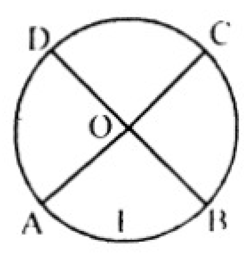
Exercise 9.1

1. Prove that, only the diameters of a circle are the intersecting chords which bisect each other.

Given: A circle having diameters $\overline{AC}$ and $\overline{BD}$ which passes through centre O.



To Prove: Diameters $\overline{AC}$ and $\overline{BD}$ bisect each other.

Proof:

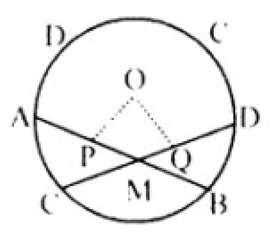
Statements	Reasons
$\overline{OA} \cong \overline{OC}$ (i)	
Similarly, $m\overline{OC} \cong \overline{OD}$ (ii)	Common
$m\overline{OA} = m\overline{OD}$ (iii)	
From (i), (ii) and (iii), we have	Radii of the same circle
$m\overline{OA} = m\overline{OB} = m\overline{OC} = \overline{OD}$	

Hence AC and BD are intersecting chords which bisect each other.

2. Two chords of a circle do not pass through the center. Prove that they cannot bisect each other.

Given:

A circle with center O having two chords $\overline{AB}$ and $\overline{CD}$.



To Prove:

M is not the mid-point of chords $\overline{AB}$ and $\overline{CD}$.

Construction:

Join O to P and Q such that $\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$.

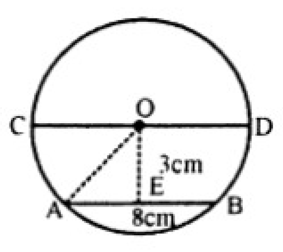
Proof:

Statements	Reasons
O is the center of the circle with $\overline{OP} \perp \overline{AB}$	Construction
Thus $\overline{OP} \perp \overline{AB}$	
Now point M lies between P and B.	
Therefore, M is not the midpoint of AB.	
Hence $\overline{AB}$ and $\overline{CD}$ cannot bisect each other.	

3. If the length of the chord AB = 8 cm. Its distance from the center is 3 cm, then measure the diameter of such circle.

Given:

mAB = 8cm, mOE = 3cm



Required:

to find the length of diameter

i.e., mCD = ?

Construction:

Join O to A and E.

Proof:

Statements	Reasons
In $\triangle$ AEO $(AO)^2 = \overline{AE}^2 + \overline{EO}^2$ $= \left[\frac{1}{2}(\overline{AB})\right]^2 + (3)^2$ $= \left[\frac{1}{2} \times 8\right]^2 + 9$ $= (4)^2 + 9$ $= 16 + 9 = 25cm$ $\Rightarrow \overline{AO} = \sqrt{25} = 5cm$	

$$m\overline{AO} = m\overline{OC} = m\overline{OD} = 5cm$$

$$\Rightarrow \overline{CD} = \overline{CO} + m\overline{OD} = 5cm + 5cm = 10cm$$

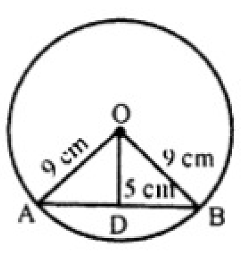
Hence

$\boxed{\text{Diameter} = 10cm}$

4. Calculate the length of a chord which stands at a distance 5cm from the center of a circle whose radius is 9cm.

Given:

$m\overline{OA} = m\overline{OB}$ = 9cm,



m $\overline{OD}$ = 5cm

Required:

m $\overline{AB}$ = ?

