$ .....(i)

R.H.S. = $A' \cap B'$

$= (U - A) \cap (U - B)$

$=(N - \emptyset) \cap (N - P)$

$=N \cap (N - P)$

$= N - P$ ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.

4. If U = {1, 2, 3, 4, ... 10), A - {1, 3, 5, 7, 9) and B = {2, 3, 4, 5, 8) then prove the following questions by Venn diagram:

(i) $A - B = A \cap B'$

Solution:

U = {1, 2, 3, 4,.....,10} A = {1, 3, 5, 7, 9}

B = {2, 3, 4, 5, 8}

(i) $A - B = A \cap B'$

L.H.S. = A - B

= {1, 3, 5, 7, 9} - {2, 3, 4, 5, 8}

= {1,7, 9} ......(i)

R.H.S. = $A \cap B'$

$= A \cap (U - B)$

= {1,3, 5, 7, 9} ∩ ({l, 2, 3,4,....., 10} - {2, 3, 4, 5, 8})

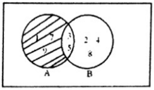
= {1, 3, 5, 7, 9} ∩ {1, 6, 7, 9, 10}

= {1, 7, 9} ......(ii)

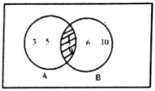
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



A B



$A \cap B'$

(ii) $B - A = B \cap A'$

Solution:

L.H.S $= B - A$

$= \{2, 3, 4, 5, 8\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 8\}$ ....(i)

R.H.S. $= B \cap A'$

$= B \cap (U - A)$

$= \{2, 3, 4, 5, 8\} \cap (\{1, 2, 3, 4, \ldots, 10\} - \{1, 3, 5, 7, 9\})$

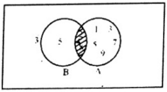
$= \{2, 3, 4, 5, 8\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 8\}$ ....(ii)

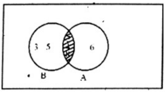
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



$B - A$



$B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

Solution:

L.H.S. $= (A \cup B)'$

$= \{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\} \cup \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots, 10\} - \{1, 2, 3, 4, 5, 7, 8, 9\}$

$= \{6, 10\}$ .....(i)

R.H.S. $= A' \cap B'$

$= (U - A) \cap (U - B)$

$= (\{1, 2, 3, 4, \ldots, 10\} - (\{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, 4, \ldots, 10\} - \{2, 3, 4, 5, 8\})$

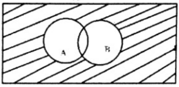
$= \{2, 4, 6, 8, 10\} \cap \{1, 6, 10\}$

$= \{6, 10\}$ .....(ii)

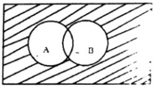
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



(A ∪ B)'



A' ∩ B'

(iv) (A ∩ B) ' = A ∪ B'

Solution:

L.H.S = $(A \cap B)'$

$= U - (A \cap B)$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - \{3, 5\}$

$= \{1, 2, 4, 6, 7, 8, 9, 10\}$ .....(i)

R.H.S. = $A' \cup B'$

$= (U - A) \cup (U - B)$

$= (\{1, 2, 3, 4, \ , 10\} - (\{1, 3, 5, 7, 9\} \cup (\{1, 2, 3, 4, \ldots\ldots, 10\} - \{2, 3, 4, 5, 8\})$

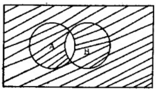
$= \{2, 4, 6, 8, 10\} - \{1, 6, 7, 9, 10\}$

$= \{1, 2, 4, 6, 7, 8, 9, 10\}$ ....(i)

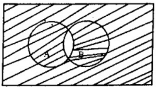
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.



