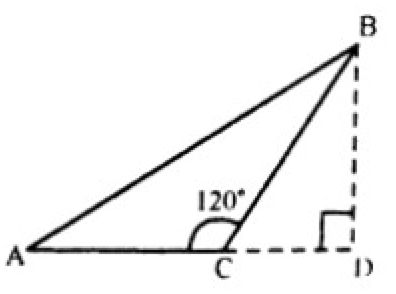
Exercise 8.1

Ql. Given m $\overline{AC}$ = 1cm, m $\overline{BC}$ = 2cm, m∠C = 120°. Compute the length AB and the area of ⊔ ABC.

Hint: $\left(\overline{AB}\right)^2=\left(\overline{AC}\right)^2+\left(\overline{BC}\right)^2+2mAC.m\overline{CD}$

Where $\left(m\overline{CD}\right)=\left(m\overline{BC}\right)\cos\left(180^\circ-m\angle C\right)$ *(Use theorem I)*

Solution:



$m\overline{AC}=1cm;\ m\overline{BC}=2cm;\ m\angle C=120^\circ$

Required: $m\overline{AB}=?$

and Area of ABC = ?

$m\overline{AB}^2=m\overline{AC}^2+m\overline{BC}^2+2m\overline{AC}m\overline{CD}$

$=(1)^2+(2)^2+2(1)\left(\overline{CD}\right)$

$=1+4+2\overline{CD}$

$=1+4+2\overline{CD}$ ____ (i)

In □ BCD,

$m\angle BCD=60^\circ$

and $m\angle CBD=30^\circ$

The side opposite to $\angle 30^\circ$ is $\overline{CD}$ which is $\frac{1}{2}\overline{CB}$, the hypotenuse of right □CDB.

$\overline{CD}=1cm$

By putting the value of $\overline{CD}$ in eq.(1)

$m\overline{AB}^2=5+2(1)(1)=5+2=7$

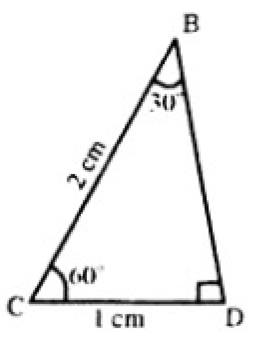
$m\overline{AB}^2=7\Rightarrow m\overline{AB}=\sqrt{7}cm=2.646cm$

$m\overline{CB}^2=m\overline{CD}^2+\overline{BD}^2$

$2^2=1^2+m\overline{BD}^2$

$mBD^2=4-1=3$

$h=m\overline{BD}=\sqrt{3}$



Area of ABC

$=\frac{1}{2}\text{base}\times\text{height}$

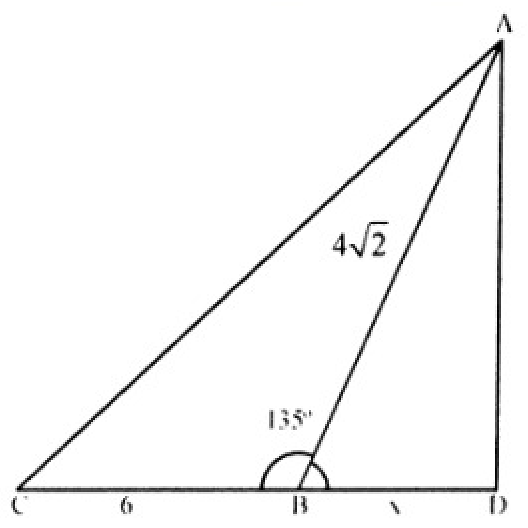
$=\frac{1}{2}m\overline{AC}\times m\overline{BD}$

$=\frac{1}{2}\times1\times\sqrt{3}$

Area of ABC $=\frac{\sqrt{3}}{2}$ sq cm

Q2. Find m $\overline{AC}$ if in □ ABC m $\overline{BC}$ = 6 cm, m $\overline{AB}$ = $4\sqrt{2}$ cm and m∠ABC = 135°.