Proof:

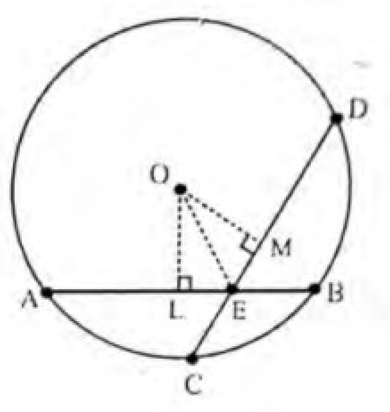
Statements	Reasons
In $\triangle$ OAD	
$m\overline{OA}^2 = m\overline{OD}^2 + m\overline{AD}^2$	
$m\overline{OA}^2 - m\overline{OD}^2 = m\overline{AD}^2$	
$9^2 - 5^2 = \left[\frac{1}{2} m(\overline{AB})\right]^2$	$\left[\because AD = \frac{1}{2}\overline{AB}\right]$
$\left[\frac{1}{2} m(\overline{AB})\right]^2 = 81 - 25$	
$\frac{1}{4} m(\overline{AB})^2 = 56$	
$\Rightarrow m\overline{AB}^2 = 56 \times 4 = 224$	
$AB = 14.97cm$	

Exercise 9.2

1. Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.

Given:

A circle with center "O". Two equal chords $\overline{AB}$ and $\overline{CD}$ (i.e. $m\overline{AB} = m\overline{CD}$) intersect each other at point E.



To Prove:

$m\overline{AE} = m\overline{ED}$ and $m\overline{EB} = m\overline{EC}$

Construction:

Draw perpendiculars $\overline{OL}$ and $\overline{OM}$ from the center "O" to the chords $\overline{AB}$ and $\overline{CD}$ respectively. L and M are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

Proof:

Statements	Reasons
In $\triangle OLE \leftrightarrow \triangle OME$	
$\overline{OL} \cong \overline{OM}$	Two equal chords of a circle are equidistant from the center.
$m\angle OLE = m\angle OME = 90^\circ$	$\overline{OL} \perp \overline{AB}$ and $\overline{OM} \perp \overline{CD}$
$m\overline{OE} \cong m\overline{OE}$	Common side
$\therefore$ $\triangle OLE \cong \triangle OME$	H.S $\cong$ H.S

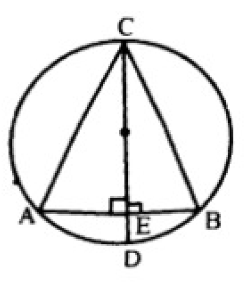
$\overline{LE} \cong \overline{ME}$ ..... (i)	Corresponding sides of congruent triangles
$m\overline{AL} = \frac{1}{2} m\overline{AB}$	
$m\overline{DM} = \frac{1}{2} m\overline{CD}$	
$m\overline{AL} = \overline{DM}$ ..... (ii)	Both are half of equal chords.
$m\overline{AL} + m\overline{LE} = m\overline{DM} + m\overline{ME}$	Adding (i) and (ii).
$m\overline{AE} = m\overline{DE}$ ..... (iii)	
Now, $m\overline{AB} = m\overline{CD}$	Given
$m\overline{AE} + m\overline{EB} = m\overline{DE} + m\overline{EC}$	
$m\overline{AE} + m\overline{EB} = m\overline{AE} + m\overline{EC}$	From (iii)
$m\overline{EB} = m\overline{EC}$	By cancellation property.

2. AB is the chord of a circle and the diameter CD is perpendicular bisector of AB . Prove that $m\overline{AC} = m\overline{BC}$.

Given:

In a circle.

AB ⊥ CD and AE ≅ EB



To Prove:

$m\overline{AC} = m\overline{BC}$

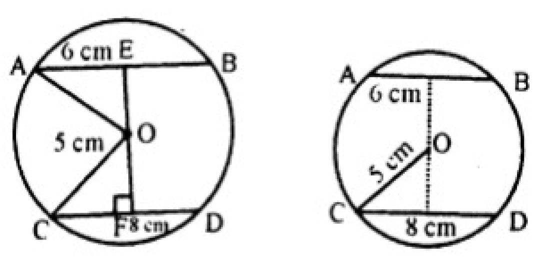
Construction:

Join C to A and B.