(A' ∪ B)'



A' ∪ B'

(v) (A - B)' = A' ∪ B

Solution:

L.H.S. = $(A - B)'$

$= U - (A - B)$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} - \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - \{1, 7, 9\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ ......(i)

R.H.S. = $A' \cup B$

$= (U - A) \cup B$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\}) \cup \{2, 3, 4, 5, 8\}$

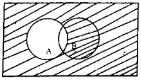
$= \{2, 4, 6, 8, 10\} \cup \{2, 3, 4, 5, 8\}$

$= \{2, 3, 4, 5, 6, 8, 10\}$ ....(ii)

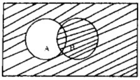
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.



(A U B)'



A' ∪ B

(vi) (B - A)' = B' ∪ A

Solution:

L.H.S. = (B - A)'

= U - (B - A)

= {1, 2, 3, 4, ...,10} - ({2, 3, 4, 5, 8} - {1, 3, 5, 7, 9})

= {1,2, 3, 4, ..., 10} - {2, 4, 8}

= {1, 3, 4, 5, 6, 7, 9, 10} .....(i)

R.H.S. = B' ∩ A

= (U - B) ∪ A

= ({1, 2, 3, 4,...., 10} - {2, 3, 4, 5, 8}) ∪ {1, 3, 5, 7, 9}

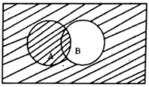
= {1, 3, 5, 7, 9, 10} ∪ {1, 3, 5, 7, 9}

= {1, 3, 5, 7, 9, 10} ....(ii)

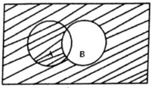
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



(A ∪ B)'



B' ∪ A

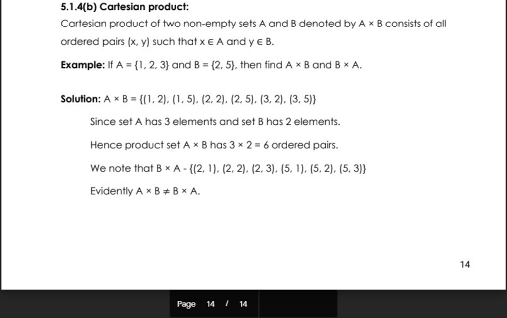
5.1.4 (viii) Ordered pairs and Cartesian product:

5.1.4 (a) Ordered pairs:

Any two numbers x and y, written in the form (x, y] is called an ordered pair. In an ordered pair (x, y), the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that (x, y) ≠ (y, x) unless x = y.

Note that (x, y) = (s, t), if x = s and y = t.



5.1.4(b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by A × B consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

Example: If A = {1, 2, 3} and B = {2, 5}, then find A × B and B × A.

Solution: A × B = {(1, 2), (1, 5), (2, 2), (2, 5), (3, 2), (3, 5)}

Since set A has 3 elements and set B has 2 elements.

Hence product set A × B has 3 × 2 = 6 ordered pairs.

We note that B × A - {(2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3)}

Evidently $A \times B \neq B \times A$.

Exercise 5.4

1. If A = {a, b} and 5 = {c, d}, then find A × B and B × A.

Solution:

$A = \{a, b\}$ and $B = \{c, d\}$

$A \times B = (a, b\} \times \{c, d\}$

$= \{(a,c), (a,d), (b,c), (b,d)\}$

$B \times A = \{c, d\} \times (a, b\}$

$= \{(c, a), (c, b), (d, a), (d, b)\}$

2. If A = {0, 2, 4), B = {-1, 3}, then find A × B, B × A, A × A, B × B.

Solution:

$A = \{0, 2, 4\}$ and $B = \{-1, 3\}$

$A \times B = \{0, 2, 4\} \times \{-1, 3\}$

$= \{(0, -1), (0, 3), (2, -1), (4, -1), (4, 3)\}$

$B \times A = \{-1, 3\} \times \{0, 2, 4\}$

$= \{(-1, 0), (-1, 2), (-1, 4), (3, 0), (3, 2), (3, 4)\}$

$A \times A = \{0, 2, 4\} \times (0, 2, 4\}$

$= \{(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)\}$

$B \times B = \{-1, 3\} \times \{-1, 3\}$

$= \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$

---

3. Find a and b, if

(i) (a - 4, 6 -2) = (2, 1)