Solution:



Let $m\overline{BD}=x$

In □ABD, we have

$\cos 45^\circ=\frac{\overline{BD}}{\overline{AB}}$

$\frac{1}{\sqrt{2}}=\frac{x}{4\sqrt{2}}$

$\sqrt{2}\ x=4\sqrt{2}$

$x=4$ cm

We know that

To Prove

$\left(m\overline{AC}\right)^2=\left(m\overline{CB}\right)^2+\left(m\overline{AB}\right)^2+2\times m\overline{CB}\times m\overline{BD}$

$=(6)^2+\left(4\sqrt{2}\right)^2+2\times6\times4$

$=36+32+48$

$=116$

$\Rightarrow m\overline{AC}=\sqrt{116}=\sqrt{4\times29}=2\sqrt{29}$ cm

EXERCISE 8.2

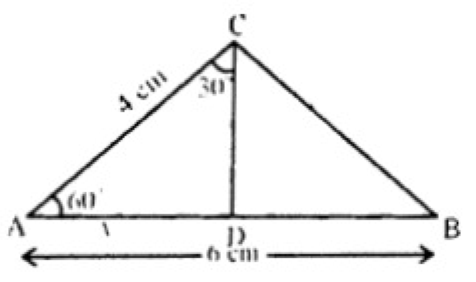
Q1. In a $\triangle ABC$ calculate m $\overline{BC}$ when m $\overline{AB}$ = 6cm, m$\overline{AC}$ = 4cm and m$\angle A$ = 60°.

Solution:

Given: mAB = 6cm; mAC = 4cm;

m$\angle A$ =60°.

Required: m$\overline{CB}$ = ?



In $\triangle ABC$, we have

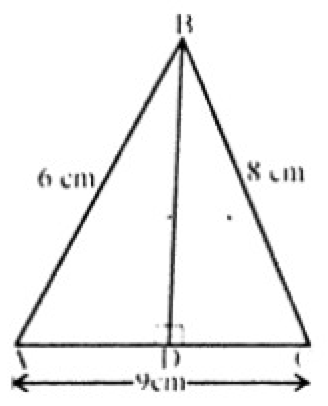
$$(\overline{BC})^2 = (\overline{AB})^2 + (\overline{AC})^2 - 2(\overline{AB}).(\overline{AD})$$
$$= (6)^2 + (4)^2 \times 2(6)(x)$$
$$= 36 + 16 - 2(6)(2)$$
$$= 52 - 24$$
$$= 28$$
$$m\overline{BC} = \sqrt{28}$$
$$= 2\sqrt{7}\text{ cm} \Rightarrow = 5.29\text{cm}$$

$$\because \cos 60^\circ = \frac{x}{4}$$
$$\frac{1}{2} = \frac{x}{4}$$
$$2x = 4$$
$$\Rightarrow x = 2$$

Q2. In a $\triangle ABC$, m$\overline{AB}$ = 6cm, m$\overline{BC}$ = 8cm ,m$\overline{AC}$ = 9cm and D is the midpoint of side $\overline{AC}$. Find length of the median $\overline{BD}$

Solution:

According to the figure, we have



$$m\overline{AD} = \overline{DC}$$
$$and \quad m\overline{AC} = m\overline{AD} + m\overline{DC}$$
$$m\overline{AC} = m\overline{AD} + m\overline{AD}$$
$$9 = 2m\overline{AD}$$
$$Or \quad 2m\overline{AD} = 9$$

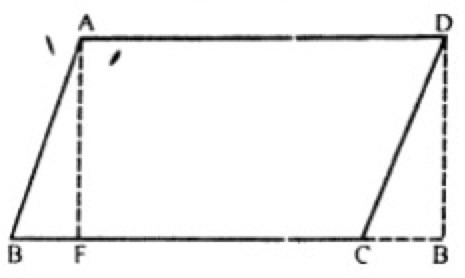
$$m\overline{AD} = \frac{9}{2} = 4.5\text{ cm}$$

We know that

$$(\overline{AC})^2 + (\overline{BC})^2 = 2\left[(\overline{AD})^2 + (\overline{BD})^2\right]$$
$$(6)^2 + (8)^2 = 2\left[(4.5)^2 + (\overline{BD})^2\right]$$
$$36 + 64 = 2(4.5)^2 + 2(\overline{BD})^2$$
$$100 = 40.5 + 2(\overline{BD})^2$$
$$Or \quad 2(\overline{BD})^2 = 100 - 40.5$$
$$2\overline{BD}^2 = 59.5$$
$$\Rightarrow \overline{BD}^2 = 29.75$$
$$\Rightarrow \overline{BD} = \sqrt{29.75} = 5.45\text{ cm}$$