Proof:

Statements	Reasons
In $\triangle ACE \leftrightarrow \triangle EBC$	
$AE \cong EB$	A diameter CD ⊥ on chord AB bisect it.
$m\angle AEC = m\angle CEB$	Given
$CE \cong CE$	Common side
$\therefore$ □AEC ≅ □EBC	S.A.S ≅ S.A.S
$\Rightarrow$ $\overline{AC} \cong \overline{BC}$	Corresponding sides of congruent triangles.
$m\overline{AC} = m\overline{BC}$	

3. As shown in the figure, find the distance between two parallel chords $\overline{AB}$ and $\overline{CD}$.



Given:

$m\overline{AB} = 6cm$ and $m\overline{CD} = 8cm$

$m\overline{OC} = 5cm$

Required:

$m\overline{EF} = ?$

In □OCF

$m\overline{OC}^2 = \overline{OF}^2 + \overline{FC}^2$

$5^2 = \overline{OF}^2 + 4^2$

$\Rightarrow \overline{OF}^2 = 25 - 16 = 9$

$\overline{OF} = \sqrt{9} = 3cm$

In □OAE

$\overline{OA}^2 = \overline{OE}^2 + \overline{EA}^2$

$5^2 = \overline{OE}^2 + 3^2$

$\Rightarrow \overline{OE}^2 = 25 - 9 = 16$

$\overline{OE} = \sqrt{16} = 4$

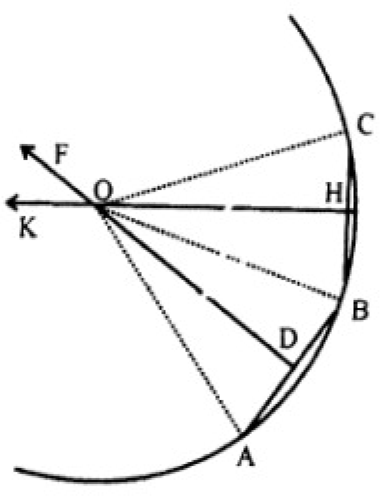
$\therefore \overline{EF} = \overline{OE} + \overline{OF} = 4 + 3 = 7cm$

THEOREM 1

9.1 (i) One and only one circle can pass through three non-collinear points.

Given:

A, B and C are three non collinear points in a plane.



To prove:

One and only one circle can pass through three non-collinear points A, B and C.

Construction:

Join A with B and B with C. AB and BC are distinct and non collinear lines. Draw $\overline{DF}$ ⊥ bisector to $\overline{AB}$ and $\overline{HK}$ ⊥ bisector to BC.

So $\overline{DF}$ and $\overline{HK}$ are not parallel rather they meet each other at some point O. Also join A, B and C with point O.

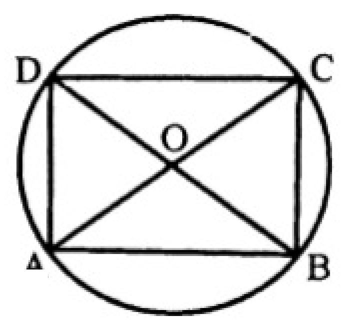
Proof:

Statements	Reasons
Every point on $\overline{DF}$ is equidistant from A and B.	DF ⊥ bisector to AB (construction)
In particular m $\overline{OA}$ = m $\overline{OB}$ (i)	By Locus Theorem
Similarly, every point on $\overline{HK}$ is equidistant from to B and C.	$\overline{HK}$ is ⊥ bisector to BC (construction)
In particular m $\overline{OB}$ = m $\overline{OC}$ (ii)	By Locus theorem 1
Now O is the only point common to $\overline{DF}$ and $\overline{HK}$ which is equidistant from A, B and C.	

i.e., mOA = mOB = mOC However, there is no such other point expect O.	Using (i) and (ii) '

Hence a circle with center O and radius $\overline{OA}$ will pass through A, B and C. Ultimately there is only one circle which passes through three given points A, B and C.