Solution:

⇒ $a - 4 = 2$ and $b - 2 = 1$

$a = 2 + 4$ $b = 1 + 2$

$a = 6$ $b = 3$

(ii) (2a + 5, 3) = (7, b - 4) Solution:

⇒ $2a + 5 = 7$ and $3 = b - 4$

$2a = 7 - 5$ $b = 4 + 3$

$2a = 12$ $b = 7$

$a = \frac{12}{2}$

$a = 6$

(iii) (3 - 2a, b - 1) - (a - 7, 2b + 5)

Solution:

⇒ $3 - 2a = a - 7$ and $b - 1 = 2b + 5$

$- 2a - a - 7 - 3$ $b - 2b = 1 + 5$

$- 3a = - 10$ $- b = 6$

⇒ $3a = 10$ $b = -6$

$a = \frac{10}{3}$

4. Find the sets - X and Y, if X × Y = {(a, a), (b, a), (c, a), (d, a)}

Solution:

$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

⇒ $X = \{a, b, c, d\}$ and $Y = \{a\}$

5. If X = {a, b, c} and 7= {d, e}, then find the number of elements in

(i) X × Y

Solution:

Since set X has 3 elements and set y has 2 elements.

Hence, product X × Y has $3 \times 2 = 6$ elements.

(ii) Y × X

Solution

Since set y has 2 elements and set X has 3 elements.

(iii) X ×X

Solution:

Since set X has 3 elements. Hence, product X × X has 9 elements.

Binary relation:

If A and B are any two non-empty sets, then a subset $R \subseteq A \times B$ is called binary relation from set A into set B, because there exists some relationship between first and second element of each ordered pair in R.

Domain of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

Range of relation denoted by Rang R is the set consisting of all the second elements of each ordered pair in the relation.

Function or Mapping:

Suppose A and B are two non-empty sets, then relation $f: A \rightarrow B$ is called a function.

If (i) Dom f = A (ii) every $x \in A$ appears in one and only one ordered pair in f.

Alternate Definition:

Suppose A and B are two non-empty sets, then relation $f: A \rightarrow B$ is called a function if (i) Dom f = A (ii) $\forall x \in A$ we can associate some unique image element $y = f(x) \in B$.

Domain, Co-domain and Range of Function:

If $f: A \rightarrow B$ is a function, then A is called the domain of f and B is called the co-domain of f.

Domain f is the set consisting of all first elements of each ordered pair in f and range f is the set consisting of all second elements of each ordered pair in f

Example:

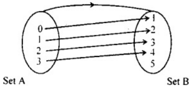
Suppose $A = \{0, 1, 2, 3\}$ and $B = \{-1, 2, 3, 4, 5\}$

Define a function $f: A \to B$

$f = \{(x, y) \mid y = x + 1 \ \forall x \in A,\ y \in B\}$ $f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$

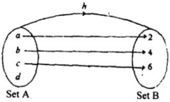
Dom $f = \{0, 1, 2, 3\} = A$

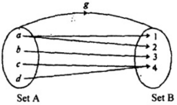
Rang $f = \{1, 2, 3, 4\} \subseteq = B$



The following are the examples of relations but not functions.

g is not a function, because an element a e A has two images in set B and A is not a function because an element d G A has no image in set B.



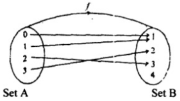


Demonstrate the following:

(a) Into function:

A function $f: A \to B$ is called an into function, if at least one element in B is not an image of some element of set A i.e.,

Range of $f \subset$ set R.



For example, we define a function $f : A \to S$ such that

$f = \{(0, 1), (1, 1), (2, 3), (3, 2)\}$

where $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$ f is an into function.