Q3. In a parallelogra ABCD prove that $(\overline{AC})^2 + (\overline{BD})^2 = 2\left[(\overline{AB})^2 + (\overline{BC})^2\right]$

Solution:



$$(\overline{BD})^2 = (\overline{CD})^2 + (\overline{BC})^2 + 2(\overline{BC})(\overline{CE}) \quad (1)$$
$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2(\overline{BC})(\overline{BF}) \quad (2)$$

Adding (1) and (2), we get

$$(\overline{AC})^2 + (\overline{BD})^2 = (\overline{CD})^2 + (\overline{BC})^2 + 2(\overline{BC})\overline{CE} + (\overline{AB})^2 + (\overline{BC})^2 - 2(\overline{BC})(\overline{BF})$$
$$= (\overline{AB})^2 + (\overline{CD})^2 + 2(\overline{BC})^2 + 2(\overline{BC})(\overline{CE}) - 2(\overline{BC})(\overline{BF})$$

In parallelogram opposite sides are congruent, so

$\overline{AB} = \overline{DC}, \overline{AD} = \overline{BC}$, and $\overline{BF} = \overline{CE}$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2(\overline{AB})^2 + (\overline{AB})^2\, 2(\overline{BC})^2 + 2(\overline{CE}) - 2(\overline{BC})\overline{CE}$$
$$(\overline{AC})^2 + (\overline{BD})^2 = 2(\overline{AB})^2 + 2(\overline{BC})^2$$

$$(\overline{AC})^2 + (\overline{BD})^2 = 2\left[(\overline{AB})^2 + (\overline{BC})^2\right]$$

Hence Proved.

THEOREM 1

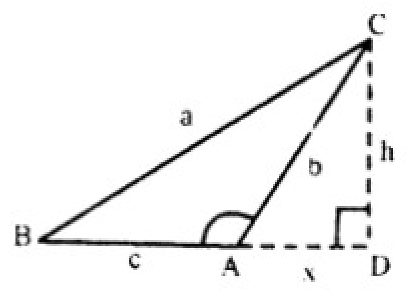
8.1 (i) In an obtuse angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.

Given:

ABC is a triangle having an obtuse angle

BAC at A. Draw $\overline{CD}$ perpendicular on $\overline{BA}$ produced.

So that $\overline{AD}$ is the projection of $\overline{AC}$ on $\overline{BA}$ produced.



Take mBC = a, mCA = b, mAB = c,

mAD = x and mCD = h.

To prove

$(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$

i.e., $a^2 = b^2 + c^2 + 2cx$

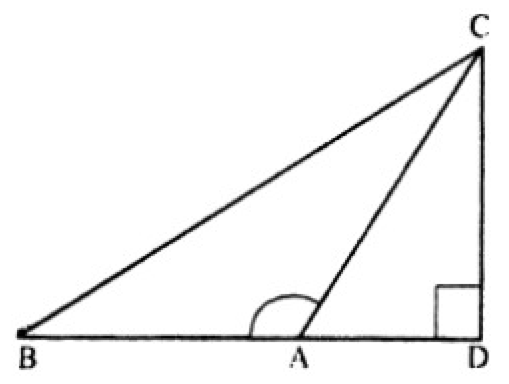
Proof:

Statements	Reasons
In $\angle$rt$\triangle$ CDA,	
$m\angle CDA = 90^\circ$	Given
$\therefore\ (\overline{AC})^2 = (AD)^2 + (CD)^2$	Pythagoras Theorem
or $b^2 = x^2 + h^2$ ___ (i)	
In $\angle$rt$\triangle$ CDB,	
$m\angle CDB = 90^\circ$	Given
$\therefore\ (\overline{BC})^2 = (BD)^2 + (CD)^2$	Pythagoras Theorem
or $a^2 = (c+x)^2 + h^2$	$\overline{BD} = \overline{BA} + \overline{AD}$
$= c^2 + 2cx + x^2 + h^2$ ___ (ii)	
Hence, $a^2 = c^2 + 2cx + b^2$	
i.e., $a^2 = b^2 + c^2 + 2cx$	Using (i) and (ii)
or $(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	

Example

In a $\triangle$ ABC with obtuse angle at A, if $\overline{CD}$ is an altitude on $\overline{BA}$ produced and in m $\overline{AC}$ = m $\overline{AB}$.