Example:
Show that only one circle can be drawn to pass through the vertices of any rectangle.



Given:
ABCD is an inscribed rectangle w. r. t. a circle with center O.
To Prove:
Only one circle can be drawn through the vertices of the rectangle ABCD.
Construction: Diagonals AC and BD of the rectangle meet each other at point O.

Proof:

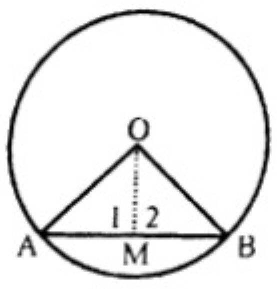
Statements	Reasons
ABCD is a rectangle.	Given
m $\overline{AC}$ = m $\overline{BD}$ (i)	Diagonals of a rectangle are equal.
$\overline{AC}$ and $\overline{BD}$ meet each other at O	Construction
$\overline{OA}$ = $\overline{OC}$ and $\overline{OB}$ = $\overline{OD}$ (ii)	Diagonals of rectangle bisect each either
⇒ $\overline{OA}$ = $\overline{OB}$ = $\overline{OC}$ = $\overline{OD}$ (iii)	Using (i) and (ii)

i.e., point O is equidistant from all vertices of the rectangle ABCD. Hence $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$ are the radii of the same circle with center O.	

THEOREM 2

A straight line, drawn from the center of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

Given:



M is the mid-point of any chord $\overline{AB}$ of a circle with center at O.

Where chord $\overline{AB}$ is not the diameter of the circle.

To prove:

$\overline{OM} \perp$ the chord $\overline{AB}$

Construction:

Join A and B with center O.

Write ∠1 and ∠2 as shown in the Figure.

Proof:

Statements	**Reasons**
In $\triangle OAM \leftrightarrow \triangle OBM$	
$m\,\overline{OA} = m\,\overline{OB}$	Radii of the same circle
$m\,\overline{AM} = m\,\overline{BM}$	Given
$m\,\overline{OM} = m\,\overline{OM}$	Common
$\triangle OAM \cong \triangle OBM$	S.S.S ≅ S.S.S
$\Rightarrow m\angle 1 = m\angle 2$ (i)	Corresponding angles of congruent triangles
i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ$ (ii)	Adjacent supplementary angles
$m\angle 1 = m\angle 2 = 90^\circ$	From (i) and (ii)

i.e., $\overline{OM} \perp \overline{AB}$	

THEOREM 3

9.1 (iii) Perpendicular from the center of a circle on a chord bisects it.

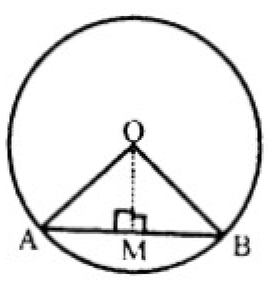
Given:

$\overline{AB}$ is the chord of a circle with center at O so that $\overline{OM} \perp$ chord $\overline{AB}$

To prove:

M is the midpoint of chord $\overline{AB}$

i.e., $m\overline{AM} = m\overline{BM}$



Construction:

Join A and B with center O.

Proof:

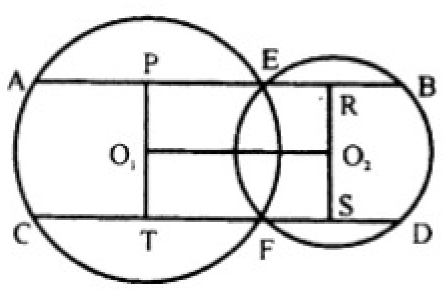
Statements	Reasons
In $\angle$rt $\triangle OAM \leftrightarrow \triangle OBM$	
$m\angle OMA = m\angle OMB = 90^\circ$	Given
hyp. $m\overline{OA}$ = hyp. $m\overline{OB}$	Radii of the same circle
$m\overline{OM} = m\overline{OM}$	Common
$\triangle OAM \cong OBM$	In $\angle$rt $\triangle$s H.S $\cong$ H.S
Hence, $m\overline{AM} = m\overline{BM}$	Corresponding sides of congruent triangels
$\overline{OM}$ bisects the chord AB.	