(b) One-one function:

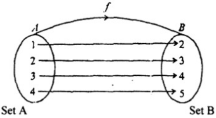
A function f: A → B is called one-one function, if all distinct elements of A. have distinct images in B, i. e., $f\ (x_1) = f\ (x_2) = \Rightarrow x_1 = x_2 \cong A$ or $\forall\ x_1 \neq x_2 \in A \Rightarrow f\ (x_1) \neq f\ \{x_2)$

For example,, if A = {0, 1, 2, 3}

and B = {1,2, 3, 4, 5}, then we define a function f: A → B such that

$f = \{(x, y)\ |\ y = x + 1, \forall\ x \in A, y \in B\}.$

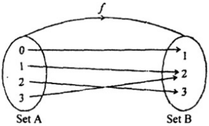
= {(0,1,), (1,2), (2, 3), (3,4)} f is one-one function.



(c) Into and one-one function: (injective function)

The function f discussed in (b) is also an into function. Thus f is an into and one-one function.

(d) An onto or surjective function:



A function f: A → B is called an onto function, if every element of set B is an image

of at least one element of set A i.e. Range of f = B.

For example, if A = {0, 1, 2, 3}

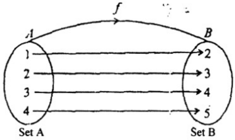
and B = {1, 2, 3}, then f: A → B such

that f = {(0, 1), (1, 2), (2, 3), (3, 2)}.

Here Rang f = {1, 2, 3} = B.

Thus, f so defined is an onto function.

(c) Bijective function or one to one correspondence:



A function f: A → B is called bijective function if f function f is one-one and onto.

For example, if $A = \{1, 2, 3, 4)$ and $B = \{2, 3, 4, 5)$

We define a function f: A → B such that $f = \{(x, y) \mid y = x + 1, \forall x \in A, y \in B)$

Then $f = \{(1, 2), (2, 3), (3, 4), (4, 5))$

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of 0 is the image of at least one element of A.

Note: (1) Even function is a relation but converse may not be true.

(2) Even function may not be one-one

(3) Every function may not be onto.

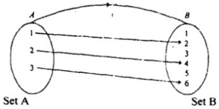
Example: Suppose $A = \{1, 2, 3)$

$B = (1, 2. 3. 4, 5, 6)$

We define a function f: $A \to B = \{(x, y) \mid y = 2x, \forall x \in A, y \in B]$

Then $f = \{1, 2), (2, 4), (3, 6))$

Evidently this function is one-one but riot an onto.



Examine whether a given relation is a function:

A relation in which each x e its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one-one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. The domain of/is A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n (A) = n (B).

Exercise 5.5

1. If L = {a, b, c}, M = {3, 4), then Find two binary relations of L × M and M × L.

Solution:

L = {a, b, c}, (3, 4)

L × M = {a, b, c} × {3,4}

= {(a, 3), (a, 4),(b, 3), (b, 4), (c, 3), (c, 4)}

Then R_1 = {(a, 3), (b, 4),(c, 3))

R_2 = ((a, 4), (b, 3),(c, 4))

= {(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)}

Here R_1 = {(3, a), (4, a),(4, c)}

R_2 = {(3, b), (4, c))

2. If Y = {-2, 1, 2), then make two binary relations for Y × Y. Also find their domain and range.

Solution:

Y = {-2, 1, 2}

Y×Y = {-2, 1, 2} × {-2, 1, 2}

= {(-2, -2)}, (-2,1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2),

(2, 1), (2, 2)}

R_1 = {(-2, -2)}, (-2, 1), (1, 2), (2, 2)