Then prove that $(BC)^2 = 2\ (AB)(BD)$



Given:

In a $\triangle$ ABC, m$\angle$A is obtuse $m\overline{AC} = m\overline{AB}$ and $\overline{CD}$ being altitude on $\overline{BA}$ produced.

To prove:

$(\overline{BC})^2 = 2(m\overline{AB})(m\overline{BD})$

Proof:

In a $\triangle$ ABC, having obtuse angle BAC at A.

Statements	Reasons
$(\overline{BC})^2 = (\overline{BA})^2 + (\overline{AC})^2 + 2(m\overline{BA})(m\overline{AD})$	By Theorem I
$= (\overline{AB})^2 + (\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	Given
$= 2(\overline{AB})^2 + 2(m\overline{AB})(m\overline{AD})$	
$(\overline{BC})^2 = 2m\overline{AB}(m\overline{AB} + m\overline{AD})$	On the line segment $\overline{BD}$
$= 2m\overline{AB}(m\overline{BD})$	Point A is between B and D

THEOREM 2

8.1 (ii) In any triangle, the square on the side opposite to acute angle is equal to sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

Given:

□ABC with an acute angle CAB at A

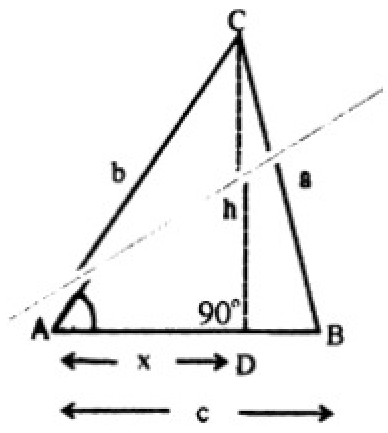
Take $m\overline{BC} = a, m\overline{CA} = b$ and $m\overline{AB} = c$

Draw $\overline{CD} \perp \overline{AB}$ so that $\overline{AD}$ is projection of $\overline{AC}$ on $\overline{AB}$

Also, $m\overline{AD} = x$ and $m\overline{CD} = h$

To Prove:

$\left(\overline{BC}\right)^2 = \left(\overline{AC}\right)^2 + \left(\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$ *i.e.,* $a^2 = b^2 + c^2 - 2cx$



Proof:

Statements	**Reasons**
In ∠rt□ CDA,	
$m\angle CDA = 90^\circ$	Given
$\left(\overline{AC}\right)^2 = (AD)^2 + (CD)^2$	Pythagoras Theorem
or $b^2 = x^2 + h^2$ ____ (i)	
In ∠rt□ CDB,	
$m\angle CDB = 90^\circ$	Given
$\left(\overline{BC}\right)^2 = \left(\overline{BD}\right)^2 + \left(\overline{CD}\right)^2$	Pythagoras Theorem
$a^2 = (c-x)^2 + h^2$	From the figure
or $a^2 = c^2 - 2cx + x^2 + h^2$ ___ (ii)	
$a^2 = c^2 - 2cx + b^2$	
Hence $a^2 = b^2 + c^2 - 2cx$	Using (i) and (ii)
i.e., $\left(\overline{BC}\right)^2 = \left(\overline{AC}\right)^2 + \left(\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$	

MISCELLANEOUS EXERCISE 8

Q1. In a △ABC, m∠A = 60°, prove that $\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - m\overline{AB}.m\overline{AC}$.

Solution:

Given:

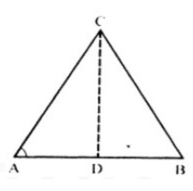
In a △ABC, m∠A = 60°

Required:

$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \overline{AB}.\overline{AC}$

Construction:

Draw $\overline{CD} \perp \overline{AB}$, so that $\overline{AD}$ the Projection of $\overline{AC}$ $\overline{AB}$.