Corollary 1: $\perp$ bisector of the chord of a circle passes through the center of a circle.

Corollary 2: The diameter of a circle passes through the mid points of two parallel chords of a circle.

Example:

Parallel lines passing through the points of in intercepted by them are equal.



Given:

Two circles have centers O_1 and O_2.

They intersect each other at points E and F.

Line segment $\overline{AB}$ || Line segment $\overline{CD}$

To Prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Draw PT and RS $\perp$ both AB and CD and join the centers O_1 and O_2.

Proof:

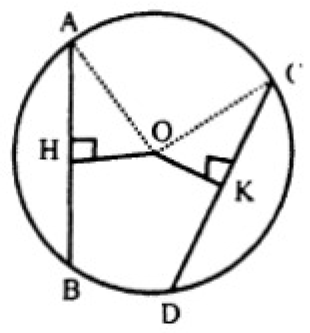
Statements	Reasons
PRST is a rectangle.	Construction
$m\overline{PR} = m\overline{TS}$ (i)	
Now $m\overline{PR} = m\overline{PE} + m\overline{ER}$	
$= \frac{1}{2}m\overline{AE} + \frac{1}{2}m\overline{EB}$	By Theorem 3
$= \frac{1}{2}(m\overline{AE} + m\overline{EB})$	
$m\overline{PR} = \frac{1}{2}(m\overline{AB})$ (ii)	$m\overline{AE} + m\overline{EB} = m\overline{AB}$

Similarly, $m\overline{TS} = \frac{1}{2}m\overline{CD}$ (iii)	
$\Rightarrow \frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{CD}$	Using (i), (ii) and (iii)
i.e., $m\overline{AB} = m\overline{CD}$	

THEOREM 4

9.1 (iv) If two chords of a circle are congruent then they will be equidistant from the center.

Given:



$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with center at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{UK} \perp \overline{CD}$.

To prove:

$m\,\overline{OH} = m\,\overline{OK}$

Construction:

Join O with A and O with C. So that we have ∠rt Δ^s OAH and OCK.

Proof:

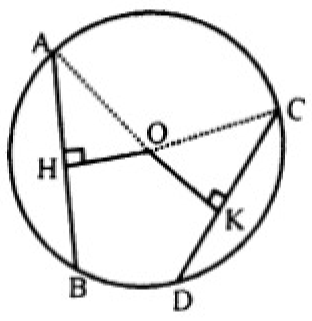
Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ (By Theorem 3)
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	
Similarly, $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ (By Theorem 3)
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence, $m\overline{AH} = m\overline{CK}$ (iv)	Using (i),(ii) and (iii)
Now in ∠rt Δ^s OAH ↔ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle

$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
□OAH ≅ □OCK	H.S postulate
$\Rightarrow m\overline{OH} = m\overline{OK}$	

THEOREM 5

9.1 (v) Two chords of a circle which are equidistant from the center, are congruent.

Given:



$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with center at O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$

To prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O. So that we can form ∠rt △s OAH and OCK.