Dom R_1 = {-2,1, 2}

Dom R_1 = {-2,1,2}

Range R_1 = {-2, 1, 2}

and R_2 = {(-2, 1), (1, I),(-2, 2)}

Dom R_2 = {-2, 1}

Range R_2 = {1, 2}

3. If L = (a, b, c} and M = {d, e, f, g}, then find two binary relations in each:

(i) L × L (ii) L × M (iii) M × M

Solution

(i) L × L

L = {a, b, c}

Now L×L = {a, b, c} × {a, b, c}

= {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

R_1 = {(a, a)}

R_2 = {(a, b), (b, b), (c, b)}

(ii) L × M

L = {a, b, c}

M = {d, e, f, g} × {a, b, c}

Now L × M = {a, b, c}, {d, e, f, g} „

= {(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)}

Here R_1 = {(a, d), (b, f)}

R_2 = {(b, e), (b, g), (c, d), (c, e)}

(iii) M × M

M = {d, e, f, g}

Now M × M = {d, e, f, g} × {d, e, f, g}

= {(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)}

Here R_1 = {(d, e), (d, f), (f, f)}

R_2 = {(d, f), (e, d), (e, e), (g, g)}

4. If set M has 5 elements, then find the number of binary relations in M.

Solution:

Number of elements in M = 5

Number of elements in M = 5

Number of elements in M × M = $2^{5\times5} = 2^{25}$

5. If L = {x | x ∈ N ∧ x ≤ 5}, M = {y | y ∈ P ∧ y < 10}, then make the following relations from L to M.

(i) R_1 = {(x, y) | y < x} **(ii) R_2 = {(x, y) | y = x}**

(iii) R_3 = {(x, y}| x + y = 6} **(iv) K_4 = {(x, y) | y - x = 2}**

Also write the domain and range of each relation.

(i) R_1 = {(x· y) | Y < x} •

Solution:

R_1 = {x | x ∈ N ∧ x ≤ 5},

Thus, L = {1, 2, 3, 4, 5}

and M = {y | y ∈ P ∧ y < 10}

Thus, M = {2, 3, 5, 7}

Now L × M = {1, 2, 3, 4, 5}, {2, 3, 5, 7}

= {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7),(5, 2), (5, 3), (5, 5), (5, 7)

R_1 = {(x, y) | y < x}

= {(3, 2) (4, 2), (4, 3),),(5, 2), (5, 3)}

DomR_1 = {3, 4, 5}

Range R_1 = {2, 3}

R_2 = {(x, y) | y = x}

(ii) R_2 = {(x, y) | y = x}

R_2 = {(2, 2), (3, 3), (5, 5)}

Dom R_2 = {2, 3, 5}

Range R_2 = {2, 3, 5}

(iii) R_3 = {(x, y}| x + y = 6}

R_3 = {(1, 5), (3, 3), (4, 2)}

Dom R_3 = {1, 3, 4}

Range R_3 = {5, 3, 2}

(iv) R_4 = {(x, y) | y - x = 2}

R_4 = {(1, 3), (3, 5), (5, 7)}

Dom R_4 = {1, 3, 5}

Range R_4 = {3, 5, 7}

Also write the domain and range of each relation.

6. Indicate relations, into function, one-one function, onto function and bijective function from the following. Also find their domain and the range.

Solution:

(i) R_1 = {(1,1), (2, 2), (3, 3), (4, 4)}

Bijective

Dom R_1 = {1, 2, 3, 4}

Range R_1 = {1, 2, 3, 4}

(ii) R_2 = {(1, 2), (2, 1), (3, 4), (3, 5)}

Relation

Dom R_2 = {1, 2, 3}

Range R_2 = {1, 2, 4, 5}

(iii) R_3 = {(b, a), (c, a), (d, a)}

Function

Dom R_3 = {b, c, d}

Range R_3 = {a}

(iv) R_4 = {(1, 1), (2, 3), (3, 4), (5, 4)}

On to function

Dom R_4 = {1, 2, 3, 4, 5}

Range R_4 = {1, 3, 4}

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

One-one function

Dom $R_5 = \{a, b, c, d\}$

Dom $R_5 = \{a, b, d, e\}$

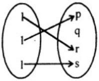
(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