Proof:

In right angle □ACD

$$\text{Cos}\,60° = \frac{m\overline{AD}}{m\overline{AC}}$$
$$\frac{1}{2} = \frac{m\overline{AD}}{m\overline{AC}} \qquad \left(\text{Cos}\,60° = \frac{1}{2}\right)$$
$$m\overline{AD} = \frac{1}{2} m\overline{AC}$$

Now, according to the theorem, we have

$$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB}.\overline{AD}$$
$$\Rightarrow \overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - \overline{AB}.\overline{AC} \qquad \because [2AD = AC]$$

Q2. In a △ABC, m∠A = 45°, prove that $\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \sqrt{2}\,m\overline{AB}.m\overline{AC}$.

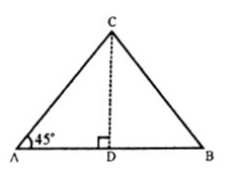
Solution:

Given:

In a △ABC, m∠A = 45°.

Required:

$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \sqrt{2}\,\overline{AB}.\overline{AC}$



Construction:

Draw CD ⊥ AB, so that AD is the projection of AC on AB.

Proof:

In right angle □ACD

$$\text{Cos}\,45° = \frac{m\overline{AD}}{m\overline{AC}}$$
$$\frac{\sqrt{2}}{2} = \frac{m\overline{AD}}{m\overline{AC}}$$
$$2m\overline{AD} = \sqrt{2}\,m\overline{AC}$$

□ABC is acuted angled at A,so according to the theorem, we have

$$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB}.\overline{AD}$$

$$\Rightarrow \overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - \sqrt{2}\,\overline{AB}.\overline{AC} \qquad \because \left[2\overline{AD} = \sqrt{2}AC\right]$$

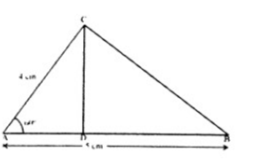
Hence proved.

Q3. In a △ABC, calculate mBC when m $\overline{AB}$ = 5 cm, m $\overline{AC}$ = 4 cm, m∠A = 60°.

Solution:

We know that when m∠A = 60° then,

$$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - \overline{AB}.\overline{AC}$$
$$= 5^2 + 4^2 - 5.4$$
$$= 25 + 16 - 20$$
$$= 21$$
$$m\overline{BC} = \sqrt{21} = 4.58\text{cm}$$

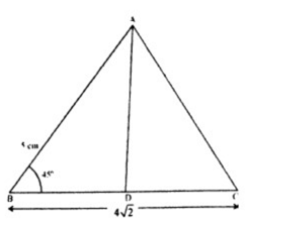


Q4. In a △ABC, calculate m $\overline{AC}$ when m $\overline{AB}$ = 5 cm, mBC = $4\sqrt{2}$ cm, m∠B = 45°.

Solution:

We know that when m∠B=45° then,

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - \sqrt{2}\overline{AB}.\overline{BC}$$
$$= (5)^2 + \left(4\sqrt{2}\right)^2 - \sqrt{2}(5)\left(4\sqrt{2}\right)$$
$$= 25 + 32 - 40$$
$$= 57 - 40 = 17$$
$$m\overline{AC} = \sqrt{17}\text{cm} = 4.123\text{cm}$$

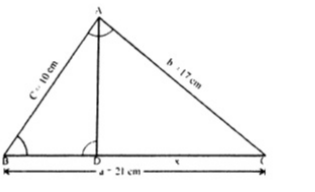


Q5. In a triangle ABC, m $\overline{BC}$ = 21 cm, m $\overline{AC}$ = 17 cm, m $\overline{AB}$ = 10 cm.

Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$.

Solution:

c = 10 cm, a = 21 cm, b = 17 m, x =?



We know that

$$c^2 = a^2 + b^2 - 2(a)(x)$$
$$(10)^2 = (21)^2 + (17)^2 - 2(21)(x)$$
$$100 = 441 + 189 - 42x$$
$$42x = 441 + 189 - 42x$$
$$42x = 730 - 100$$
$$42x = 630$$
$$x = \frac{630}{40} = 15\text{cm}$$

Q6. In a triangle ABC, mBC = 21 cm. mAC = 17 cm, mAB = 10 cm.