Proof:

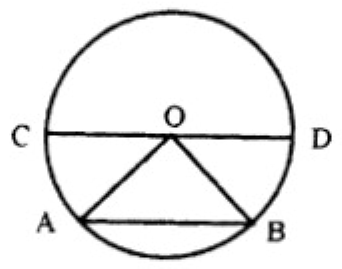
Statements	Reasons
In ∠rt △s OAH ↔ OCK	
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle
$m\overline{OH} = m\overline{OK}$	Given
□OAH ≅ □OCK	H.S postulate
So $m\overline{AH} = m\overline{CK}$ (i)	Corresponding sides of congruent triangls
But $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	$\overline{OH} \perp \overline{AB}$ (Given)
Similarly, $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (iii)	$\overline{OK} \perp \overline{CD}$ (GIven)
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
	Using (ii) and (iii)

$\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	
Or $m\overline{AB} = m\overline{CD}$	

Example:

Prove that the largest chord in a circle is the diameter.

Given:



$\overline{AB}$ is a chord and $\overline{CD}$ is the diameter of a circle with center point O.

To prove:

If $\overline{AB}$ and $\overline{CD}$ are distinct, then $m\overline{CD} > m\overline{AB}$.

Construction:

Join O with A and O with B then form a △OAB.

Proof:

Sum of two sides of a triangle is greater than its third side.

In △OAB ⇒ $m\overline{OA} + m\overline{OB} > m\overline{AB}$ ..(i)

But $\overline{OA}$ and $\overline{OB}$ are the radii of the same circle with center O.

So that $m\overline{OA} + m\overline{OB} = m\overline{CD}$ ..(ii)

⇒ Diameter $\overline{CD}$ > chord $\overline{AB}$ using (i) & (ii).

Hence, diameter CD is greater than any other chord drawn in the circle.

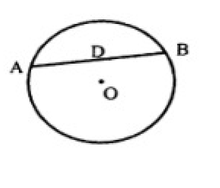
Miscellaneous Exercise 9

Q1. Multiple Choice Questions:

Four possible answers are given for the following questions.

Tick (✓) the correct answer.

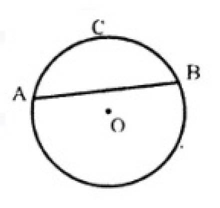
(i) In the circular figure, ADB is called



(a) an arc **(b)** a secant

(c) a chord **(d)** a diameter

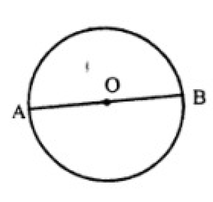
(ii) In the circular figure, $\overset{\frown}{ACB}$ is called



(a) an arc **(b)** a secant

(c) a chord **(d)** a diameter

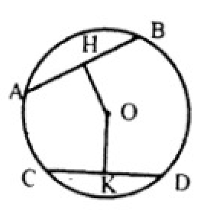
(iii)In the circular figure, AOB is called



(a) an arc **(b)** a secant

(c) a chord **(d)** a diameter

(iii) In a circular figure, two chords AB and CD are equidistant from the center. **They will be:**



(a) parallel **(b)** non congruent

(c) congruent **(d)** perpendicular

(v) Radii of a circle are

(a) all equal **(b)** double of the diameter

(c) all unequal **(d)** half of any chord

(vi) A chord passing through the center of a circle is called:

(a) radius **(b)** diameter

(c) circumference **(d)** secant

(vii) Right bisector of the chord of a circle always passes through the

(a) radius **(b)** circumference

(c) center **(d)** diameter

(viii) The circular region bounded by two radii and the corresponding arc is called

(a) circumference of a circle **(b)** sector of a circle

(c) diameter of a circle **(d)** segment of a circle

(ix) The distance of any point of the circle to its center is called

(a) radius **(b)** diameter

(c) a chord **(d)** an arc

(x) Line segment joining any point of the circle to the center is called

(a) circumference **(b)** diameter

(c) radial segment **(d)** perimeter

(xi) Locus of a point in a plane equidistant from a fixed point is called

(a) radius **(b)** circle

(c) circumference **(d)** diameter

(xii) The symbol for a triangle is denoted by

(a) ∠ **(b)** △

(c) ⊥ **(d)** ⊙

(xiii) A complete circle is divided into

(a) 90 degrees **(b)** 180 degrees

(c) 270 degrees **(d)** 360 degrees

(xiv) Through how many non collinear points, a circle can pass?