Relation

Dom $R_6 = \{1, 2, 3\}$

Dom $R_6 = \{2, 3, 4\}$

(vii) $R_7 =$

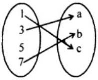


one-one function

Dom $R_7 = \{1, 2, 3\}$

Dom $R_7 = \{r, p, s\}$

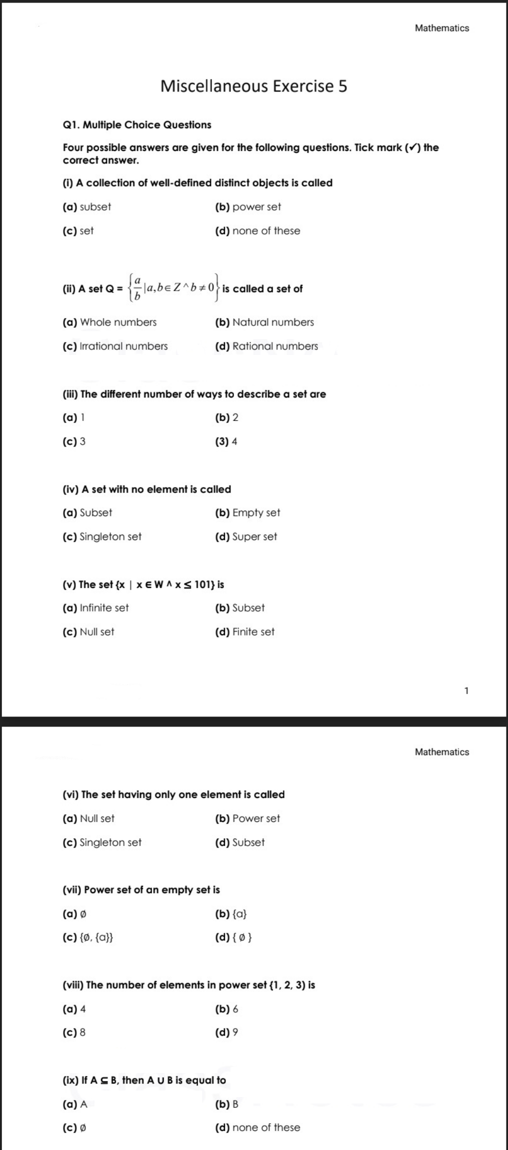
(viii) $R_8 =$



Relation

Dom $R_8 = \{1, 3, 7\}$

Dom $R_8 = \{c, a, b\}$



Miscellaneous Exercise 5

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick mark (✓) the correct answer.

(i) A collection of well-defined distinct objects is called

(a) subset **(b)** power set

(c) set **(d)** none of these

(ii) A set Q = $\left\{\frac{a}{b} | a, b \in Z \wedge b \neq 0\right\}$ **is called a set of**

(a) Whole numbers **(b)** Natural numbers

(c) Irrational numbers **(d)** Rational numbers

(iii) The different number of ways to describe a set are

(a) 1 **(b)** 2

(c) 3 **(3)** 4

(iv) A set with no element is called

(a) Subset **(b)** Empty set

(c) Singleton set **(d)** Super set

(v) The set {x | x ∈ W ∧ x ≤ 101} is

(a) Infinite set **(b)** Subset

(c) Null set **(d)** Finite set

(vi) The set having only one element is called

(a) Null set **(b)** Power set

(c) Singleton set **(d)** Subset

(vii) Power set of an empty set is

(a) ∅ **(b)** {a}

(c) {∅, {a}} **(d)** { ∅ }

(viii) The number of elements in power set {1, 2, 3) is

(a) 4 **(b)** 6

(c) 8 **(d)** 9

(ix) If A ⊆ B, then A ∪ B is equal to

(a) A **(b)** B

(c) ∅ **(d)** none of these

(x) If $A \subseteq B$, then $A \cap B$ is equal to

(a) A
(b) B
(c) $\emptyset$
(d) none of these

(xi) If $A \subseteq B$, then $A - B$ is equal to

(a) A
(b) B
(c) $\emptyset$
(d) $B - A$

(xii) $(A \cup B) \cup C$ is equal to

(a) $A \cap (B \cup C)$
(b) $(A \cup B) \cap C$
(c) $A \cup (B \cup C)$
(d) $A \cap (B \cap C)$