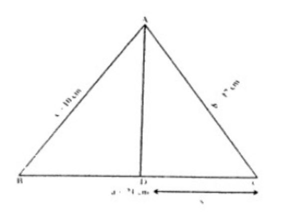
Calculate the projection of AB upon BC.

Solution:

c = 10 cm, a = 21 cm, b = 17m, x =?

We know that

$$b^2 = a^2 + c^2 - 2ax$$
$$(17)^2 = (10)^2 + (21)^2 - 2(21)(x)$$
$$289 = 100 + 441 - 42x$$
$$289 = 541 - 42x$$
$$42x = 541 - 289$$
$$42x = 252$$
$$x = \frac{252}{42} = 6\text{cm}$$

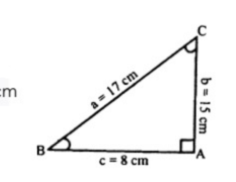


Q7. In a △ABC, a = 17 cm, b =15 cm and c = 8 cm find m∠A.

Solution:

Given:

In a △ABC, a = 17 cm, b = 15 cm and, c = 8 cm



Required:

m∠A = ?

By Pythagoras theorem.

$$a^2 = b^2 + c^2$$
$$(17)^2 = (15)^2 + (8)^2$$
$$289 = 225 + 64$$
$$289 = 289$$

So, it satisfied, that given values are the sides of a right angled triangle.

$\therefore m\angle A = 90°$

Q8. In a △ABC, a = 17 cm, b = 15 cm and c = 8 cm find m∠B.

Solution:

Given:

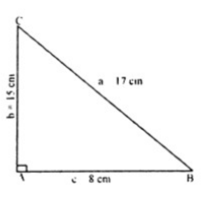
In a △ABC; a = 17 cm, b = 15 cm and c = 8 cm

Required:

m∠B =?

We know that it is right angled triangle.

$$\sin(m\angle B) = \frac{b}{a} = \frac{15}{17} = 0.882$$
$$m\angle B = \sin^{-1}(0.882) = 61.90°$$



Q9. Whether the triangle with sides 5 cm, 7 cm, 8 cm is acute, obtuse or right angled.

Solution:

Given:

a = 5cm; b = 7 cm; c = 8 cm

Case I:

$$c^2 = a^2 + b^2$$
$$(8)^2 = (5)^2 + (7)^2$$
$$64 = 25 + 49$$
$$64 \neq 74$$

It is not right angled triangle.

Also

$74 > 64$ i.e.

$a^2 + b^2 > c^2$

The result shows that the given triangle is an acute angled triangle.

Q10. Whether the triangle with sides 8 cm, 15 cm, 17 cm is acute, obtuse or right angled.

Solution:

a = 8; b = 15; c = 17

Case I:

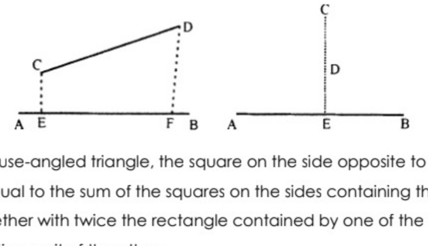
$$c^2 = a^2 + b^2$$
$$(17)^2 = (8)^2 + (15)^2$$
$$289 = 64 + 225$$
$$289 = 289$$

Hence,It is right angled triangle.

Summary

✓ The projection of a given point on a line segment is the foot ⊥ of drawn from the point on that line segment. If $\overline{CD} \perp \overline{AB}$, then evidently D is the foot of perpendicular $\overline{CD}$ from tire point C on the line segment AB.

✓ The projection of a line segment $\overline{CD}$ on a line segment AB is the portion $\overline{EF}$ of the latter intercepted between foots of the perpendiculars drawn from C and D. However, projection of a vertical line segment $\overline{CD}$ on a line segment AB is a point on $\overline{AB}$ which is of zero dimension.



✓In an obtuse-angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.

✓In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing that acute angle

diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

✓ In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side (Apollonius' Theorem).