(a) one **(b)** two

(c) three **(d)** none

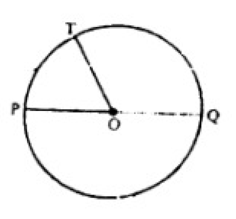
Answers:

(i)	c	**(ii)**	a	**(iii)**	d	**(iv)**	c	**(v)**	a
(vi)	b	**(vii)**	c	**(viii)**	b	**(ix)**	a	**(x)**	c
(xi)	b	**(xii)**	b	**(xiii)**	d	**(xiv)**	c		

Q2. Differentiate between the following terms and illustrate them by diagrams.

(i) A circle and a circumference.

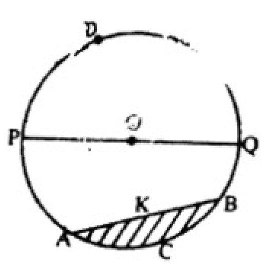
Ans: Circle and circumference



A circle is the locus of a moving point P in a plane which is always equidistant from some fixed-point O. The fixed-point O not lying on the circle is called the center. Whereas the boundary traced by moving point P is called circumference of the circle.

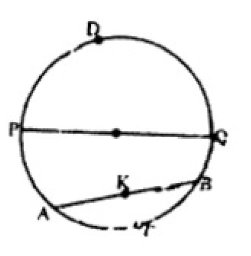
(ii) A chord and the diameter of a circle.

Ans: Chord and the diameter of a circle



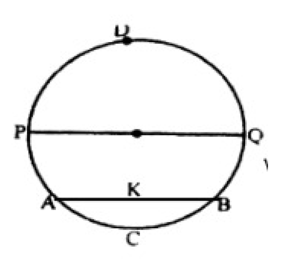
A chord AKB of a circle is a straight line joining any two points on the circumference of a circle. Whereas diameter POQ is the chord passing through the center a circle.

(iii) A chord and an arc of a circle.



Ans: An arc ACB of a circle is any portion of its circumference. A chord AKB of a circle is a straight line joining any two points on the circumference of a circle.

(iv) Minor arc and major arc of a circle.

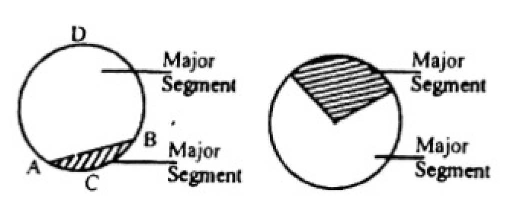


Ans: If the above figure, the smaller arc $\overset{\frown}{ACB}$ is minor arc, $\overset{\frown}{ADB}$ is major arc of a circle.

(v) Interior and exterior of a circle.

Ans: The internal area or part of a circle is called interior of a circle and external part or area of a circle is called exterior of a circle.

(vi) A sector and a segment of a circle.



Ans: A sector of a circle is the center in the plane figure bounded by two radii and the intercepted between them. Any pair of radii divides a circle into two sectors. Segments is the portion of a circle bounded by an arc and a corresponding chord. Any chord divides a circle into two segments.

Summary

✓ $2\pi r$ is the circumference of a circle with radius r.

✓ πr^2 is the area of a circle with radius r.

✓ Three or more points lying on the same line are called collinear points otherwise they are non-collinear points.

✓ The circle passing through the vertices of a triangle is called its circumcircle whereas ⊥ bisectors of sides of the triangle provides the center.

✓ One and only one circle can pass through three non-collinear points.

✓ A straight line, drawn from the center of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

✓ Perpendicular from the center of a circle on a chord bisects it.

✓ If two chords of a circle are congruent, then they will be equidistant from the center.

✓ Two chords of a circle which are equidistant from the center are congruent.