(xiii) $A \cup (B \cap C)$ is equal to

(a) $(A \cap B) \cap (A \cup C)$
(b) $A \cap (B \cap C)$
(c) $(A \cap B) \cup (A \cap C)$
(d) $A \cup (B \cup C)$

(xiv) If A and B are disjoint sets, then $A \cup B$ is equal to

(a) A
(b) B
(c) $\emptyset$
(d) $B \cup A$

(xv) If number of elements in set A is 3 and in set B is 4, then number of elements in $A \times B$ is

(a) 3
(b) 4
(c) 12
(d) 7

(xvi) If number of elements in set A is 3 and in set B is 2, then number of binary relations in $A \times B$ is

(a) 2^3
(b) 2^6
(c) 2^8
(d) 2^2

(xvii) The domain of $R = \{(0, 2), (2, 3), (3, 3), (3, 4))$ is

(a) $\{0, 3, 4\}$
(b) $\{0, 2, 3\}$
(c) $\{0, 2, 4\}$
(d) $\{2, 3, 4\}$

(xvii) The range of R = {(1, 3), (2, 2), (3, 1), (4, 4)) is

(a) (1, 2, 4) **(b)** (3, 2, 4)

(c) (1, 2, 3, 4) **(d)** (1, 3, 4)

(xix) Point (-1, 4) lies in the quadrant

(a) I **(b)** II

(c) III **(d)** IV

(xx) The relation {(1, 2), (2, 3), (3, 3), (3, 4)) is

(a) onto function **(b)** into function

(c) not a function **(d)** one-one function

Answers.

(i)	c	**(ii)**	d	**(iii)**	c	**(iv)**	b	**(v)**	d
(vi)	c	**(vii)**	d	**(viii)**	c	**(ix)**	b	**(x)**	a
(xi)	c	**(xii)**	c	**(xiii)**	a	**(xiv)**	d	**(xv)**	c
(xvi)	b	**(xvii)**	b	**(xviii)**	c	**(xix)**	b	**(xx)**	c

Q2. Write short answers of the following questions.

(i) Define a subset and give one example.

Ans: Set A is said to be subset of a set B, denoted by A ⊆ B, if and only if each element of A is an element of B.

For example, if A = {2, 3, 4} and B = {1, 2, 3, 4, 5} then A is subset of B.

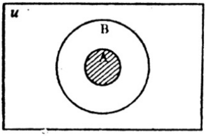
(ii) Write all the subsets of the set {a, b}.

Ans: Let A = {a, b}

Subsets of A are: ∅, {a}, {b}, {a, b}

(iii) Show A ∩ B by Venn diagram. When A ⊆ B.

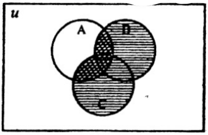
Ans:



A ∩ B

(iv) Show by Venn diagram A ∩ (B ∪ C).

Ans:



A ∩ (B ∪ C)

Double shaded region shows A ∩ (B ∪ C).

(v) Define intersection of two sets.

Ans: The intersection of two sets A and B written as A ∩ B is the set consisting of all the common elements of a and B.

(vi) Define a function.

Ans: Suppose A and B are two non-empty sets, then relation of A → B is called one-one function, if all distinct elements of A have distinct images in B i.e.

(vii) Define one-one function.

Ans:

$$f(x_1)=f(x_2)\Rightarrow x_1=x_2\in A$$

$$or$$

$$\forall x_1\neq x_2\in A\Rightarrow f(x_1)f(x_2)$$

(viii) Define an onto function.

Ans: A function *f*: A→B is called an onto function, if every element of set B is an image of at least one element of set A i.e. Range f = B.

(ix)Define a bijective function.

Ans: A function f : A → B is called bijective function if function f is one-one and onto.

f = {(l, 2), (2, 3), (3, 4), (4, 5)}.

This function is one-one because distinct element of A has distinct images in B. This

is an onto function also because every element of B is the image of at least one element of A.

(x) Write De Morgan's laws.

Ans:

(i) $(A \cap B)' = A' \cup B'$

(ii) $(A \cup B)' = A' \cap B'$

Q3. Fill in the blanks

(i) If $A \subseteq B$, then $A \cup B =$ ________.

(ii) If $A \cap B = 0$ then A and B are ________.

(iii) If $A \subseteq B$and $B \subseteq A$ then ________.

(iv) $A \cap (B \cup C) =$ ________.

(v) $A \cup (B \cap C)=$ ________.

(vi) The complement of U is ________.

(vii) The complement of $\emptyset$ is ________.

(viii) $A \cap A^c =$ ________.

(ix) $A \cup A^c =$ ________.

(x) The set $\{x \mid x \in A \text{ and } x \in B) =$ ________.

(xi) The point (-5, -7) lies in ________ quadrant.

(xii) The point (4, - 6) lies in ________ quadrant.

(xiii) The y co-ordinate of every point is ________ on-x-axis.

(xiv) The x co-ordinate of every point is ________ on-y-axis.

(xv) The domain of {(a, b), (b, c), (c, d)} is ________ .

(xvi) The range of {(a, a), (b, b), (c, c)} is ________ .

(xvii) Venn-diagram was first used by ________ .

(xviii) A subset of A × A is called the ________ in A.

(xix) If: $A \rightarrow B$ and range of f = B, then f is an ________ function.

(xx) The relation ((a, b), (6, c), (a, d)} is ________ a function.

Answer:

(i)	B	**(ii)**	Disjoint	**(iii)**	A = B
(iv)	$(A \cap B)$	**(v)**	$\emptyset$	**(vi)**	U
(vii)		**(viii)**	$\emptyset$	**(ix)**	U
(X)	A\B	**(xi)**	III-quad	**(xii)**	IV quad
xiii)	zero	**(xiv)**	0	**(xv)**	{a, b, c}
(xvi)	{a, b, c}	**(xvii)**	John Venn	**(xviii)**	Binary relation
(xix)	Onto	**(xx)**	Not		

Summary

✓ A set is the well-defined collection of distinct objects with some common properties:

✓ Union of two sets A and B denoted by $A \cup b$ is the set containing elements which either belong to A or to B or to both.

✓ Intersection of two sets A and B denoted by $A \cap B$ is the set of common elements of both A and B. In symbols $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

✓ The set difference of B and A denoted by B - A is the Set of all those elements of B which do not belong to A.

✓ Complement of a set A w.r.t. universal set U denoted by $A^c = A' = U - A$ contains all those elements of U which do not belong to A.

✓ British mathematician John Venn (1834 - 1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

✓ An ordered pair of elements is written according to a specific order for which the order of elements is strictly maintained.

✓ Cartesian product of two non-empty sets A and B denoted by A × B consists of all ordered pairs (x, y) such that $x \in A$, $y \in B$.

✓ If A and B are any two non-empty sets: then a non-empty subset $R \subseteq A \times B$ is called binary relation from set A into set B.

✓ If A and B are two non-empty sets, then relation $f: A \Rightarrow B$ is called a function if (i)

Dom f = set A (ii) every $x \in A$ appears in one and only one ordered pair $\in f$.

✓ Dom f is the set consisting of all first elements of each ordered pair $\in f$ and Rang of f

is the set consisting of all second elements of each ordered pair $\in f$.

✓ A function $f: A \rightarrow B$ is called an into function if at least one element in B is not an image of some element of set A i.e., range of $f \subseteq B$.

✓ A function $f: A \rightarrow B$ is called an onto function if every element of set B is an image of at least one element of set A i.e., range of $f = B$.

✓ A function $f: A \rightarrow B$ is called one-one function if all distinct elements of A have distjnct images in B.

✓ A function $f: A \rightarrow B$ is called bijective function if f function f is one-one and onto.